dynamics of mixing in the stirred reactor of Fig. 1.

Another critical current problem in automatic process control is the precise treatment of integrated control systems. Frequency response techniques have been used widely and quite successfully, but not for design to specific performance quality. The damped frequency response offers enough promise to merit further study. For nonlinear systems the approaches via the theorems of Lyapunov, with the extensions described by Letov, appear to be practical first steps.

The third current problem area in the field of automatic process control is concerned broadly with optimization at all levels of effort. What are practical criteria of optimality? What search procedures for finding optima are themselves optimum? The answers to these and similar questions must be given in greater refinement. Some aspects of these questions are of concern to economists, and some are of concern to mathematicians and computer programmers. But all of the questions concern the control engineer.

There is considerable activity in the field of automatic process control in this country and in Japan, France, England, and Germany. It is somewhat disquieting to know that in matters of automatic control theory the Soviet Union leads us and appears to be increasing the lead at the present time (*10*).

Future Trends

At the present time the fully automatic process plant where all operations, including direction and execution, are handled by machines does not exist. There are no major theoretical or technical barriers, but the details of application must be worked out in each case. With the continuing acceleration of activity in research, development, and application in the field of automatic process control, the automatic plant will soon become a reality. The long-range sociological effects will be enormous, but they can be good.

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An Econometric Model of Economic Development

The model used by Indian planners poses a paradox concerning the best allocation of scarce capital.

Martin Bronfenbrenner

Mathematical economics is nearly as old as economics itself—substantially older than, say, Adam Smith's *Wealth of Nations* (*1*). Yet it did not become intellectually respectable, at least in the Anglo-Saxon world, until the Great Depression of the 1930's. In retrospect, the founding of the Econometric Society and the appearance of its journal, *Econometrica,* in 1932 seem a kind of watershed within the economics profession. Mathematical economics still appears to most outsiders an esoteric and new-fangled sort of science or pseudoscience.

The earliest mathematical-economic studies were largely devoid of empirical content, just as the earliest statistical-economic studies were largely devoid of economic analysis. It was the econometricians (and their ancestors) who brought these two strands together. An early example, not otherwise particularly fortunate, was W. Stanley Jevons's sunspot theory of business fluctuations (*2*).

Most of the earlier econometric studies, Jevons's being by no means the sole exception, dealt mainly with the supply of or the demand for individual commodities one at a time. Later they developed into studies of a few related markets, and later still they reverted to Jevons-type grand schemes of "ambitious equation systems attempting to represent the dynamic properties of an entire economy" (*3*).

When an econometric study involves more than one equation (usually fitted statistically) at a time, the set of equations is called an economic model (*4*). To cite one simple case, a demand equation and a supply equation form a two-equation model of market price determination. When all the variables of an equation system or model refer to the same point in time, there is no need to "date" them and the model is called *static*. When different variables relate to different points in time, the model must be *dated* and is called *dynamic* (*5*). Suppose, for example, that the amount of wheat q_t^d demanded in period t depends on its price p_t during that period, but that, since wheat production requires time, the amount q_t^s supplied in period t depends on its price in the preceding period $(t-1)$. Then the three equations:

$$q_t^d = f(p_t)$$
$$q_t^s = g(p_{t-1})$$
$$q_t^d = q_t^s$$

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whose symbols require no further explanation, form a three-equation dynamic model of the wheat market [This model is not in fact as simple as it looks (*6*).] When no explicit account is taken of errors of observation and errors of omission of causal factors not entering into the model as written, the model is called *exact*. When error terms enter explicitly, the model is called *stochastic*. The model of the wheat market given above is exact; to make it stochastic, we may rewrite the first two equations (demand and supply, respectively):

$$q_t{}^d = f(p_t) + u_t$$

$$q_t{}^s = g(p_{t-1}) + v_t$$

where the u and v are assumed to have mean values of 0 and standard deviations σ_u and σ_v, respectively, which statisticians call *standard errors of estimate*. It has been shown that, unless u and v are independent both of each other and of p and q, statistical investigation of these two equations separately will usually give *biased* estimates of the demand and supply functions f and g; avoidance of biased estimates is one reason why so much contemporary econometric work uses stochastic models (*7*).

