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Structure of Nuclei and Nucleons

Extension of electron-scattering studies to higher energies gives a new model of the neutron and proton.

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I am very conscious of the high honor that has been conferred on me, and I wish to thank the Swedish Academy of Science sincerely for this recognition. It is a privilege and a pleasure to review the work which has brought me here and which concerns a very old and interesting problem.

Over a period of time of at least 2000 years man has puzzled over and sought an understanding of the composition of matter. It is no wonder that his interest has been aroused in this deep question, because all objects he experiences, including even his own body, are in a most basic sense special configurations of matter. The history of physics shows that whenever experimental techniques advance to the extent that matter, as then known, can be analyzed by reliable and proved methods into its "elemental" parts, newer and more powerful studies subsequently show that the "elementary particles" have a structure themselves. Indeed this structure may be quite complex, so that the elegant idea of elementarity must be abandoned. This observation provides the theme of my lecture.

In recent times the structure of matter has been shown to arise from various combinations of the "atoms" of the periodic system. The picture of the now-familiar atom was first sketched by Rutherford, Bohr, Pauli, and others and later developed in great detail by many of their colleagues. The efforts of these scientists have led to an understanding of the cloud of electrons which surrounds the dense center of the atoms, the so-called nucleus. In the nucleus practically all the mass of the atom resides in an extremely concentrated form. The nucleus itself was an invention of the aforementioned physicists, and in the year 1919 the first vague ideas concerning the sizes of nuclei were worked out. By studying the deviations from Coulomb scattering of alpha particles, Rutherford showed that a nuclear radius was of the order of 10^5 times smaller than an atomic radius. Subsequently other investigators demonstrated, by means of studies of alpha-particle radioactivity, neutron-capture cross sections, and comparisons of the energy of decay of mirror nuclei, that consistent values for nuclear size parameters could be obtained. All useful methods showed that if a nucleus could be represented by a model of a uniformly charged sphere, the radius R of the sphere would be given by the relation

$$R \cong 1.40 \times 10^{-13} A^{1/3}\,\text{cm} \qquad (1)$$

where A is the mass number of the nucleus.

This is the point from which the present studies began. Although much of what I wish to say concerns nucleon structure (a nucleon is a proton or a neutron), the method of investigation that my co-workers and I have employed had its origins in the study of larger nuclei. Consequently, a historical approach beginning with the larger nuclei not only seems natural but also may be didactically sound. I shall therefore review briefly the method used in studying nuclear sizes and shall at the same time give some of the results, which may not be without interest themselves.

Method of High-Energy Electron Scattering

We have used the method of high-energy electron scattering. In essence the method is similar to the Rutherford scattering technique, but in the case of electrons it is presently believed that only a "simple" and well-understood interaction—the electromagnetic or Coulomb interaction—is involved between the incident electron and the nucleus investigated. Under these conditions quantum electro-dynamics and Dirac theory teach us how to calculate a differential elastic-scattering cross section. It can be shown that the differential cross section corresponding to a beam of electrons scattering against a point nucleus of small charge Ze, lacking spin and magnetic moment, is calculable by the Born approximation and takes the following form:

$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2(\theta/2)}{\sin^4(\theta/2)} \frac{1}{1 + \frac{2E}{Mc^2}\sin^2(\theta/2)} = \sigma_{NS} \qquad (2)$$

in the laboratory system of coordinates (*1*). This is the "Mott" scattering cross section, where E is the incident energy, θ is the scattering angle, and M is the mass of the struck nucleus. Other symbols in Eq. 2 have their usual meanings. If a nucleus has a finite size, and is thus not merely a point, the scattering cross section is decreased below the value of the scattering from a point. The decrease can be described in terms of a factor, represented by F, which is

The author is professor of physics at Stanford University, Stanford, California. This is the lecture which he delivered in Stockholm, Sweden, on 10 December 1961 when he received the Nobel prize in physics, a prize which he shared with Rudolf Mössbauer. Dr. Mössbauer's lecture will appear in a subsequent issue.

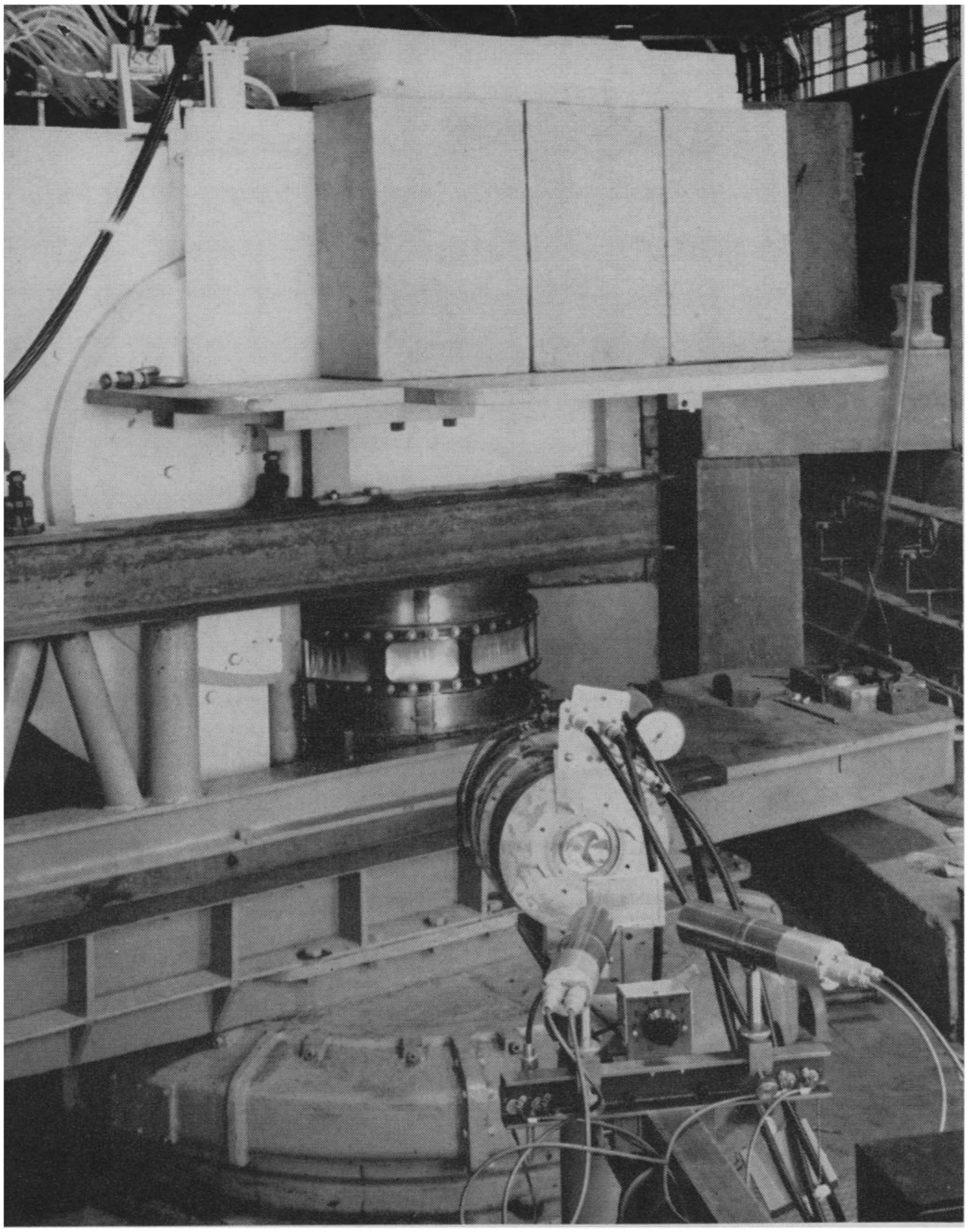


Fig. 1. The first electron-scattering apparatus, built at Stanford University. The semicircular 190-Mev spectrometer is shown at the left on its gun-mount support. The upper platform carries lead and paraffin shielding that encloses the Cerenkov counter. The brass scattering chamber is shown below, with the thin window encircling it. Early forms of electron monitors appear in the foreground. The spectrometer itself is about 4 feet high.

