waves are developing can vary to a certain extent without producing any change in the actual pattern formed. However, a threshold would be reached, involving the appearance of an additional peak or peaks and respacing of other peaks in the field. The respacing of bristles in the present experiments cannot be explained easily without considering changes in the number and shapes of the peaks of the prepattern. Since such phenotypes were seen only at the two extremes of the mutant expression, it is reasonable to suggest that the primary effect of the *ocelli-less* gene is to alter the amount and distribution of precursor, and that this alteration in turn may change the prepattern.

K. C. SONDHI
Division of Genetics, New England Institute for Medical Research, Ridgefield, Connecticut

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9 April 1962

Theory of Homologous Sublattices and Intracrystalline Equilibria

Abstract. A method is presented by which the isothermal-isobaric distribution of certain atomic or ionic species among two sublattices of a crystal may be calculated from the heterogeneous exchange equilibria of the species between two coexisting crystalline phases, one of which is the crystal under investigation. The method depends on structural homologies between the crystals. Quantitative data on the distribution of Mg^{2+} and Fe^{2+} in the mineral cummingtonite are found to agree with the theory.

Although a quantitative concept of order-disorder for alloys dates to the work of Gorsky (*1*) and of Bragg and Williams (*2*), few methods based upon easily measurable parameters have been proposed to treat the distribution of atomic or ionic species within complex phases. The purpose of this paper is to present a simple, essentially thermodynamic approach to certain types of distribution equilibria in crystals.

In order to illustrate the method it is convenient to discuss a particular type of crystal structure which has been investigated. It should, however, be apparent from this illustration how the method may be extended to certain other types of structures.

Consider two stably coexisting crystals with the formulas

$$(A,B)_n^{\alpha}\,(A,B)_m^{\beta}\,\mathrm{F}$$

and

$$C_n^{\alpha'}\,(A,B)_m^{\beta'}\,G$$

In these formulas A, B, and C represent three different species which are distributed among the sublattices α, β, α', and β' of the two crystals. F and G represent essentially nonparticipating framework structures, and n and m are small whole numbers. The species A and B are somewhat similar in properties such as electronegativity and atomic or ionic radii, whereas C must differ from A and B in one or more of these fundamental properties. This difference causes C to be nearly confined to the α' sublattice. Although this model may appear somewhat narrowly restricted in properties it does in fact describe several important groups of compounds which exhibit broad miscibility gaps.

In the heterogeneous system consisting of at least these two crystals the exchange equilibrium between the coexisting crystals may be represented by the reaction

$$\begin{aligned}(1/m+n)\,A_n^{\alpha}\,A_m^{\beta}\,F &+ \\ 1/m\,C_n^{\alpha'}\,B_m^{\beta'}\,G &\rightleftharpoons \\ (1/m+n)\,B_n^{\alpha}\,B_m^{\beta}\,F &+ \\ 1/m\,C_n^{\alpha'}\,A_m^{\beta'}\,G & \qquad (1)\end{aligned}$$

The compositions of the coexisting phases may be determined by chemical analyses. These analyses are expressed as activities which involve the mole fractions of the total A-B content of each phase. For example, the activities of the components of the F phases are

$$a_A^F = X_A^F\,f_A^F$$

and

$$a_B^F = (1 - X_A^F)\,f_B^F \qquad (2)$$

where $X_A^F = A/(A+B)$ in $\alpha + \beta$ and f_A^F and f_B^F are the corresponding activity coefficients. Similarly for the G phase:

$$a_A^G = X_A^G\,f_A^G$$

and

$$a_B^G = (1 - X_B^G)\,f_B^G \qquad (3)$$

But $X_A^G = A/(A+B)$ in β' only since α' is filled with C.

However, our interest here is in the homogeneous intracrystalline equilibrium:

$$A^{\alpha} + B^{\beta} \rightleftharpoons B^{\alpha} + A^{\beta} \qquad (4)$$

which governs the distribution of A and B among the α and β sublattices of the F crystal. Consequently it is necessary to find alternative expressions for the activities which are functions of the mole fractions of the sublattices. For A in the F-crystal we have, for example:

$$a_A^F = f\,(X_A^{\alpha}, X_A^{\beta}) \qquad (5)$$

which is equivalent to the first relation of Eq. 2. One type of expression is given directly by the stoichiometric relations of the total A-B lattice $(\alpha + \beta)$ to the sublattices. Thus

$$X_A^F = (1-\mathrm{p})\;X_A^{\alpha} + p\;X_A^{\beta}$$

and

$$X_B^F = (1-p)\,(1-X_A^{\alpha}) + p(1-X_A^{\beta}) \qquad (6)$$

where $\mathrm{p} = m/(m+n)$. These expressions may be substituted into the activi-