How Models Are Used

How are mathematical-economic econometric models used? To start from the ivory tower and work outward, they are used to verify "literary" economic theories and to suggest which of their postulates or assumptions are really crucial to their conclusions. Consider, for example, the Keynesian system, which reaches the conclusion that a competitive capitalist economy need not tend toward full employment (*8*). Lord Keynes himself was neither a mathematical economist nor an econometrician, but a number of writers, including particularly Lawrence Klein (*9*), have concluded after econometric study that various economic relations do in fact have the general forms Keynes assumed they did. On the more strictly mathematical front, Franco Modigliani has asserted that Keynes's results require an unexpressed assumption that money wages are held rigid and never fall (*10*), while Klein insists that, even without rigid wages, the Keynesian result of "under-employment equilibrium" requires only that interest rates be positive (*11*).

Two of the most important postwar applications of mathematical economics and econometrics have been in the eminently practical domains of short-run forecasting and long-range development planning. The leading short-run forecasting models have been based upon the work of Klein and Goldberger (*12*) for the U.S. economy, as modified in the University of Michigan Research Center in Quantitative Economics. The leading development-planning models have been based upon elaborate input-output tables for leading industries, developed for the United States by Wassily Leontief (*13*) and applied to developing countries by Hollis Chenery on the Western and by Oskar Lange on the Eastern side of the Iron Curtain (*14*). These tables indicate in considerable detail the interdependencies between different industries, both as sources for materials (inputs) and outlets for products (outputs).

India's Development Model

The example considered here is also a development-planning model, clearly mathematical and marginally econometric. It was developed with special reference to India (and in much less detail than some of the examples just mentioned) by P. C. Mahalanobis and his staff. It formed the theoretical framework of both India's Second Five-Year Plan (1956–61) and the subsequent Third Five-Year Plan currently in operation.

Mahalanobis is director of the Indian Statistical Institute in Calcutta and editor of the international statistical journal *Sankhyā*. Originally trained as a physicist, and eminent primarily as a mathematical statistician, he rather scorns the title of economist. He nevertheless became, more than any other single man, the guiding spirit of Indian development planning, on both the ideological and the technical sides. Few other formal development models have been as influential for actual development policy in any country as those devised by Mahalanobis. This fact explains the increasing world-wide interest in his work, even among economists like myself who profess no special interest or expertise in Indian problems.

Mahalanobis's thinking is distilled in a series of three papers in *Sankhyā* (*15–17*). This discussion centers upon the first and simplest of the three (*15*). I shall develop a model even further simplified, with the principal properties of the Mahalanobis model and in a form accessible to readers without extensive mathematical training (*18*). Except where statements are made to the contrary, my assumptions and conclusions are intended to parallel those of Mahalanobis.

Paradoxical Income Growth Equation

Let income during any time period t be denoted by Y_t, the sum of contemporaneous investment I_t and consumption C_t, measured in monetary units of constant purchasing power. This is to say, the income of a society consists of its members' expenditures for investment and consumption goods, or, in symbols:

$$Y_t = I_t + C_t$$

Let the capital stock K be the "limiting" or "strategic" resource on the side of production—not an unreasonable assumption in an overpopulated, underemployed country like India. More specifically, let the magnitudes of investment and consumption (I_t, C_t) be related to the capital stock at the end of the *preceding* time period or the beginning of the current one (K_{t-1}) in such wise that:

$$I_t = \lambda_i \beta_i K_{t-1}$$

and

$$C_t = \lambda_c \beta_c K_{t-1}$$

The terms in λ are allocations. A certain proportion of the capital stock is allocated, either by the market or by the economic planners, to investment or to consumption goods. [The sum of the allocations ($\lambda_i + \lambda_c$) must equal unity.] As for the terms in β, these are gross *productivities* of capital, measured as proportions of the capital stock produced in period t in each of the two "sectors" of the economy. They are normally fractional, but there is no condition on their sum.

Mahalanobis defines the I and C sectors, into which many economists since Marx's day have seen fit to divide the economy, as including raw materials and parts. They therefore represent the outputs of investment and consumption goods *with their respective subsidiaries* (*19*). (The production not only of clothing but of textiles of which clothing is made, and not only of food but of cans and bottles, is included in C; this reclassification restricts I largely to the output of heavy industry.)