called the "form factor" or "structure factor." Thus, in the Born approximation,

$$\frac{d\sigma}{d\Omega} = \sigma_{NS}\, F^2(q) \qquad (3)$$

and this is the elastic-scattering cross section for a finite nucleus (*2*). Here q is the momentum-energy transfer, defined by the relation

$$q = \frac{(2E/\hbar c)\sin(\theta/2)}{[1 + (2E/Mc^2)\sin^2(\theta/2)]^{1/2}} \qquad (4)$$

The parameter q is relativistically invariant and is a very important quantity in electron-scattering studies. The form factor F takes account of the interference between scattered wavelets arising from different parts of the same, finite nucleus and therefore is responsible for diffraction effects observed in the angular distribution. The quantity F is actually given by

$$F = \frac{4\pi}{q}\int_0^\infty \rho(r)\,(\sin qr)\, r dr \qquad (5)$$

in the event that the nucleus exhibits spherical symmetry. The quantity $\rho(r)$ is the electric charge density function, in which r represents the distance from the center of the nucleus to the volume element where ρ is measured. A mathematical inversion of Eq. 5 allows one to deduce the form of $\rho(r)$ if $F(q)$ is known over a large range of values of q.

Of course, since we used the Born approximation and therefore specified small values of the atomic number, the foregoing description of the basic formulas of the electron-scattering process is only an approximate one. More exact methods of finding the scattering cross section have been developed by many authors (*3*). These calculations of more precise types employ the "phase-shift" methods and are applicable to heavy nuclei as well as light ones. The qualitative physical ideas involved in the determination of nuclear structure can be adequately described by the Born approximation method (Eq. 3). Nevertheless, quantitative results definitely require the more elaborate phase-shift methods, and simple—and in this case, closed—formulas cannot be given to describe the scattering cross section.

Early electron-scattering experiments were carried out at the University of Illinois in 1951 (*4*) at an incident electron energy of about 15.7 Mev. Such experiments showed that nuclear radii obeyed an approximate relationship of the type given in Eq. 1. However, few details of nuclear shape or size could be discerned because the energy of the electrons was relatively low and the corresponding de Broglie wavelength of the electrons was larger than the typical size of the nucleus.

In 1953 higher-energy electrons became available at Stanford University and at the University of Michigan, and experiments on various nuclei were carried out (*5*). Phase-shift interpretations of the Stanford experiments (*6*) showed that the rule expressed in Eq. 1 was approximately true, but that in reality the nuclear charge density distribution could not be described in terms of a single size parameter R. If one attempted so to describe it, at the expense of an inferior fit between experiment and theory, the resulting R would have to be made 20 percent smaller than the value of the radius in Eq. 1. Mu-mesonic atom studies (*7*) showed, a bit earlier, that a similar conclusion was required for a one-parameter description of the size of the nucleus. Two parameters could not be determined from the mu-mesonic atom investigations.

Figure 1 is a photograph of the first high-energy electron-scattering equipment. This apparatus gave the results just discussed and was employed up to an energy of about 190 Mev. An obsolete naval gun mount was used as the rotating platform for the heavy equipment, which weighed about 5 tons. The type of geometry employed in a modern electron-scattering experimental area is shown in Fig. 2. A photograph of the corresponding magnetic spectrometers and associated equipment is shown in Fig. 3. A large form of gun mount is used to carry the spectrometers, whose total weight is approximately 250 tons. Each of the two magnetic spectrometers in this apparatus is similar to the well-known Siegbahn double-focusing instrument. The two spectrometers may be used in coincidence experiments as well as "in parallel." The massive equipment of Fig. 3 can bend and focus 1.0-Bev electrons and is required in order to resolve the elastic-scattering process from the many types of inelastic-scattering processes occurring in electron-nucleus collisions. An example of the resolution obtained in early experiments is shown in Fig. 4 for the case of a carbon target (*8*). When an angular distribution in carbon is measured one may observe (see, for example, Fig. 5) the position of a diffraction minimum. The value of the angle at this minimum gives immediately an indication of the nuclear size if one employs results similar to those of Eqs. 2 through 5, modified appropriately in terms of the phase-shift method. The solid line in Fig. 5 shows the result of a theoretical calculation of the scattering cross section. From the theoretical calculation one may deduce the charge density distribution, which may be seen in Fig. 6. It is clear that a study of the inelastic-scattering peaks corresponding to the excited states of C^{12} (or other nuclei) can be studied by the electron-scattering method. In fact, Fig. 5 shows also the angular dependence of the scattering of the 4.4-Mev level in C^{12}. The subject of inelastic level scattering is not relevant to our present topic, and thus I shall not pursue this matter any further here.

One last example is that of the nucleus of the gold atom. The elastic electron scattering was studied at the four different energies shown in Fig. 7. The solid lines again show the results of theoretical calculations by Ravenhall and Yennie from which the charge density distribution ρ can be obtained (see Fig. 6).

Charge Density Distributions in Nuclei

The electron-scattering method was employed in the manner I have described and resulted in the determination of two-parameter descriptions of nuclear charge density distributions. Studies of the charge density distributions in various nuclei culminated in the evolution of a simple scheme of construction of most spherical nuclei (*9*). Such nuclei could be represented by a charge density function of the type shown in Fig. 8. The exact shape of this density function is not of overriding importance; rather, the distance c from the center of the nucleus to the 50-percent point and the interval t between the 90-percent and 10-percent ordinates are the two important parameters that determine the behavior of the scattering cross sections. A trapezoidal distribution with the same values of the two parameters would also suffice to describe the experimental results in the medium and heavier nuclei when the fitting procedure is limited by the accuracy obtained in the experiments. Higher accuracy will probably distinguish between these possibilities but such studies are only beginning now.

The results of many of these experiments covered a large range of nuclei and demonstrated (*9*) that two simple rules can be used to summarize the scheme of construction of spherical nuclei:

$$\begin{aligned} c &= (1.07 \pm 0.02) \times 10^{-13} A^{1/3}\ \text{cm} \\ t &= (2.4 \pm 0.3) \times 10^{-13}\ \text{cm} = \text{constant} \end{aligned} \tag{6}$$

The first equation gives the principal parameter governing the size of a nucleus and describes the behavior with

