Effect of Isotropic Confining Pressure on Hydrostatic Pressure of Water in Unsaturated Soil

Abstract. A theory has been developed for the response of the pore-water pressure u_w to pressure applied isotropically to unsaturated soil. The results indicate that the response is determined by two shrinkage characteristics of the soil.

Croney *et al.* (*1*) have given a theoretical relationship equating the shrinkage slope,

$$n = (\partial V_b/\partial V_w)_p,$$

of unsaturated soils to the pressure coefficient,

$$\alpha = (\partial u_w/\partial p)_{V_w},$$

of the pore-water pressure u_w with respect to the confining pressure p. (V_b, the bulk volume of the soil, is equal to the sum of V_s, V_w, and V_a, the volume of soil, water, and air, respectively.) Croney *et al.* imply that soils which have reached their shrinkage limit, $n = 0$, will not change their pore-water pressure or their "suction" with changes of confining pressure. Furthermore, saturated soils, with shrinkage slopes of unity, should show a 1:1 response of u_w to the confining pressure. The theory of Croney *et al.*, if valid, has important implications in soil mechanics because it relates the piezometric properties of unsaturated soil to their volumetric properties.

An independent derivation, given here, shows that α is not equal to n, and that an additional term should be included (see Eq. 7).

A cylindrical sample of saturated soil, having a volume V_b, was encased in thin rubber, supported on a suction plate, and capped by a coarse filter plate which permitted exchange of air between soil and atmosphere. The whole assembly was placed in a triaxial cell whose outer chamber was filled with water and attached to a burette, making it possible to measure increments of volume (ΔV_b) accurately. The sample was subjected to progressively increasing suction through the filter plate until equilibrium was attained at each of several pore-water pressures. Finally, air pressure was applied to the water in the outer chamber ($\Delta p = 0.085$ kg/cm^2) and Δu_w was measured by an automatic null-point tensiometer.

The shrinkage curves are shown in Fig. 1. The vertical deviation from the 45-degree reference line represents the volume of air that has entered the soil at atmospheric pressure ($= u_a$). Tables in the upper left of each diagram compare the values of n for the final desorption step with those of α obtained subsequently. The pressure coefficient α was approximately equal to the shrinkage slope in all cases except that shown in Fig. 1*b* where $\alpha = 0.70$ and $n = 0.95$. A repetition of the discordant case gave $\alpha = 0.71$ and $n = 0.99$. The discrepancy forced us to re-examine the theoretical basis for the supposed equality of α and n.

If, at the end of desorption and before application of pressure, in a state of equilibrium and at constant temperature T, a virtual displacement is made such that water is absorbed by the sample, the sum of the virtual work terms is

$$-u_w dV_w + p dV_b - u_a dV_a - \Pi dV_w = 0 \quad (1)$$

where the separate terms represent contributions to the Helmholtz free energy F. The quantity Π, whose dimensions are those of pressure, represents the work done by the internal forces of the system per unit volume of water absorbed and comprises the work of interaction of the components during the virtual displacement (*2*).

