Maneuvers on Lattices

I was very happy to see the recent editorial, "A children's zoo of math" [Science 136, 617 (1962)]. The maneuvers on lattices are most intriguing. David Page performs a most valuable service for all those interested in math and in teaching math in bringing this new approach to our attention. I have, however, one criticism which I am sure Page will not object to. In the editorial it is implied that the idea of maneuvers on lattices is the invention of a mathematician. I would say that this is an incorrect assumption, for this method was really invented by the children, although without Page's wonderful combination of curiosity, tolerance, and encouragement, I am sure this invention by a class of sixth-grade children never would have come into actuality.

I say this because I was there when the invention was born, at one of the colloquia sponsored by the Center for Cognitive Studies at Harvard under the direction of Jerome Bruner and George Miller. This meeting took place during the 1960-61 academic year; the class was a sixth grade from the Shady Hill School in Cambridge. I recall clearly the feelings of wonder, amazement, and sudden puzzlement that overcame me and, I think, others there, during the birth of this method. It started when Page put on the board some of the numbers of an ordinary decimal lattice system. Then he asked for suggestions as to what should be filled in. And while I sat there confidently, and perhaps smugly, expecting the children to work out some regular decimal lattice, possibly with Page's help, they did no such thing, but instead developed the system of lattice and lattice operations that your editorial mentions. I recall sitting and waiting for Page to tell the children they were wrong in some of the answers they were getting, but he did not, and, with the skill of a most experienced obstetrician, he helped the new system into aliveness-despite quite obvious

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puzzlement on his own face as to what the kids were getting at.

Because his expression of feelings was at first so genuinely one of being puzzled I will be greatly surprised if it turns out that Page had already invented the method and was helping these youngsters to learn from him through skillful guidance. If this should be true it would be the second time that I would have felt stupid in response to his matrix mathematics-but I do not believe that this will be so, for, as a psychiatrist, my judgment of emotions and their genuineness should be greater than my skill at, and ability to evaluate. mathematics-a field I would love to have gone into but never felt quite bright enough to tackle.

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The puzzlement on the instructor's face during that class at Harvard was genuine. Gray witnessed the class during which students took me for probably the fastest and most bewildering ride in my teaching experience. I had certainly not planned or even considered the ideas which those children invented. In fact, many of their suggestions were so sophisticated that I have not yet found time to work out adequately what may be entailed by them.

However, it would not be correct to call this class period the birth of the general idea or method contained in "Maneuvers on Lattices" (but as far as the awestruck mood of the teacher was concerned, it might as well have been!). Other classes of children, extending back more than a year, had already shown me that "Maneuvers" was a sufficiently interesting topic to be worth our attention that day. To hint at the true origin of most of these ideas, I quote from an introductory statement in the paper upon which the editorial was based: "The author prefers not to estimate the fraction of content here which is attributable to the collection of children in schools who have worked with it."

I appreciate Gray's kind words and, clearly, I agree fully with the spirit of his criticism. But I must add a word of caution. Although the eternal, inventive zeal of children is a main source of inspiration to me and to this project, the notion of "children leading the teacher" can be warped into a dangerous doctrine: that the teacher needs, mostly, training in "educational methodology" because children can be depended upon to supply any needed content ad lib. Actually, the teacher needs as much background in mathematics as possible -otherwise he will be taken for a variety of unfortunate rides by students. And on the days when students do not catch fire and have no suggestionswhen teaching is a plodding affair-the teacher must be prepared to plod responsibly in the direction of mathematics.

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Water Research by the Geological Survey

I've just read with interest, and appreciation, the 1 June issue of *Science* with its sympathetic account of our efforts to formalize the Geological Survey's long-continued research programs in the field of water [**136**, 767 (1962)].

Because so much of the article accurately reports the history of the plan for an Institute for Water Research, it is perhaps ungenerous of me to regret that I did not have the opportunity to discuss the subject with the author, D. S. Greenberg. Several inaccuracies might have been avoided had there been such an opportunity. One that I think does need correction is the implication that no basic research in hydrology had been carried out in the Survey until recently. Although I'm sure we would all agree that the research was not as extensive as we would have liked, anyone familiar with the reports issued by the Survey from the time of Major Powell's directorship, written by such individuals as G. K. Gilbert and O. E. Meinzer, will recognize that this implication is not warranted.

And I should like to correct the suggestion that the Survey is bewildered and frustrated. These may be temporary or passing reactions on the part