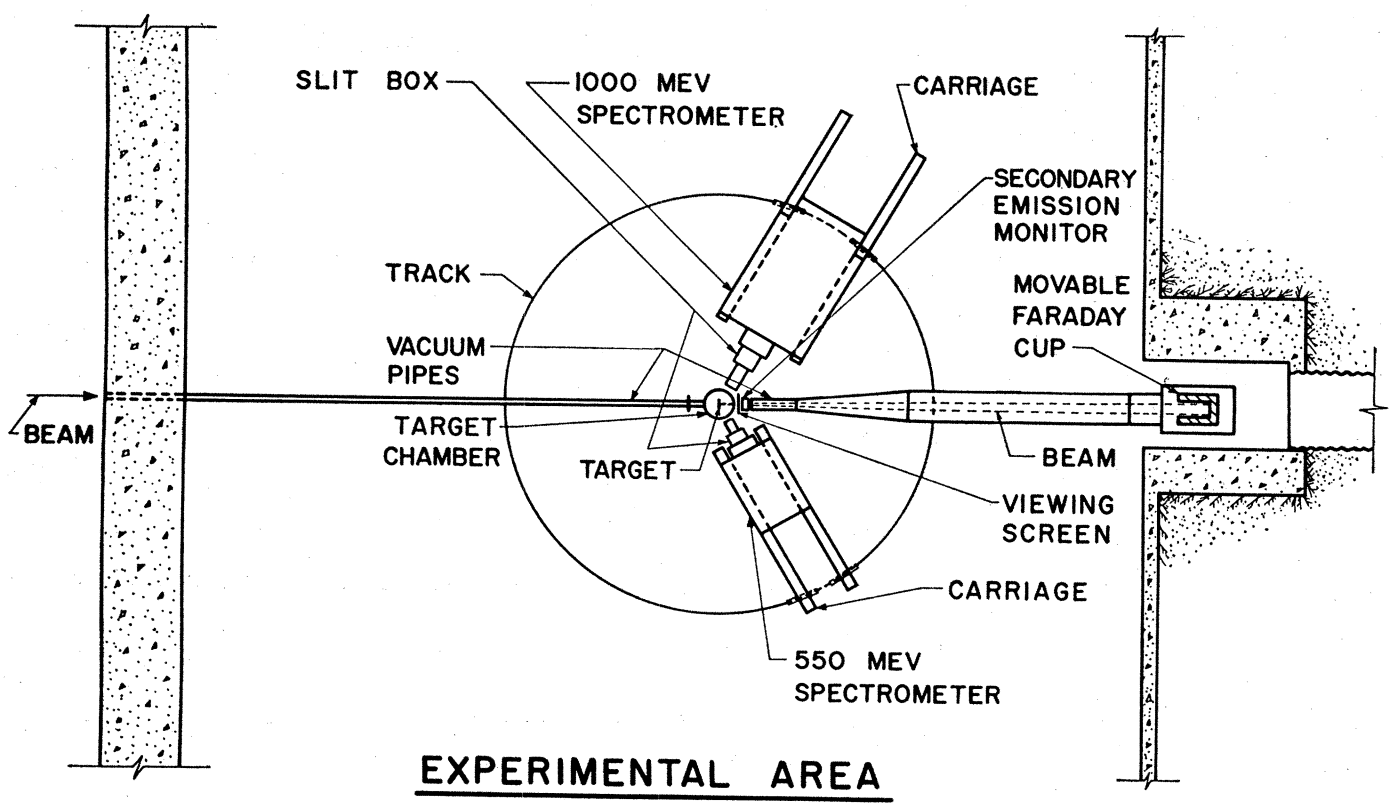


Fig. 2. Schematic diagram of a modern electron-scattering experimental area. The track on which the spectrometers roll has a radius of approximately 13.5 feet.

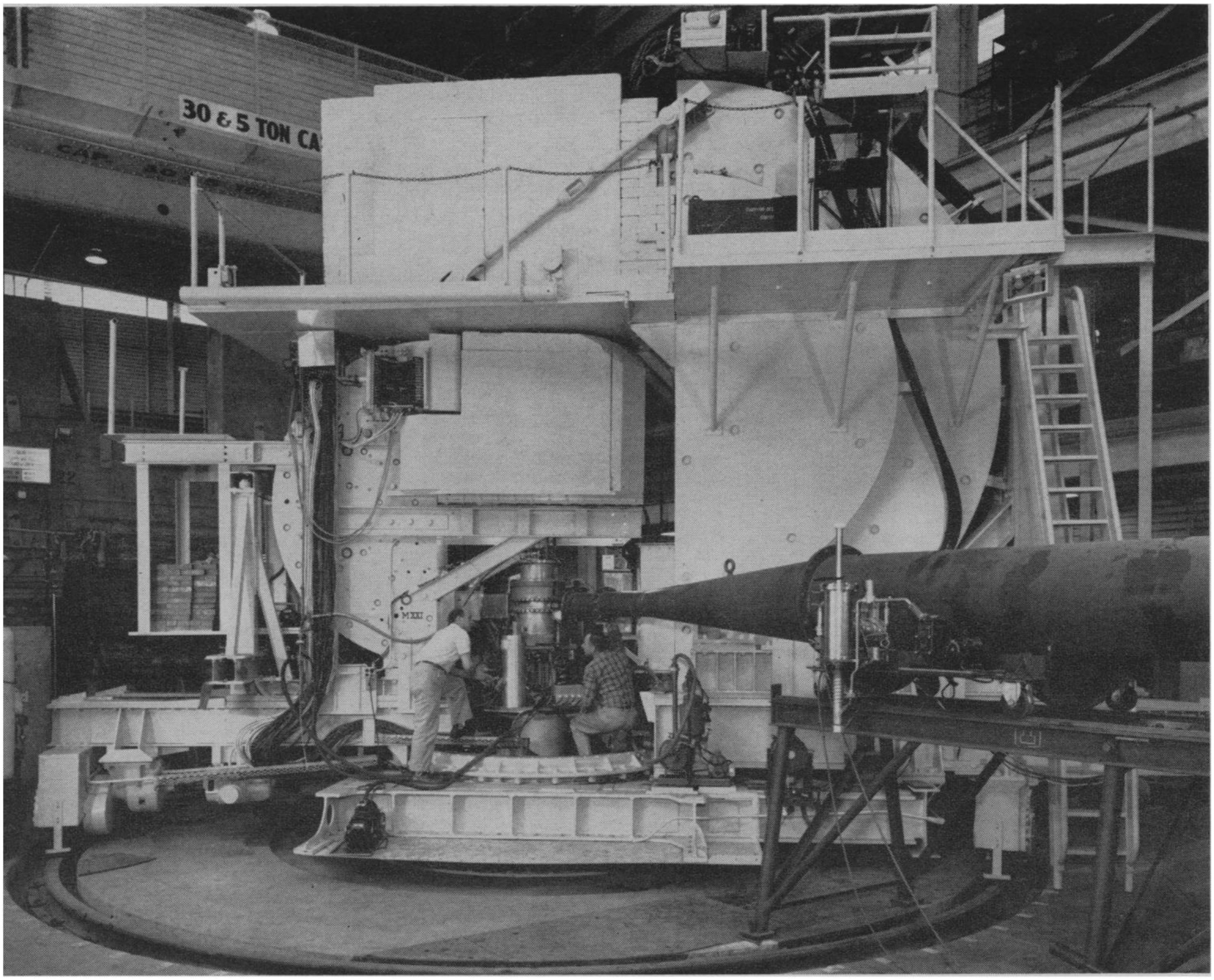


Fig. 3. A recent photograph of the double spectrometer system. The shield of the smaller spectrometer can be removed easily with the aid of an auxiliary stand. The long tube in the foreground is the vacuum pipe leading to the Faraday cup. The auxiliary stand and the Faraday cup are not shown.

increasing A of a kind of "mean" nuclear radius. The second equation states that the thickness of the nuclear skin is constant. The second rule implies that there is some property of nuclear matter that causes the outer nuclear regions to develop an essentially constant surface thickness. The two rules together are responsible for the approximate constancy of the central charge density of nuclei. The latter property is illustrated in Fig. 6, where a summary of the charge distributions found by the electron-scattering method is presented for various nuclei. Except for the extremely light nuclei of hydrogen and helium, the constancy of the central nuclear density is clearly represented in the figure.