Assuming that the volume V_s of the solid phase remains constant, we write

$$dV_b = dV_a + dV_w. \quad (2)$$

Using Eq. 2 to eliminate dV_a from Eq. 1, we obtain

$$(p - u_a)dV_b - (u_w - u_a)dV_w - \Pi dV_w = 0, \quad (3)$$

and dividing by dV_w, we get

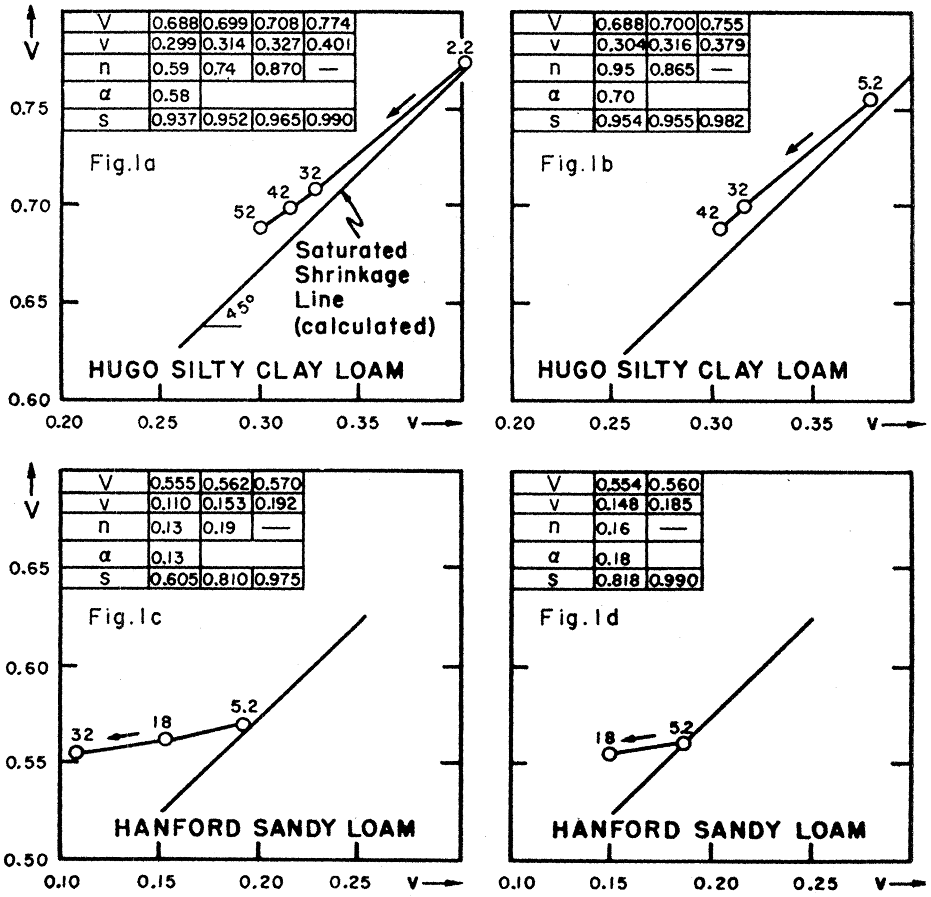
$$(p - u_a)(\partial V_b/\partial V_w)_{p,u_a,T} = (u_w - u_a) + \Pi. \quad (4)$$

(The substitution of $(\partial V_b/\partial V_w)_{p,u_a,T}$ for dV_b/dV_w is justified by the conditions of constant p, u_a and T prescribed for Eq. 1.)

Differentiating Eq. 4 partially with respect to p, we get

$$(\partial u_w/\partial p)_{V_w,u_a,T} = (\partial V_b/\partial V_w)_{p,u_a,T} + (\partial V_b/\partial V_w)_{p,u_a,T} + (p - u_a)[\partial/\partial p(\partial V_b/\partial V_w)_{p,u_a,T}]_{V_w,u_a,T}, \quad (5)$$

recognizing that u_a is constant and that Π is constant for a particular state of



V	0.688	0.699	0.708	0.774
v	0.299	0.314	0.327	0.401
n	0.59	0.74	0.870	—
α	0.58			
S	0.937	0.952	0.965	0.990

V	0.688	0.700	0.755
v	0.304	0.316	0.379
n	0.95	0.865	—
α	0.70		
S	0.954	0.955	0.982

V	0.555	0.562	0.570
v	0.110	0.153	0.192
n	0.13	0.19	—
α	0.13		
S	0.605	0.810	0.975

V	0.554	0.560
v	0.148	0.185
n	0.16	—
α	0.18	
S	0.818	0.990

Fig. 1. Shrinkage curves for two soils. V = specific bulk volume in cubic centimeters per gram; v = volume of water per unit weight of soil in cubic centimeters per gram; $n = \Delta V/\Delta v$; $\alpha = \Delta u_w/\Delta p$; S = degree of saturation. Numbers beside points represent applied suction ($u_a - u_w$) in centimeters of mercury. *Erratum*: The first entry for v in Fig. 1*a* should read 0.295, not 0.299.

the system with V_w constant. Commuting the mixed partial and representing the last term in Eq. 5 by r, we write

$$r = (p - u_a) \times [\partial/\partial V_w \, (\partial V_b/\partial p)_{V_w, u_a, T}]_{p, u_a, T} \quad (6)$$

The mixed partial represents, physically, the rate of variation of the coefficient of compression of the soil $\partial V_b/\partial p$ with changing water content. By substitution, Eq. 5 may be rewritten

$$\alpha = n + r. \quad (7)$$

Evidently the pressure coefficient is determined, not only by the shrinkage slope n, as claimed by Croney *et al.* (*1*), but also by the quantity r defined by Eq. 6.

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Very Small Diatoms: Preliminary Notes and Description of Chaetoceros galvestonensis

Abstract. Several species and genera of very small diatoms have been isolated from Gulf of Mexico waters at Galveston. A quantitative and qualitative study of their distribution has not yet been made. The organisms were isolated in unialgal cultures, and very rapid growth rates were observed. Environmental and experimental implications are pointed out. A new species, *Chaetoceros galvestonensis*, 1.5 by 3.0 microns in broad view, is described and figured.