Mahalanobis assumes that the entire national income is produced domestically, on the grounds that India should become independent of outside sources as soon as possible for "fundamental" goods of all sorts. His model also reduces the entire economic problem to one of supply. Demand is nowhere taken into account. The Plan, meaning the Government, is to absorb or distribute any surpluses which may arise, and alleviate any deficits by rationing.

Let us define both Y_t and Y_{t-1} in terms of our earlier definitions of I and C, remembering that $Y_t = I_t + C_t$:

$$Y_t = (\lambda_i\beta_i + \lambda_c\beta_c)K_{t-1}$$

and

$$Y_{t-1} = (\lambda_i\beta_i + \lambda_c\beta_c)K_{t-2}$$

These equations permit us to define a percentage growth rate of income, which we shall call g (Mahalanobis does not use this symbol):

$$g = \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{(\lambda_i\beta_i + \lambda_c\beta_c)(K_{t-1} - K_{t-2})}{(\lambda_i\beta_i + \lambda_c\beta_c)K_{t-2}}$$

But the difference between the value of the capital stock at two periods of time is simply the net investment (gross investment less depreciation) in the interval. In symbols, $K_{t-1} - K_{t-2} = I_{t-1}$. We also know that $I_{t-1} = \lambda_i\beta_i K_{t-2}$. Making these substitutions and carrying out all cancellations, we obtain an extremely simple result, which may be called the Mahalanobis paradox:

$$g = \lambda_i\beta_i \qquad (1)$$

The paradoxical result is that the growth rate of national income as a whole depends only upon the allocation of capital to the investment industries and upon its productivity there. It makes no difference whatever how productive the scarce capital stock may be in the consumption sector.

To obtain a growth path or equation for income, let us rewrite Eq. 1 as a difference equation, making use of the definition of the growth rate g:

$$Y_t - Y_{t-1} = \lambda_i\beta_i Y_{t-1}$$

or

$$Y_t - (1 + \lambda_i\beta_i)Y_{t-1} = 0$$

This has two equivalent solutions (*20*), one in terms of the pre-plan income Y_o and the other in terms of the pre-plan capital stock K_o; the latter is generally more interesting (*21*).

$$Y_t = Y_o(1 + \lambda_i\beta_i)^t \qquad (2a)$$

or

$$Y_t = (\lambda_i\beta_i + \lambda_c\beta_c)K_o(1 + \lambda_i\beta_i)^{t-1} \qquad (2b)$$

Equation 2*b* can be used to show that income will eventually be higher with a high λ_i than with a low one, even though capital may be more productive in consumption than in investment industries. This form of the Mahalanobis paradox is illustrated arithmetically by Mahalanobis himself.

He supposes the alternative productivities of capital in India to be 0.1 and 0.3 in the investment and consumption sectors, respectively ($\beta_i = 0.1$, $\beta_c = 0.3$), although he feels the actual differences in productivity are in fact considerably smaller. Then he works out alternative growth patterns for $\lambda_i = 0.1$ ($\lambda_c = 0.9$) and for $\lambda_i = 0.3$ ($\lambda_c = 0.7$). The first of these allocation systems corresponds roughly to that of India's First Five-Year Plan (1951–56), the second to that recommended by Mahalanobis for India's Second Five-Year Plan. We shall use the same coefficients and Eq. 2*b* to derive the same paradox which Mahalanobis obtained from his more complex equivalent.

With $\lambda_i = 0.1$, the growth-path equation (Eq. 2*b*) may be written as Eq. 3*a*. With $\lambda_i = 0.3$, it becomes Eq. 3*b*:

$$Y_t = 0.28\,K_o(1.01)^{t-1} \qquad (3a)$$

$$Y_t = 0.24\,K_o(1.03)^{t-1} \qquad (3b)$$

For low values of t—that is, in the early years of development—Eq. 3*a*, with its predominantly consumption-goods allocation of capital, gives higher values for income than does Eq. 3*b*, precisely because capital is more productive in the consumption sector. For higher values of t, however, or "in the long run," Eq. 3*b* forges ahead. The critical value of t at which Eqs. 3*a* and 3*b* give equal values of Y_t is a little less than 9 years after the start of the plan (*22*). To take the long view, then, Eq. 3*b* does better than Eq. 3*a*, whatever the initial value of the society's original stock of capital.