Form Factors of the Proton

The results obtained with heavier nuclei indicated that the electron-scattering method could also be applied to the very light nuclei and even to the proton itself. Accordingly, in early 1954 experiments were initiated on hydrogen and helium. The first targets employed high-pressure, thin-wall gas chambers and were designed by the late Eva Wiener. In the latter part of 1954 it was first realized that the experiments on hydrogen demonstrated that the proton was an object of finite size and not merely a point object. In fact, the size was found to be surprisingly large (*10*) and could be described in terms of a root-mean-square radius of value $(0.74 \pm 0.24) \times 10^{-13}$ centimeter. It is an interesting fact that more recent determinations of the root-mean-square proton charge radius appear to converge on a value of $(0.79 \pm 0.08) \times 10^{-13}$ centimeter. Figure 9 shows the first evidence of finite size in the proton; it has been drawn from Hofstadter and McAllister (*10*). The first experiments leading to these conclusions were carried out at relatively low energies ($\sim$ 190 Mev).

Now the proton is known to have a spin and a magnetic moment. The magnetic moment will affect the scattering behavior appreciably at values of $\hbar q$ (Eq. 4) in the range equal to or larger than about 0.2 Mc, where M is the mass of a nucleon. The magnetic type of scattering causes a leveling off in the decrease of the elastic cross section as a function of the scattering angle at high energies of the incident electrons. As may be seen in Fig. 9, the experimental data fell below the theoretical curve for a proton possessing a point charge and a point magnetic moment. This behavior can be understood in terms of the theoretical scattering law developed by M. Rosenbluth (*11*) in 1950. This law described the composite effect of charge and magnetic moment scattering and is given by the following equation:

$$\frac{d\sigma}{d\Omega} = \sigma_{NS}\left\{ F_1^2 + \frac{\hbar^2 q^2}{4M^2c^2} \times \left[2(F_1 + KF_2)^2 \tan^2(\theta/2) + K^2F_2^2\right]\right\} \quad (7)$$

where σ_{NS} is taken from Eq. 2 with $Z = 1$. In the Rosenbluth equation the quantity $F_1(q)$ is the Dirac form factor, representing the proton's charge and its associated Dirac magnetic moment. The quantity $F_2(q)$ is the Pauli form factor and represents the anomalous magnetic moment of the proton. K in Eq. 7 indicates the static value (1.79) of the anomalous magnetic moment in units of the nuclear magneton.

Although one may speak qualitatively of size and shape factors of the proton in the low-energy limit, it is more consistent and more desirable, from a quantitative point of view, to discuss only the two phenomenological form factors $F_1(q)$ and $F_2(q)$. Actually, all the electromagnetic structure of the proton is, in principle, described by the behavior of these quantities as functions of q. Note that for the proton, $F_1(0) = F_2(0) = 1.00$. Meson theory should be able to make definite assertions about F_1 and F_2, starting from the foregoing values.

In our subsequent discussion we shall concentrate on determining the two phenomenological quantities (F_1, F_2) from the experimental data, so that the form factors can be compared with theory. Experimental determination of the form factors can be accomplished, for example, by using the method of intersecting ellipses (*12*) or by other, equivalent methods based on the relativistic idea that each F is a function *only* of q, and not of E or θ separately.

The early work on the proton was confirmed by subsequent studies at higher energies (~ 600 Mev) (see *13*, *14*), but these energies were still low enough so that the assumption $F_1 \cong F_2$ could be employed. It was noted in the latter experiments that F_1 was slightly greater than F_2 at values of $q^2 = 4f^{-2}$, where f = fermi = 10^{-13} centimeter. The value of one fermi corresponds to $(197\ \text{Mev})^{-1}$.

Recently the extension of the experimental measurements to higher energies (~ 1.0 Bev) showed that indeed $F_1 > F_2$ (see *14*). The appropriate detailed behavior is shown in Fig. 10 (see *15*) and represents the most recent Stanford

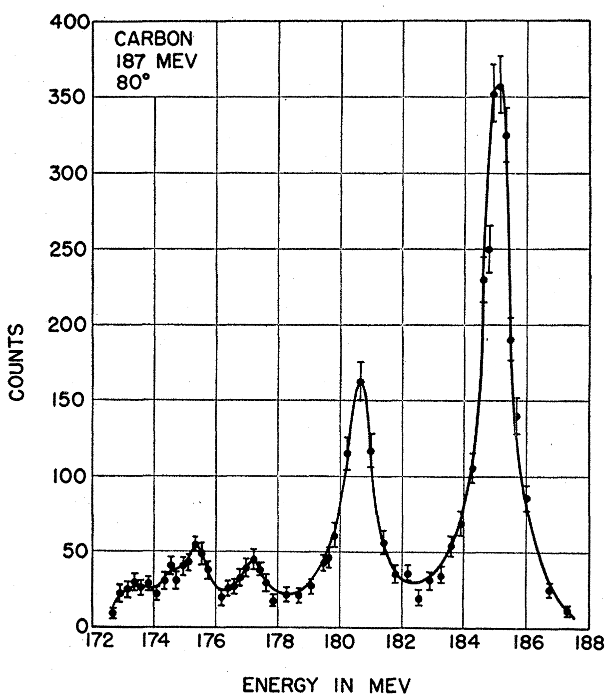


Fig. 4 (above). The elastic-scattering peak from carbon at an abscissa near 185 Mev and the inelastic-scattering peaks from the excited states of C^{12}. The peak near 180.7 Mev is associated with the 4.43-Mev level. [From J. H. Fregeau and R. Hofstadter (*8*)]

