Table 1. Number and percentage of successes on each trial for the entire sample of experiment 1 and theoretical probabilities obtained by least squares. The $i$th trial occurs after the $i$th reinforcement; the theoretical probabilities were calculated with the least squares equation $P_{n+1} = 0.902 - 0.902(1 - 0.513)^n$.

| Trial | No. of successes | Percentage of successes | Theoretical percentages |
|---|---|---|---|
| 0 | 2 | 1.8 | 0 |
| 1 | 50 | 44.6 | 46.3 |
| 2 | 82 | 73.2 | 68.8 |
| 3 | 87 | 77.7 | 79.8 |
| 4 | 94 | 83.9 | 85.1 |
| 5 | 95 | 84.8 | 87.7 |
| 6 | 99 | 88.3 | 89.0 |
| 7 | 100 | 89.2 | 89.6 |
| 8 | 105 | 93.8 | 89.9 |
| 9 | 106 | 94.6 | 90.0 |

were connected so that this first and all subsequent shocks could be terminated only by 50 deg head position toward the opposite side. Subsequent shocks were started at 1-minute intervals. If, during any shock, the head was over 50 deg on the opposite side from the position which terminated shock, a failure was recorded for that trial. If a shock was terminated without error, a success was recorded. Ten trials were administered.

Experiments 2 and 3 were similar, except that they utilized 96 Wistar and Sprague Dawley female rats and 96 male Sprague Dawley rats, respectively. Twenty trials were administered in experiments 2 and 3.

The fact that our data are consistent with the model which assumes that each reinforcement adds an increment of probability of response which is a constant fraction of what potentially remains to be learned is exhibited in Table 1. Here a least squares fit is obtained for the data of experiment 1, yielding $a = 0.90$ and $\theta = 0.513$ for the parameters of the linear model. Similarly close fits may be obtained for experiments 2 and 3. On the other hand, it will be shown that our data are not consistent with an all-or-none model in which $\alpha$ is the probability that an animal which has not learned succeeds by chance, $\beta$ is the probability that an animal which has learned fails by chance, and $P$ is the constant probability of completely learning after any one reinforcement. By summing all possible ways that success can occur for the first time on the $n$th trial, we obtain the following expression for $Q_n$, the probability that the first success occurs on the $n$th trial:

$$\begin{aligned}Q_n = {} & P\beta^{n-1}(1-\beta) + (1-P)(1-\alpha)P\beta^{n-2}(1-\beta) + \dots \\ & + (1-P)^{n-1}(1-\alpha)^{n-1}P(1-\beta) + (1-P)^n(1-\alpha)^{n-1}\alpha \\ = {} & \frac{P(1-\beta)}{(1-P)(1-\alpha)-\beta} \left\{ [(1-P)(1-\alpha)]^n - \beta^n \right\} \\ & + (1-P)^n(1-\alpha)^{n-1}\alpha\end{aligned}$$

In particular,

$$Q_1 = P(1-\beta) + (1-P)\alpha$$

$$Q_2 = P(1-\beta)[\beta + (1-P)(1-\alpha)] + (1-P)^2(1-\alpha)\alpha$$

Since $Q_2$ is almost as large as $Q_1$ in each of the three experiments (see Table 2), there are no values of the parameters $\alpha$, $\beta$, and $P$ which provide a reasonably good fit to these points. It follows from the model equation that the first two points will be equal only if $\alpha$ is small and if the value of $P$ is close to that of $\beta$. This discrepancy between the model and the observed data is illustrated in Table 2, where the column 6 contains theoretical values of $Q_n$ calculated with $\alpha = 0.00$, $\beta = 0.06$ (the mean probability of failure on trials 10–19 in experiments 2 and 3), and $P = 0.50$ (the approximate value of $\theta$ in the least squares fit to group data in Table 1). This provided the best fit among several attempts performed with a desk calculator.