Returning to Eq. 1, which defines the growth rate g, we obtain another result which Mahalanobis stresses—namely, the minimum value of the investment allocation λ_i consistent with a "respectable" rate of growth. If 3 percent per year is the growth rate required to forge ahead of the Indian population explosion and increase per capita income appreciably in India, and if Mahalanobis's estimate of 0.1 for β_i (capital productivity in the investment sector) is approximately correct, it follows from Eq. 1 that the investment allocation λ_i must be at least 0.3 or 30 percent of the capital stock. (Mahalanobis's own suggested λ_i is 33 percent, slightly higher than this minimum.) For minimal growth rates higher than 3 percent, the required λ_i is higher still.

Additions of increments of capital (net new investment) between investment and consumption goods should also bear some relation to the country's "saving ratio," the proportion of its income not used in consumption. This feature does not enter the Mahalanobis model explicitly, but we can see in a general way that the higher λ_i is, the higher must be the saving ratio required to support it without strain. There is clearly a maximum value of λ_i beyond which the population could not survive the hard early years of the development process. This maximum condition is known to the economic planner (at least within limits) and is omitted from the development model itself. A dilemma in many "democratic" development plans, which may possibly apply to the Indian one, is that the maximum λ_i which the populace accepts without heavy doses of propaganda laced with terror may fall short of the minimum λ_i which keeps income ahead of population. Models of the Mahalanobis type may bring this dilemma into focus but suggest no methods for avoiding it. When it applies, in other words, the model provides no third alternative to stagnation or dictatorship.

Consumption Growth Equation

Following in Mahalanobis's footsteps, we can derive a growth equation for consumption as well as for income as a whole. Here again the Mahalanobis paradox holds, on Mahalanobis's assumptions: the level of consumption is eventually increased by concentration of capital in the investment sector, even when capital is more productive in the consumption sector.

As in our development of the income growth equation, we have:

$$C_t = \lambda_c \beta_c K_{t-1}$$

and

$$C_{t-1} = \lambda_c \beta_c K_{t-2}$$

Defining g' as the percentage rate of growth of total consumption, we obtain the result:

$$g' = \frac{C_t - C_{t-1}}{C_{t-1}} = \frac{(\lambda_c \beta_c)(K_{t-1} - K_{t-2})}{(\lambda_c \beta_c) K_{t-2}}$$

Again, the difference between the two capital stock figures in the numerator at right is I_{t-1}, defined earlier as $\lambda_i \beta_i K_{t-2}$. Making this substitution and carrying out the cancellations indicated, we obtain a simple and interesting parallel to Eq. 1:

$$g' = \lambda_i \beta_i \qquad (4)$$

Equation 4 says that the growth rate of consumption is identical with that of income and, therefore, also depends only upon the allocation of capital to investment industries and upon its productivity in these industries. The Mahalanobis paradox, in other words, applies once more.

Like Eq. 1, Eq. 4 may be rewritten as a difference equation (use being made this time of the definition of g'):

$$C_t - (1 + \lambda_i \beta_i) C_{t-1} = 0$$

Solutions of this equation, in terms of pre-plan consumption C_o and pre-plan capital stock K_o are:

$$C_t = C_o (1 + \lambda_i \beta_i)^t \qquad (5a)$$

and

$$C_t = (\lambda_c \beta_c) K_o (1 + \lambda_i \beta_i)^{t-1} \qquad (5b)$$

To illustrate the Mahalanobis paradox, let us again suppose the productivities λ_i and λ_c to be 0.1 and 0.3, respectively, and let us work out alternative growth paths for $\lambda_i = 0.1$ ($\lambda_c = 0.9$) and $\lambda_i = 0.3$ ($\lambda_c = 0.7$). In the first case, Eq. 5*b* may be rewritten as Eq. 6*a*, while in the second it becomes Eq. 6*b*:

$$C_t = 0.27\, K_o (1.01)^{t-1} \qquad (6a)$$

and

$$C_t = 0.21\, K_o\, (1.03)^{t-1} \qquad (6b)$$

For low values of t and in the early years of planned development, Eq. 6*a* gives higher values for consumption than Eq. 6*b*, but for higher values of t and in the long run, Eq. 6*b* forges ahead. The critical value of t, at which Eqs. 6*a* and 6*b* give equal values of C_t, is slightly more than 29 years for all values of the initial capital stock K_o (*23*). As might be expected, this period is substantially longer than the corresponding critical period for income as a whole, which we derive as not quite 9 years, and which Mahalanobis estimates at 15 years.