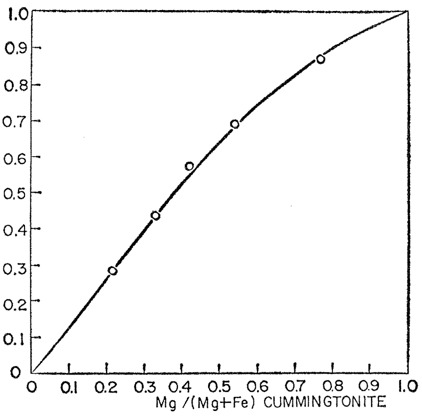

Fig. 1. Distribution of Mg^{2+} and Fe^{2+} among coexisting cummingtonites and actinolites expressed as atomic fractions.

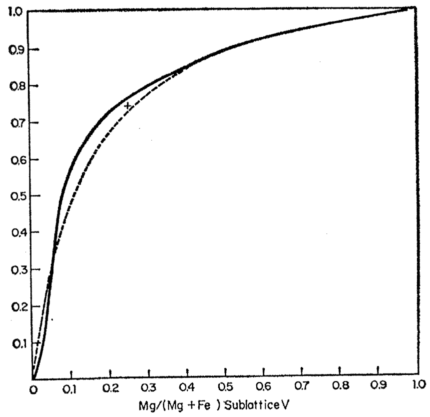

Fig. 2. Theoretical and observed distributions of Mg^{2+} and Fe^{2+} among the V and W sublattices of cummingtonite expressed as atomic fractions. The full curve is the more exact theoretical relation, whereas the dashed curve is the simplified approximation (see text). The cross is a value observed by Ghose with x-ray diffraction.

ties in Eq. 2 to yield exact functions of the sublattices.

An alternative expression, Eq. 5, for the activities in the *F*-crystal is suggested by the law of mass action in which the concentration of each species is raised to the power of the stoichiometric coefficient of the sublattice. Thus we are inclined to try

$$a_A^F = (X_A^{\alpha})^{1-p}\ (X_A^{\beta})^p \qquad (7)$$

and

$$a_B^F = (1 - X_A^{\alpha})^{1-p}\ (1 - X_A^{\beta})^p \qquad (8)$$

An expression for the equilibrium constant for reaction No. 1 above may be deduced from the data of the heterogeneous system using the expressions Eq. 2 and Eq. 3, if the activity coefficients in these expressions can also be deduced. If, in addition $f_A^G = f_B^G = 1$, we have from Eqs. 1, 3, 7, and 8:

$$K_a = \frac{(X_B^{\alpha})^{1-p}\ (X_B^{\beta})^p\ (X_A^{\beta'})}{(X_A^{\alpha})^{1-p}\ (X_A^{\beta})^p\ (X_B^{\beta'})} \qquad (9)$$

If now it can be assumed that the sublattices β and β' are energetically identical with respect to *A* and *B*, one may set

$$X_A^{\beta'} = X_A^{\beta},\ X_B^{\beta'} = X_B^{\beta} \qquad (10)$$

since

$$X_A^G = X_A^{\beta'},\ X_B^G = X_B^{\beta'}$$

Whether or not Eq. 10 is a valid assumption will depend to a large extent on the degree of structural homology possessed by β and β'. It would seem that this requirement would be quite strict for ionic crystals because of the relatively long-range forces involved.

In the absence of the requisite strict homology, relations such as the following might be found useful:

$$X_A^{\beta'}\ L_A^{\beta'} = X_A^{\beta}\ L_A^{\beta} \qquad (11)$$

L_A^{β} and $L_A^{\beta'}$ in these expressions being functions of the sublattices analogous to activity coefficients.

If, however, the expressions in Eq. 10 hold, we have by substitution into Eq. 9

$$K_a^{1/(1-p)} = \frac{(1 - X_A^{\alpha})\ X_A^{\beta}}{(1 - X_A^{\beta})\ X_A^{\alpha}} \qquad (12)$$

which is the simplest possible distribution relation for *A* and *B* among the sublattices α and β of the *F*-crystal. The expression No. 1 is thus a simplified alternative expression to the more complex exact expression for the distribution obtainable from Eqs. 2, 3, and 6. The two types of expressions may, of course, be compared to each other and with experimental data such as that derived from x-ray diffraction studies.