Table 2. Percentage of animals achieving their *first* success on each trial and theoretical values for all-or-none model. The $i$th trial occurs after the $i$th reinforcement; the theoretical values were calculated for $\alpha = 0.00$, $\beta = 0.06$ and $P = 0.50$.

| Trial | Experiment 1 | Experiment 2 | Experiment 3 | Mean | Theoretical values |
|---|---|---|---|---|---|
| 1 | 44.6 | 37.5 | 47.9 | 43.5 | 47.0 |
| 2 | 35.0 | 33.5 | 36.4 | 34.8 | 26.3 |
| 3 | 10.7 | 19.8 | 9.4 | 13.2 | 13.4 |
| 4 | 5.4 | 4.2 | 5.2 | 4.9 | 6.7 |
| 5 | 2.7 | 1.0 | 0.0 | 1.3 | 3.3 |
| 6 | 0.0 | 3.1 | | 1.0 | 1.7 |
| 7 | | 0.0 | | | 0.8 |

It may also be seen from Table 2 that there is an increase in the proportion of initial successes in rats which have previously failed. This is consistent with the linear model mentioned above. The constant proportions previously reported may be due to utilization of more complex human subjects, to a high initial probability $P_0$, and/or to a rate of conditioning $\theta$ so low as to have negligible effect over the short series of trials reported by Estes (*1*). The all-or-none model could perhaps describe our data if we assume that $P$ is not constant. This is reasonable in view of the shock escape situation which necessarily provides different amounts of shock on successive trials.

Corresponding to the modifications suggested in the preceding paragraph, more general models are being investigated. One of the major purposes of this note is to point out that the constant probability, all-or-none model, which has been widely advocated, is not appropriate to our experimental data (*6*).

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## Direct Determination of Density of Solids

*Abstract.* A method and apparatus for directly determining the density of solids is described. Essentially, density is determind by simultaneous measurement of the solid's volume by liquid displacement and of its weight by liquid head increase.

Standard laboratory methods to determine the density of an irregular insoluble solid are based on the measurement of liquid displacement upon total immersion to determine volume and on the use of a balance to determine weight. Procedures which can be adapted for field work utilize sink-float determinations in liquids of appropriate densities (*1*). A simple but effective way of determining the density of solids in one rapid operation with a portable unit has been devised in our laboratories.

When a solid is immersed in a liquid, it displaces a volume of liquid equal to its volume. A solid floating between two liquids, one heavier and one lighter than the solid, displaces the weight of heavy liquid equal to its weight corrected for buoyancy of the solid in the lighter liquid. Thus, if a solid is placed in a cylinder containing two liquids, one with a density greater than and one with

Table 1. Density determinations with special apparatus compared to the same determinations from conventional weight–liquid displacement measurements. Values are the averages of six determinations; the standard deviation (S. D.) is that of an individual measurement.

| Special apparatus | | Conventional measurement | |
|---|---|---|---|
| Density (g/ml) | S. D. (g/ml) | Density (g/ml) | S. D. (g/ml) |
| *Stainless steel* | | | |
| 8.05 | ±0.10 | 7.82 | ±0.07 |
| *Aluminum* | | | |
| 2.802 | ± .058 | 2.771 | ± .047 |
| *Plastic-coated lead* | | | |
| 10.88 | ± .39 | 11.22 | ± .15 |
| *Small stones (70) simultaneously* | | | |
| 2.651 | ± .056 | 2.702 | ± .026 |

a density less than that of the solid, the total liquid level will be increased by an amount equivalent to the volume of the solid immersed, and the level of the denser liquid will be raised by an amount equivalent to the corrected weight of the solid. A broad range of densities may be measured by using water for the lighter liquid and mercury for the denser liquid. However, the increase in level of the heavy liquid in the cylinder is small when many common solids are immersed. An apparatus which amplifies the weight-indicating excursion so that it may be read with greater precision is shown in Fig. 1. The apparatus consists of a large cylinder with a small attached side arm. The side arm has a reduced diameter in its upper section into which only the lighter liquid is allowed to rise. All cylinders have uniform cross-sections.