"Literary" Criticism Not Enough

Mahalanobis proceeds much further in the direction of detailed concreteness than we have followed him, and other writers have gone further on balance than Mahalanobis. Neither Mahalanobis nor any other development planner known to me has produced development models immune from criticism. "Planning" has not turned out to be the magic wand that its more sentimental advocates expected. Some of the criticisms of the Mahalanobis plan in particular, both in India and abroad, have been quite sharp, particularly after the Second Five-Year Plan led to price inflation in India and to depletion of India's foreign assets, without being fulfilled in its entirety (*24*). It should, however, be noted that old-fashioned "literary" criticisms by the older generation of Indian economists, in the main unable to follow Mahalanobis's mathematics and statistics, have failed to disturb the widespread faith in the "scientific" character of Mahalanobis's work, which itself is based on this same mathematical and statistical basis. The younger generation of Indian economists is more nearly able to meet Mahalanobis on his own ground but has not been heeded, precisely because of its youth.

It is hard to fight something with nothing and, likewise, to improve upon something with nothing. Insofar as Mahalanobis's model for India is modified or even rejected in the future, it will be modified or rejected by economists capable of using his tools approximately as capably as he has used them himself.

References and Notes

1. Irving Fisher, in his "Bibliography of mathematical economics," which is appended to the English version of the earliest mathematical-economic "masterpiece," A. A. Cournot's *Recherches sur la théorie mathématique des richesses* (1838), lists all contributions known to him that are dated prior to 1897. The oldest, Giovanni Ceva's *De re nummaria*, dates from 1711, whereas the first edition of the *Wealth of Nations* appeared in 1776 [A. A. Cournot, *Researches into the Mathematical Theory of Wealth*, N. T. Bacon, trans. (Macmillan, New York, 1929), p. 173]. Statistical economics is even older. Sir William Petty's *Political Arithmetick*, published in 1690 and probably written 18 years earlier, is regarded by many (as it was by Karl Marx) as the world's first serious work in both economic statistics and economics proper. Compare E. Roll, *History of Economic Thought* (Prentice-Hall, Englewood Cliffs, N.J., ed. 3, 1959), p. 99 ff.
2. Jevons's study "The periodicity of commercial crises and its physical explanation" (1878) formed a chapter in his *Investigations in Currency and Finance* (1884). It has been reprinted in A. H. Hansen and R. V. Clemence, *Readings in Business Cycles and National Income* (Norton, New York, 1953), pp. 83–95. In the general history of economic thought Jevons is best known for his development of "marginal utility" as a basis for the theory of demand and thus for the theory of value as a whole [W. S. Jevons, *Theory of Political Economy* (Macmillan, London, ed. 4, 1924, originally published in 1871), chaps. 2–4].
3. T. Koopmans, *Three Essays on the State of Economic Science* (McGraw-Hill, New York, 1957), p. 201.
4. A good elementary exposition of the principles of model construction appears in E. F. Beach, *Economic Models: An Introduction* (Wiley, New York, 1957).
5. The distinction made between "statics" and "dynamics" along these lines is attributable primarily to J. R. Hicks [*Value and Capital* (Oxford Univ. Press, Oxford, England, 1939), p. 115].
6. The resulting pattern of price and quantity movements resembles a converging or diverging cobweb. For a standard study, see M. Ezekiel, "The cobweb theorem" [*Quart. J. Economics* (1938)], reprinted in G. Haberler, Ed., *Readings in Business Cycle Theory* (Blakiston, Philadelphia, 1944), pp. 422–442.
7. Theorems regarding bias and the allied problem of "identification" arise largely from the classical contribution of T. Haavelmo, "The probability approach in econometrics" [*Econometrica*, suppl. (1944)]. A standard text built up around this approach is L. R. Klein, *Textbook of Econometrics* (Row, Peterson, Evanston, Ill., 1953).
8. J. M. Keynes, *General Theory of Employment Interest and Money* (Harcourt Brace, New York, 1936); D. Dillard, *The Economics of John Maynard Keynes* (Prentice-Hall, New York, 1948); A. H. Hansen, *A Guide to Keynes* (McGraw-Hill, New York, 1953).
9. L. R. Klein, "The empirical foundations of Keynesian economics," in K. K. Kurihara, *Post-Keynesian Economics* (Rutgers Univ. Press, New Brunswick, N.J., 1954), pp. 277–319. A similar task of empirical verification for a rival set of hypotheses, the quantity theory of money, has been attempted by a University of Chicago group and reported in M. Friedman, Ed., *Studies in the Quantity Theory of Money* (Univ. of Chicago Press, Chicago, 1956).
10. F. Modigliani, "Liquidity preference and the theory of interest and money," *Econometrica* (1944), reprinted in F. A. Lutz and L. W. Mints, Eds., *Readings in Monetary Theory* (Blakiston, Philadelphia, Pa., 1951), pp. 186—240.
11. L. R. Klein, *The Keynesian Revolution* (Macmillan, New York, 1947), pp. 80–90, 198–206; see also Klein's "Theories of effective demand and employment," *J. Polit. Econ.*, 108 (1947).
12. ——— and A. S. Goldberger, *An Econometric Model of the United States, 1929–1952* (North-Holland, Amsterdam, 1955); subsequent modifications at the University of Michigan have been carried out primarily by Daniel B. Suits and his associates.
13. W. W. Leontief, *The Structure of the American Economy, 1919–1939* (Oxford Univ. Press, New York, ed. 2, 1951).
14. H. B. Chenery and P. G. Clark, in *Interindustry Economics* (Wiley, New York, 1959), include (chaps. 10, 11) applications to southern Italy and to several Latin-American countries. Oskar Lange, in *Introduction to*