A system which has been investigated (*3, 4*) is comprised of the two naturally coexisting monoclinic amphiboles cummingtonite and actinolite in which Mg^{2+}, Fe^{2+}, and Ca^{2+} correspond to *A*, *B*, and *C*. The nonparticipating framework $Si_8O_{22}(OH)_2$ is virtually identical in both minerals as shown by x-ray diffraction. The formulas are:

$$(Mg,\ Fe)_2^{V}\ (Mg,\ Fe)_5^{W}\ Si_8O_{22}(OH)_2$$
(cummingtonite)
$$Ca_2^{V}\ (Mg,\ Fe)_5^{W}\ Si_8\ O_{22}\ (OH)_2$$ (actinolite)

in which the sublattices are distinguished by the superscripts V and W. Because of differences in ionic radii Ca^{2+} is almost entirely restricted to the V sublattice of actinolite, whereas Mg^{2+} and Fe^{2+} are distributed among the V and W sublattices of cummingtonite and the W sublattice of actinolite. The W sublattice of both minerals consists of Mg^{2+} and Fe^{2+} ions surrounded by six oxygen ions in octahedral coordination. The mean metal-oxygen interionic distance varies only 0.03 A from cummingtonite (*5*) to actinolite (*6*). The V sublattice, on the other hand, consists of a highly distorted polyhedron in which the interionic distances vary as much as 0.30 A and the mean interionic distance as much as 0.15 A between the two minerals.

Because of the close structural homologies of the W sublattices of the two minerals, a consequence largely of the identity of the Si-O framework, a relation such as Eq. 10 was assumed to hold. The observed distribution of Mg^{2+} and Fe^{2+} among the coexisting actinolites and cummingtonites is presented in Fig. 1. The curve which is fitted to these points is also substantiated by additional data (*3*). The derived distribution constant is $K = 1.80$ (*3*). The corresponding calculated distribution among the V and W sublattices is presented in Fig. 2. In this figure the full curve is the more exact one and was derived from relations of the type of Eqs. 2, 3, and 6. The dashed curve shows the simple approximation analogous to Eq. 1, the equilibrium constant being $(1.80)^{7/2}$, as may easily be shown. Comparison of Figs. 1 and 2 shows how Mg^{2+} is concentrated in the actinolite-like W sublattice of cummingtonite. Unfortunately, only one independent determination of this distribution exists for cummingtonite. This distribution, obtained from x-ray diffraction by Ghose (*5*), is shown in Fig. 2 as a cross. The close correspondence with the theoretical curves may be somewhat fortuitous, but the range in which the point falls seems to favor the model.

Quantitative applications of the method require rather close homologies between the structures involved. However, it seems probable that the theory might provide semiquantitative or qualitative predictions about distributions in a variety of crystals. Such predictions may be of considerable utility in the interpretation of x-ray diffraction and other data. I am reporting one such semiquantitative application, in this case to the pyroxene minerals (*4*).

R. F. MUELLER*
Department of Earth Sciences, University of California, La Jolla

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* Present address: Department of Geophysical Sciences, University of Chicago, Chicago 37, Illinois.

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Narcotic Antagonists as Analgesics

Abstract. 2-Dimethylallyl-5,9-dimethyl-2′-hydroxybenzomorphan (Win 20,228) was found to be a weak antagonist of morphine and meperidine, whereas 2-allyl-5-ethyl-2′-hydroxy-9-methyl-6,7-benzomorphan (Win 19,362) and 2-allyl-2′-hydroxy-5,9-dimethyl-6,7-benzomorphan (Win 19,631) were about three times as potent as nalorphine. Preliminary clinical trials indicate that Win 19,362 is about twice as potent as morphine as an analgesic but, like nalorphine, is capable of producing severe psychic side effects. Win 20,228 is estimated to be about half as active as morphine as an analgesic, but no bizarre psychic effects were reported at any dose tested. Studies of addiction liability with Win 20,228 in monkeys suggest that this compound will not support morphine addiction.

Lasagna and Beecher (*1*) and Keats and Telford (*2*) showed that nalorphine (N-allylnormorphine), which is a potent antagonist of morphine clinically and in experimental animals, is, in itself, a potent analgesic in man. The subsequent report by Isbell (*3*) that this drug was unable to support morphine addiction or induce primary addiction in man broke the bond which previously linked strong analgesia with addiction liability. Nalorphine produced serious side effects, particularly of a psychic nature, and thus proved to be unacceptable as an analgesic agent. Further studies by Keats and his group (*4*) on a series of antagonists de-