During experimental studies on filterable organisms isolated from surface waters of the Gulf of Mexico, several species and genera of unusually small diatoms have been found. Whatman No. 52 paper was used for the filtration. One species of *Chaetoceros* measured 1.5 microns on the apical axis and 3 microns on the pervalvar axis. Other genera have been collected with individual cells as small as 0.75 micron in diameter and forming spiraled and straight chains (the extreme range for most diatoms hitherto reported is from 10 to 200 microns). All of the diatoms appear to be normal, with well-developed chromatophores, nuclei, pyrenoid bodies, and oil globules. The various types have been isolated as unialgal cultures in standard culture media originally formulated for certain dinoflagellates. They proliferate profusely and subculture readily. In the case of several species an inoculum of 0.1 ml introduced into 10 ml of culture medium produced homogenous turbidity in less than 48 hours, and by the end of the fourth day the diatoms were forming green deposits in the bottoms of the tubes. Morphological observations were made with the aid of phase microscopy at a magnification of 1250.

Efforts to learn more about the nutritional requirements and the effects of physical factors on growth rates are being made at the present time. One example of these organisms is described below as a new species, and other descriptive material is being prepared.

The existence of populations of such small organisms in the environment and in experimental work may be significant. The small size and consequent high surface-to-volume ratio of these cells, together with their capability for rapid multiplication, have two important, closely related implications for the oceanographer. First, these factors might enable the diatoms to reproduce rapidly even under conditions of minimal nutrient concentration; and second, because of their power of intense utilization of nutrients, they might cause a very rapid depletion of an adequate concentration of nutrients.

In addition, these cells might have effects on light scattering, sound scattering, and heat absorption; because of the large amount of metabolic and degradative organic residues produced, their effect on viscosity and surface tension of the water certainly needs investigation.

Their importance as possible contaminants in light and dark bottles is obvious to those who are studying marine productivity by gasometric methods. Their minuteness and their failure to respond to bacterial sterility tests make it possible for them to be present as contaminants in unialgal cultures of larger diatoms and dinoflagellates. Our experience indicates that they will multiply over a wide range of temperatures (2° to 25°C).

Because their size approaches that of bacteria it is possible that very small diatoms offer a new approach to problems in the nutrition of filter-feeding organisms such as oysters, mussels, and certain adult and immature crustaceans that feed on plankton.

It is impossible to estimate the quantitative aspects of these implications in the open sea until some method for measuring the density of the organisms can be developed. This may be difficult to do because of lack of methods for separating such small organisms from larger diatoms, protozoa, and other plankton. Meanwhile we hope to gain insight into their potential importance by experimental procedures.

Chaetoceros galvestonensis sp. n. (Fig. 1).

Cells solitary. Apical axis 1.5 microns, pervalvar axis 3 microns. Broad girdle view rectangular with valve surface slightly convex. Valve view ellipsoidal; a bristle located at each corner of broad girdle view; bristles about 2 microns long, comparatively thick. Two large L-shaped chromatophores, one at each end of frustule; large pyrenoid body subject to pronounced brownian movement. Resting spores not observed. Cells quite uniform in dense culture material. Isolated from surface water outside of surf line, Galveston Beach. Specific name for Galveston Island, site of first location (*1*).

Chaetoceros galvestonensis sp. n. (Fig. 1).

Cellulae solitariae. Axis apicalis 1.5 micra, pervalvaris 3 micra. Aspectus lato-zonalis rectangularis facie valvari subconvexa, in quoque angulo setifer setis longitudine ca. 2 micra sat crassis.

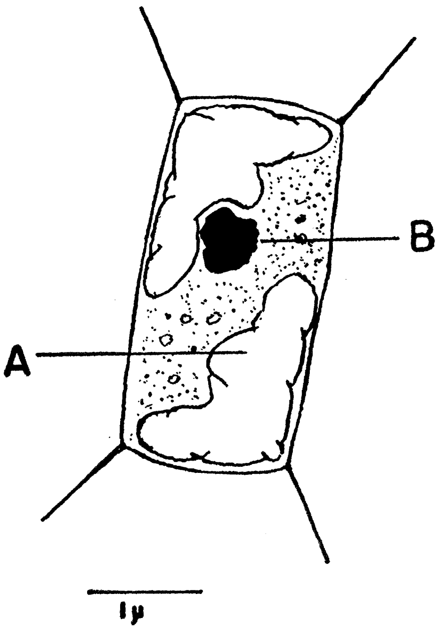

Fig. 1. *Chaetoceros galvestonensis*. *A*, chromatophores; *B*, pyrenoid body.