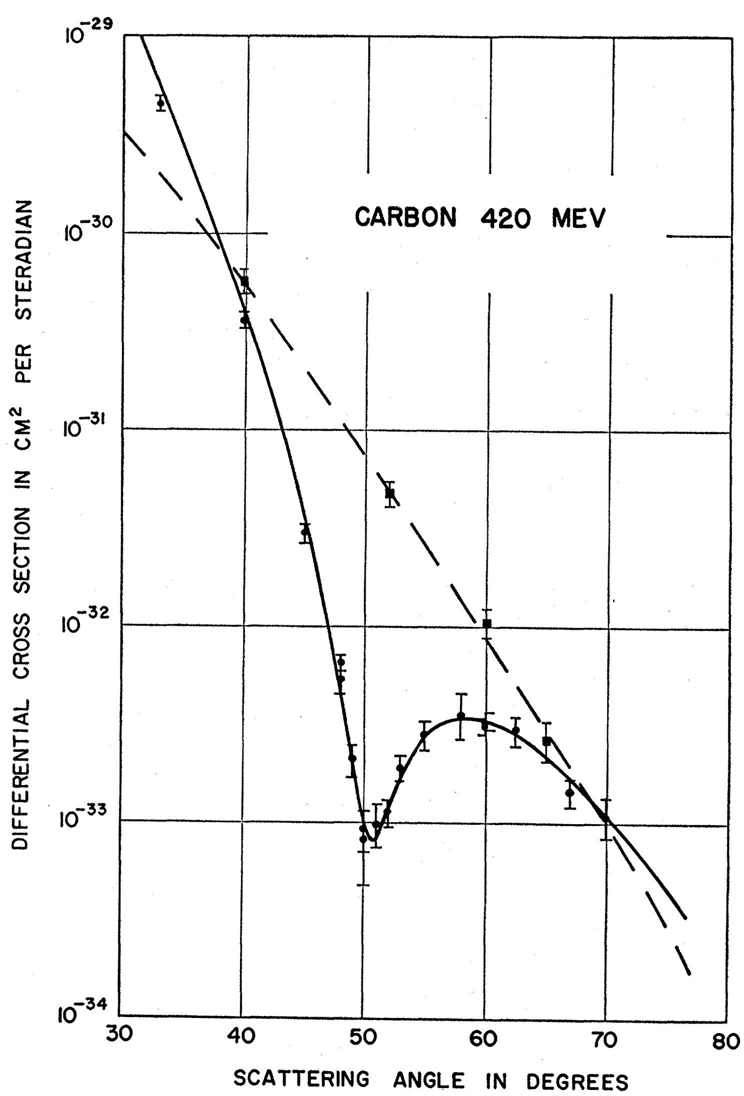


Fig. 5 (right). The elastic and inelastic curves corresponding to the scattering of 420-Mev electrons by C^{12}. The solid circles, representing experimental points, show the elastic-scattering behavior, while the solid squares show the inelastic-scattering curve for the 4.43-Mev level in carbon. The solid line through the elastic-scattering data shows the type of fit that can be calculated by phase-shift theory for the model of carbon shown in Fig. 6.

experimental data on this subject. I will describe the possible theoretical significance of these results after brief discussions of, first, some tests of the Rosenbluth equation and, second, the experimental determinations of the form factors of the neutron.

Various tests of the validity of the Rosenbluth equation were made in these experiments by examining whether F_1 and F_2 are really functions of q alone. In all cases studied for which q^2 was less than 25f^{-2}, complete consistency in F_1, F_2 values at different energies and angles was observed; thus, the Rosenbluth equation was checked and found to be valid below $q^2 = 25\text{f}^{-2}$ (*15*). At the highest values reached in these experiments—namely $q^2 \cong 31\text{f}^{-2}$—the Stanford cross sections could not be combined with the cross sections at the same value of q in recently reported Cornell experiments (*16*) to give *real* values of F_1 and F_2. If this observation can be confirmed, the possibility exists that quantum electrodynamics may fail at high momentum transfers, or that two-photon exchange processes, heretofore neglected, are needed to correct the Rosenbluth equation, or that some other fundamental aspect of the scattering theory needs improvement. This is an interesting question for the future to decide.

Form Factors of the Neutron

Let us now turn to the question of the neutron. According to relativistic quantum electrodynamics the neutron possesses Dirac and Pauli form factors. Proton and neutron form factors may be referred to, respectively, as F_{1p}, F_{2p}, F_{1n}, F_{2n}. Static values of the neutron form factors are known to be $F_{1n}(0) = 0$, $F_{2n}(0) = 1.00$. F_{1n} is also known, from early neutron scattering experiments, to vary as q^4 at small values of q in an expansion of F_{1n} as a function of q^2. This relationship is commonly expressed by saying that within experimental error, the root-mean-square radius of the neutron is zero. Thus, the neutron is not only a neutral body from the point of view of electric charge but has a power expansion of F_{1n} that starts off as a function of q^2 with zero slope! Consequently, it is most difficult to determine F_{1n} (and also F_{2n}) of the neutron. The difficulty is compounded by the experimental fact that neutron targets are obtained only by using the deuteron as a neutron carrier, for free neutrons in large numbers are unobtainable in confined spaces. A neutron is in vigorous motion in the deuteron, and this additional complication must be taken into account somehow. It is necessary at this point to introduce a relativistic theory of the deuteron to allow properly for the effects of the motion of the bound neutron. Of course, at the present stage of development of relativity theory, the deuteron problem can be solved only in an approximate way. Hence we can see that the experimental elucidation and determination of the form factors of the neutron present formidable difficulties.

Many of these difficulties were over-

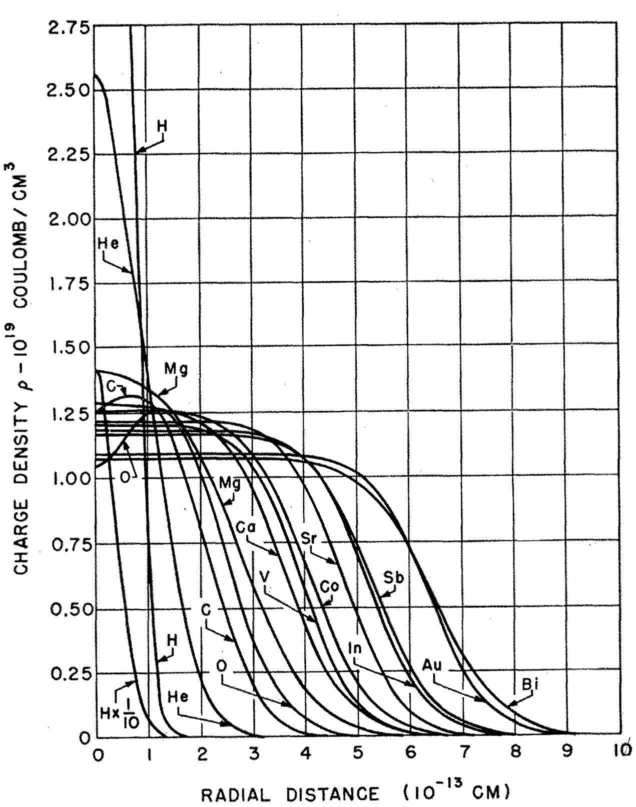


Fig. 6. A summary of the approximate charge density distributions found for various nuclei studied by electron-scattering methods. The central densities are the least well determined features of the curves. Note, however, the large disparity between the *average* central densities of the proton and of all other nuclei. The alpha particle (He^4) is also a unique case and exhibits a much higher central density than the heavier nuclei.

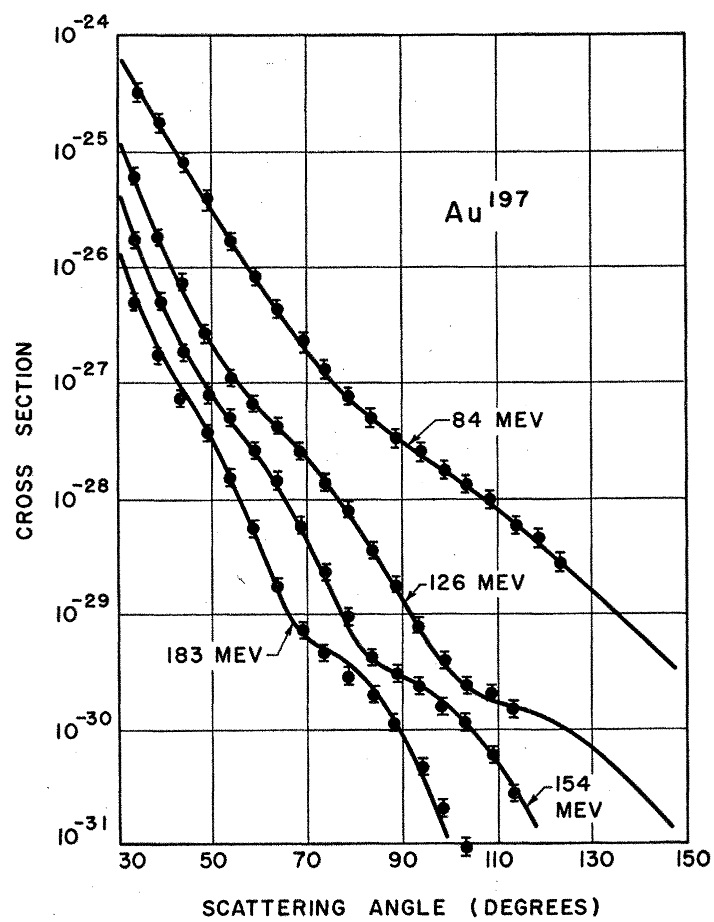


Fig. 7. The points represent experimental data observed by scattering electrons of the appropriate incident energies from gold nuclei (*9*). The solid lines are calculated angular distributions for a model of the gold nucleus similar to the model shown in Fig. 6.

come in work that Yearian and I did in which we used a difference method to compare the scattering from the deuteron and from the proton (*17*). We first showed that the neutron could not be represented as a point nucleon and that its magnetic moment was distributed in a manner similar to that of the proton. In Fig. 11 are shown data of the type from which such conclusions were drawn. The spread-out deuteron peak shows the effect of the motion of the proton and neutron in the deuteron, and this wide peak may be compared with the sharp peak of the free proton. In the work in which the finite size of the neutron was discovered, the neutron form factor F_{1n} was assumed to be approximately zero and F_{2n} had the behavior described earlier.