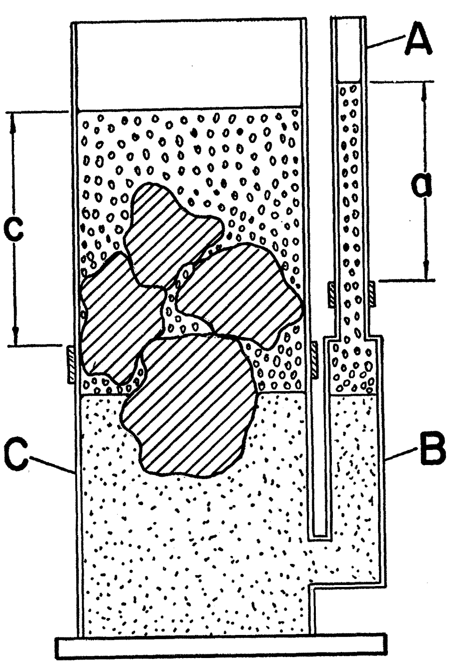


Fig. 1. Density gage. The tops of the clips on the cylinders designate the positions of the tops of the liquids before the solid was immersed. See text for explanation of diagram.

To make a density determination, the solid is introduced into the large cylinder. Its volume is measured by the increases in the two liquid levels. The increase in weight in the large cylinder from the added solid is balanced by an increase in liquid head in the smaller cylinder. The displacement of the level of the heavier liquid in the larger lower section of the side arm is amplified by a much greater displacement of the level of the lighter liquid in the top section of the side arm. Clips are shown designating the levels of the liquids before introduction of the solid.

In Fig. 1, $A$, $B$, and $C$ represent the cross-sectional areas of the top side arm, the bottom side arm, and the main cylinder, respectively; and $a$ and $c$ represent the increases in liquid levels in the side arm and the main cylinder, respectively. The volume $V$ of the solid is

$$V = aA + cC \qquad (1)$$

The increased pressure in the large cylinder is equal to the weight of the solid per unit cross section of the large cylinder less the pressure from the heavy liquid pushed into the side arm. If the densities of the lighter and heavier liquids are represented by $d$ and $D$ respectively, and if the ratios of the cylinder diameters are defined as $R \equiv A/B$ and $r \equiv A/C$, then the weight $W$ of the solid is

$$W = aC\,[D(r+R) + d(1-R)] \qquad (2)$$

and the density of the solid is

$$\rho = \frac{W}{V} = \frac{D(r+R) + d(1-R)}{r + \frac{c}{a}} \qquad (3)$$

With any apparatus and pair of fluids, $\rho$ is a function of only $c/a$, the ratio of liquid-level changes. The calculation may be easily made from the formula; or the density may be read on a simple nomograph constructed with the level changes on parallel axes and the density on the diagonal axis. The introduced solid may be in several pieces; and it need not be uniform in density. The determination furnishes the mean density of all the material introduced.

After the measurement is made, the liquids may be poured into a container through a sieve that will retain the solids. The liquids may then be reintroduced into the apparatus and used for another measurement. The appropriate amount of each fluid may be empirically determined so that both may be rapidly poured from a single container kept for that purpose into the larger cylinder, the lighter liquid first. Standard solids may be introduced to check for density changes of the liquids from dissolved substances. A convenient apparatus used in our laboratories has cylinder diameters of 2, 6, and 16 mm, respectively, and uses 8 ml of mercury. With this apparatus density determinations were made on samples of stainless steel, aluminum, plastic-coated lead, and a heterogeneous group of 70 small pebbles with an average weight of 235 mg each. The same samples were weighed and their volumes were determined by visual estimation of water displacement in a 25-ml graduated cylinder. All samples had volumes of from 4 to 8 ml. The results are shown in Table 1. A major cause of the differences between the results obtained by the two techniques may well be nonuniformity of bore in the lower portion of the side arm which was made from stock Pyrex tubing.

The method described here does not increase the accuracy of density measurements, but in many instances it is more convenient than conventional methods. The apparatus needs little, if any, adjustment. The speed and simplicity of its operation and the ease with which it can be transported have made this apparatus especially useful in the field.

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