Econometrics [E. Lepa, trans. (Pergamon, London, 1959)], includes an appendix by K. Porwit in which data for Poland are applied to Polish problems.
15. P. C. Mahalanobis, "Some observations on the process of growth of national income," *Sankhyā*, 307 (1952).
16. ———, "The approach of operational research to planning in India," *ibid.*, 3 (1955); ———, "Draft plan-frame for the second five-year plan," *ibid.*, 63 (1955). These articles and that cited in *15*, all of them written after several visits to the Soviet Union, presumably embody some of what Mahalanobis considers the lessons of both Marxist theory and Soviet experience. In fact, as E. D. Domar has pointed out, Mahalanobis's work was partly foreshadowed, as early as 1928, by the work of the neglected Soviet economist G. A. Feldman, although Mahalanobis learned of Feldman's paper only after the appearance of his own model (see *17*).
17. E. D. Domar, "A Soviet model of growth," in *Essays in the Theory of Economic Growth* (Oxford Univ. Press, New York, 1957).
18. See M. Bronfenbrenner, *Economic Development and Cultural Change* (1960), pp. 45–51, for an earlier discussion along similar lines, which includes considerably more criticism along with the exposition.
19. Because of these definitions, Mahalanobis's *I* and *C* sectors differ from the Marxian "Departments" I and II of *Das Kapital*. Interestingly enough, Feldman also reorganized the Marxian "departmental" structure in the same way in his 1928 article (see *17*, pp. 226–228).
20. Let us define a "dummy" or meaningless variable X such that $Y_t = aX^t$. Substituting aX^t for Y_t and aX^{t-1} for Y_{t-1}, we find $X = (1+\lambda_i\beta_i)$, while a becomes equal to Y_o in order that $[a(1+\lambda_i\beta_i)^o]$ should equal the known base period or pre-plan value Y_0. For a fuller explanation, prepared with the interests of economists in mind, see W. J. Baumol, *Economic Dynamics* (Macmillan, New York, ed. 2, 1959), chap. 9.
21. Equation 2 corresponds to a considerably more complex result of Mahalanobis's Eq. 1 in "Some observations on the process of growth of national income" (*15*, p. 309)—a result which he does not derive. A derivation, attributable primarily to Marcel K. Richter of the University of Minnesota (currently at the University of California, Berkeley) is given in M. Bronfenbrenner, *Economic Development and Cultural Change* (1960), p. 48. The Mahalanobis original has, however, one important economic advantage over my simplification. The present formulation assumes implicitly that the existing capital stock can be reallocated readily between sectors of the economy when λ coefficients are changed. Mahalanobis's own development provides greater realism on this point at the price of greater mathematical cumbersomeness. (Irving Morrissett of Purdue University first pointed out to me this advantage of the actual Mahalanobis model over the modification given here.)
22. To obtain the critical value of t, set the logarithms of the right-hand sides of Eqs. 3*a* and 3*b* equal to each other and solve for $t-1$: $\log 0.28 + \log K_0 + (t-1) \log 1.01 = \log 0.24 + \log K_0 + (t-1) \log 1.03$. The resulting value of t is 8.86 years. Mahalanobis, with his more elaborate equation, derives 15 years as his critical value of t (see *15*, p. 309).
23. Mahalanobis's estimate of the critical period for consumption is also 29 years; thus, approximately a generation of austerity is required if the more ambitious heavy-industry planning pattern is adopted (see *15*, p. 310).
24. One critique readily available to American readers is that by the Japanese economist R. Komiya [*Rev. Econ. and Statistics*, 29 (1959)]; Komiya's point of departure is a four-sector Mahalanobis model considerably later and more elaborate than the model considered here. In a letter written to me (dated 26 December 1961), Mahalanobis explains certain of the crudities of his two- and four-sector models. "I used [these models] mainly to get a clearer understanding of priorities in economic planning, and also to get some dimensional ideas of the pattern of investment for the second five-year plan. My exercises had an extremely practical aim; there was urgent need of making immediate decisions, and we simply did not have time to develop any refined theory."

