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A Children's Zoo of Math

In preparing material for children, a mathematician can go the keeper of a children's zoo one better. He is not limited to tame or domestic varieties of what nature offers, but can invent his own creatures. David Page of the University of Illinois Arithmetic Project has developed several such creatures, and an idea of this work may be gained from a glimpse at the latest addition to the menagerie, a kind of "algebra" called "maneuvers on lattices." The topic is sufficiently rich to offer scope for genuinely creative thinking, yet so simple that even a child can do it. It may be started early in elementary school, but it has possibilities for all later grades, even through high school.

The most natural way to begin maneuvers on lattices is to consider the most natural of lattices:

30 31 32 . . .
20 21 22 23 24 25 26 27 28 29
10 11 12 13 14 15 16 17 18 19
0 1 2 3 4 5 6 7 8 9

Now comes the secret code. Let "5↑" stand for the number directly above 5, namely 15, "6↑" for 16, "7↑" for 17, and so on. Arrows may also point in other directions: "15→" for 16, "15←" for 14, and so on. Next comes the deciphering of messages. More than one arrow may be used at a time: for example, "15↑↑" or "15→→→↑→↓." The problem is to figure out what numbers the more complicated expressions represent.

At first children will work out the answers step by step, making laborious use of the lattice, but soon they will make some discoveries. One child will discover that he can save time by counting all the arrows pointing in a given direction; a second will note that in expressions like "↑↓↑" certain pairs can be canceled out; and a third will find that the expression "↑→↓←" takes you around a loop. Here the teacher is working for shortcuts, generalities, and even matters of mathematical elegance.

Another kind of problem concerns the edges. Expressions like "29→" or "10←" may initially be regarded as nonsense, as meaningless, because they take you off the lattice. But a rule not to go off the lattice is very restrictive, and so there will soon be a search for appropriate meanings for these expressions. Some typical definitions, starting with the most natural one for an adult, are "29→ = 30" or *Reversed typewriter* (up a line at the right margin, then over to the left margin); "29→ = 28" or *Bounce back the way you came*; and "29→ = 0" or *Go directly to zero and start over.*

Not all definitions will prove equally acceptable to everyone. There will be strong objections and strong preferences, and the reasons must be brought out. Thus, not all of the above definitions preserve a rule implicit in the earlier use of the code: namely, that "29→↑ = 29↑→," or as a mathematician would say, the arrows commute. Here the teacher is working for the idea of generalizing a mathematical theory, that is, enlarging it to include more objects or more operations than are in the original system, with an eye to whether the original rules still hold or must be modified or dropped.

And so improvement in mathematics teaching need not be limited to seeking better ways to teach, say, the multiplication tables or to introducing selected aspects of more advanced subjects, say, set theory, at an earlier stage, but may include devising new mathematical topics. Experience in the classroom so far is encouraging, but it does show that teachers must develop a special kind of tolerance. Teachers must expect that children will be coming up with their own answers, with answers not in the book, just as at the playground children are forever finding new ways to go down the slide, and up it, too.—J.T.