It may be noted parenthetically that it was on the basis of the foregoing results that Nambu (*18*) postulated the existence of a new heavy neutral meson, now known as ω-meson. Events of the past year have brilliantly confirmed the existence of this meson (*19*). A pion-pion resonance (ρ-meson) responsible for the magnetic behavior of the nucleon form factors was also postulated, by Frazer and Fulco (*20*), on the basis of these experiments. This resonance was also found recently (*21*).

The foregoing conclusions about the behavior of the neutron, and also the assumption that $F_{1n} \cong 0$, have been confirmed recently (*16, 22*). More detailed studies (*23*), as yet unpublished, support this description of the neutron form factors. These results are shown in Fig. 12. In work reported (*16, 22*), F_{1n} was found to be small and positive. However, Durand (*24*) has recently shown that the theory of the deuteron used in the early work to derive the values of the neutron form factors can be improved. When the improved formula is employed, the slightly positive values of the form factor F_{1n} are relatively unaffected in the low q^2 region, but in the range $6\text{f}^{-2} < q^2 < 20\text{f}^{-2}$ the values of F_{1n} are reduced to approximately zero, within experimental error (*23*). Because the neutron measurements are so fraught with both experimental and theoretical difficulties we must still regard these new, more accurate results, particularly for F_{1n}, as preliminary.

Recent Results on Nucleons

Figure 12 shows the most recent Stanford results for both the proton (*15*) and the neutron (*23*). An ambiguity exists in the choice of sign of F_{1n}. Figure 12 shows data for the positive F_{1n} values and the corresponding F_{2n} values. Figure 13 shows the neutron data for the negative F_{1n} values and the corresponding values of F_{2n}. Theoreticians prefer the choice of the positive F_{1n} values, but as a purely experimental problem the negative F_{1n} values must be considered possible until proved untenable. The dashed parts of the curve in Fig. 12 refer to probable behavior at low q^2, and the steep fall of F_{2n} that corresponds to the negative F_{1n} values would be very surprising and is not expected.

If the choice of positive values of F_{1n} is made—and these values seem more likely—an understanding of all the proton and neutron data can be obtained along the lines of the heavy-meson or pion-resonance theory of Bergia *et al.* (*25*) or the equivalent interpretation by Herman and me (*22*) of the early data in terms of Clementel-

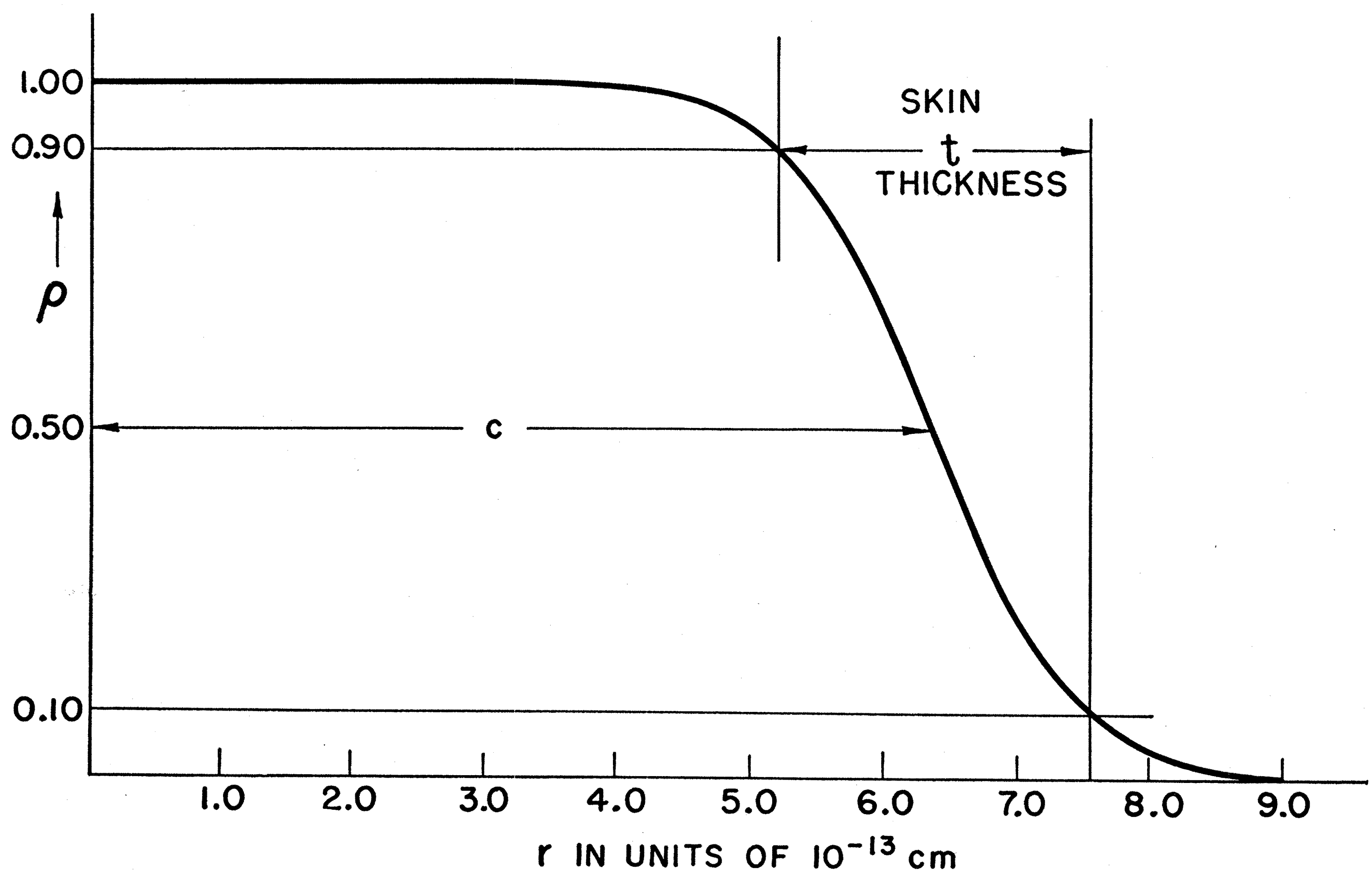


Fig. 8. The shape and parameters which describe an approximate model of the gold nucleus. This type is called the Fermi model (*9*).

Villi form factors, using Yukawa clouds of different ranges and delta functions. These initial and approximate theoretical interpretations are probably correct in principle but incomplete in detail, and it now seems likely that it is necessary to add to them the effects on the form factors of a third heavy meson (η-meson) (*26*). Such a particle has recently been discovered by A. Pevsner *et al.* (27). Its existence was also predicted by Sakurai (*28, 28a*).