Fay-Cooper Cole, Architect of Anthropology

Fay-Cooper Cole was born in Plainwell, Michigan, on 8 August 1881 and died suddenly after a brief illness in Santa Barbara, California, on 3 September 1961. He was not only an eminent anthropologist and authority on the peoples and cultures of Malaysia, and one of the founders of modern archeology, but also one of the great administrators and developers of men and institutions. In addition, he was a foremost interpreter of anthropology and social science to the general public, continuing in that role after his retirement as chairman of the department of anthropology of the University of Chicago.

Cole attended the University of Southern California briefly at the end of the century and then shifted to Northwestern University, where he graduated in 1903. After postgraduate work at the University of Chicago, he joined the staff of the Field Museum as an ethnologist under George A. Dorsey, who had an important influence upon his career in anthropology.

Public interest in the Far East had been aroused through the acquisition of the Philippines, and Cole soon had an opportunity to carry out a research and collecting assignment in northern Luzon. Before leaving for the field he spent a period in graduate work under Franz Boas at Columbia University and then went on to Berlin for further training under Felix von Luschan, the great German physical anthropologist. In the meantime he had married Mabel Cook, who was to participate fully in all his future activities; together they set off for a 2-year study of the Tinguian and neighboring tribes in the northern Philippines. The resulting papers and monographs, and the accompanying collections, established Cole as a highly promising scholar.

His first important monograph, *A Study of Tinguian Folklore*, served as his doctoral dissertation at Columbia in 1914 and was published in the Museum series the following year. In it he compared the life as reflected in myths and legends with that of the present day and found differences which we are only beginning to be able to explain. Publication of his major monograph, *The Tinguian: Social, Religious and Economic Life of a Philippine Tribe*, was delayed by World War I. The monograph appeared in 1922 and was an important contribution, both in its theoretical scope and in the completeness of its coverage of Tinguian life.

A second expedition to Mindanao, in 1910–12, to study the pagan Bukidnon and the tribes of Davao Gulf was cut short by malignant malaria which very nearly cost the Coles their lives. On their return Cole was made assistant curator of Malayan ethnology at the Field Museum. He prepared a monograph on *The Wild Tribes of the Davao District, Mindanao*, which was published in the Museum series, and he began to take an active role in public affairs and professional activities.

While on his last major expedition to Indonesia, in 1922–23, Cole received offers from both Northwestern University and the University of Chicago and was asked by each to develop a program in anthropology. On his return he taught at both institutions before deciding that his future lay with Chicago.

Coming to the University of Chicago in 1924 as an assistant professor in the department of sociology and anthro-