Attempts are now being made to fit the data of Fig. 12 in terms of the heavy meson theory in a way similar to that reported in *22* and *25* but with three mesons (ρ, ω, η) instead of two. I hesitate to show the results of the studies since the exact mass values of the heavy mesons are not yet definite and small variations of these values affect the relative importance of the mesons in the form factor equations in a sensitive way. Furthermore, it would not be surprising if new heavy mesons are discovered in the near future, and these might also contribute to the form factors. Suffice it to say that approximate agreement with the data of Fig. 12 can be obtained with the set of three mesons (ρ, ω, η).

If we now attempt to summarize the recent progress in nucleon structure determinations and in their interpretation, we may say that the proton and neutron are two different aspects of a single entity—the nucleon. The third component of isotopic spin distinguishes between the two particles. Isotopic form factors can be developed in a well-known way (*22*) from the proton and neutron form factors. A phenomenological and qualitative interpretation of the nucleon form factors then shows that the same charged mesonic clouds appear in both the neutron and the proton. In the proton the clouds add together, and in the neutron the clouds cancel.

Fig. 11 (right). Experimental comparison of the scattering from the moving proton and neutron in the deuteron (curve *C*) and the scattering associated with free protons (curve *A*) (*17*). Region *B*, The bremsstrahlung tail of the proton curve. At *D* are electrons which have been scattered after producing pions in deuterium, and also other low-energy electrons. From the scattering data near *C* the form factors of the neutron can be obtained. The proton peak is used for comparison measurements. No correction has been applied in the figure for the difference in density of liquid deuterium and liquid hydrogen.

It is a bit too early to give the final and definitive details of the mesonic clouds or of their heavy-meson compositions, since, as already indicated, such details are now being worked out. However, it is possible, and even likely,

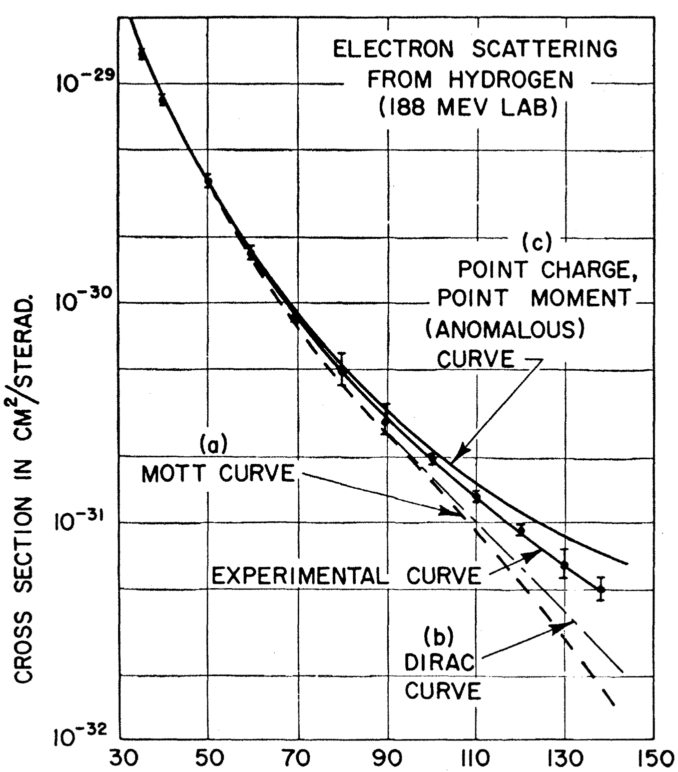


Fig. 9. Electron scattering from the proton at an incident energy of 188 Mev. *a*, The theoretical Mott curve for a spinless point proton; *b*, the theoretical curve for a point proton with a Dirac magnetic moment alone; *c*, the theoretical behavior of a point proton having the anomalous Pauli contribution in addition to the Dirac value of the magnetic moment. The deviation from *c* of the experimental curve represents the effect of form factors for the proton and indicates structure within the proton. The best fit in this figure indicates a root-mean-square radius close to 0.7 $\times$ 10^{-13} centimeter.

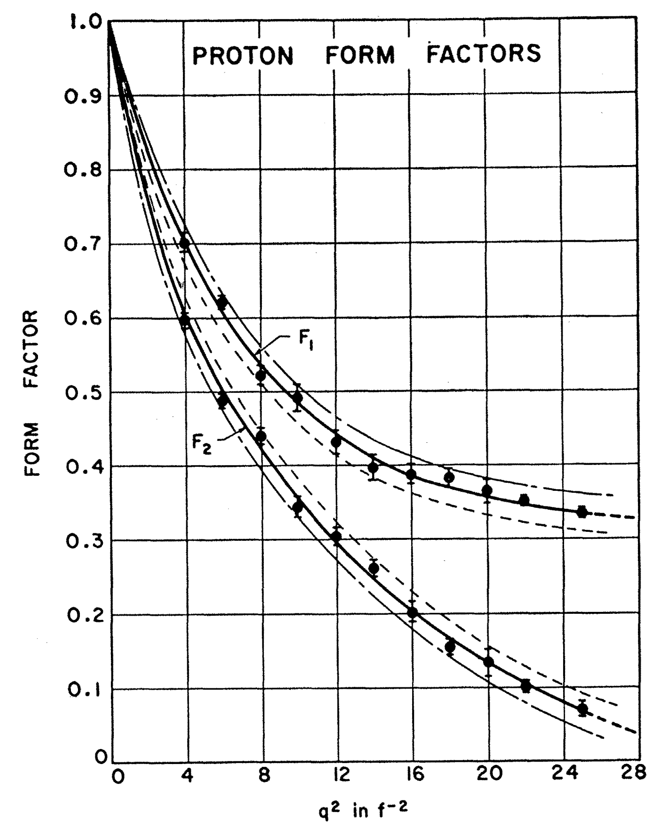
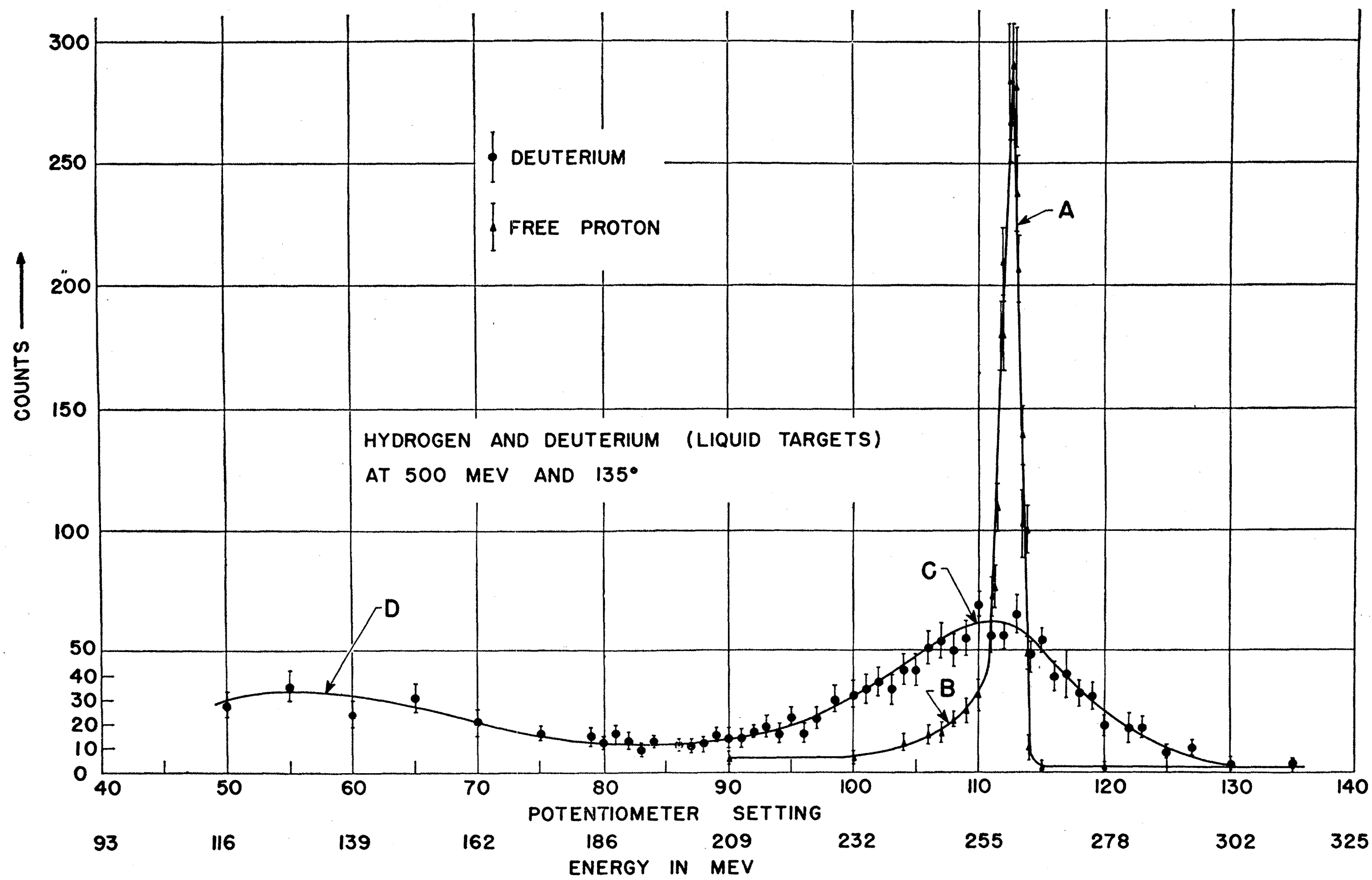
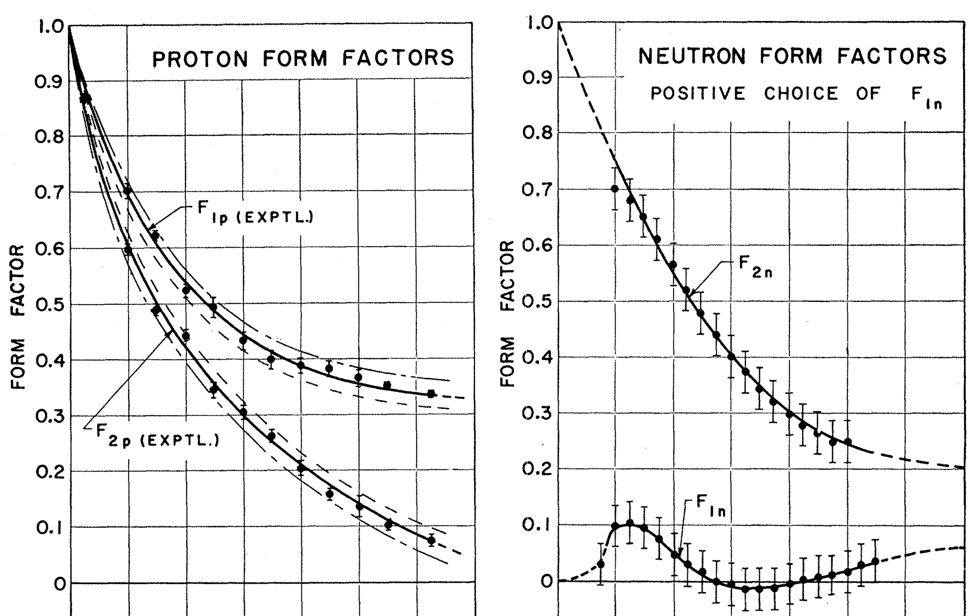


Fig. 10. The most recent Stanford experimental data on the form factors of the proton (*15*). There are two dashed curves lying between the central-value experimental curves (solid lines). If the error limits are correlated so that they move in opposite directions, as indicated by the dashed lines, the corresponding cross sections will remain consistent with experiment. A similar statement holds for the two curves of long and short dashed lines, lying outside F_1 and F_2, the central-value experimental curves. The question of correlated error needs additional study, but the dashed inner and outer curves are thought to give reasonable error limits of F_1 and F_2.





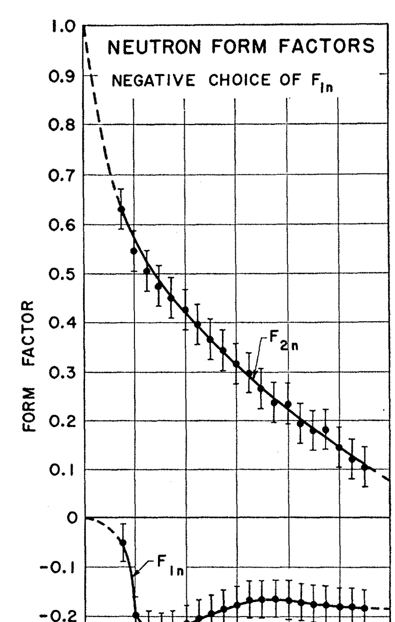


Fig. 12. The most recent Stanford results (*15, 23*) for both the neutron and the proton for positive values of F_{1n}. The regularity of the neutron curves arises from the fact that the experimental deuteron curves were smoothed before the corresponding data were put into the theoretical formulas from which the form factors are deduced. The four curves of this figure can be fit approximately with dispersion-theory or Clementel-Villi curves corresponding to the newly discovered heavy mesons. It is interesting to note that the newer neutron data agree very well with older results (*17*) and that many of the present conclusions could have been drawn in 1958.

Fig. 13. Graph similar to Fig. 12, right. It gives the set of values of F_{1n} and F_{2n} for the negative choice of F_{1n}. At present the curves of Fig. 12 appear to fit the Clementel-Villi curves better than those of Fig. 13 do.

that the next year or so should witness a crystallization of the "final" values of the nucleon structure parameters in terms of the models afforded by the new heavy-meson picture of the proton and neutron. The fact that new research is needed in order to clarify this picture is symptomatic of the general problem of the structure of elementary particles.

Conclusion

In concluding this discussion it may be appropriate to return to the theme introduced earlier and raise the question once again of the deeper, and possibly philosophical, meaning of the term "elementary" particle. As we have seen, the proton and neutron, which were once thought to be elementary particles, are now seen to be highly complex bodies. It is almost certain that physicists will subsequently investigate the constituent parts of the proton and neutron—the mesons of one sort or another. What will happen from that point on? One can only guess at future problems and future progress, but my personal conviction is that the search for ever-smaller and ever-more-fundamental particles will go on as long as man retains the curiosity he has always demonstrated (*29*).

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28a. *Note added in proof*: Compare recent paper by P. L. Bastien, J. P. Berge, O. I. Dahl, M. Ferro-Luzzi, D. H. Miller, J. J. Murray, A. H. Rosenfeld, M. B. Watson, *Phys. Rev. Letters* **8**, 114 (1962).

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