most of the abnormal development which follows exposure to oxygen-deficient atmospheres.

The significance of these data may extend beyond that of interest in chick embryos subjected to hypoxia. Vascular anomalies have been described in mouse fetuses subjected to anoxia (*7*). A comparable syndrome of edema, subcutaneous blisters, hematomas, and maldevelopment of adjacent tissues has been described in mammalian and avian embryos in deficiencies of pantothenic and linoleic acids, after administration of the redox dye trypan blue, as well as in Little and Bagg's famous mutant strain of mice (*8*). Is it not possible that some of these agents (and others too) and hypoxia have a common mode of action—that is, an interference with oxidative metabolism and the accumulation of metabolites?

CASIMER T. GRABOWSKI

Department of Zoology, University of Miami, Coral Gables, Florida

References and Notes

1. The studies reported here were supported by a grant from the Easter Seal Research Foundation. The investigation was initiated at the University of Pittsburgh and continued at the University of Miami. I acknowledge the technical assistance of J. Beck, R. Moyers, and M. Milan in various phases of this program.
2. C. T. Grabowski and J. Paar, *Am. J. Anat.* **103**, 313 (1958).
3. ———, *Anat. Record* **133**, 281 (1959).
4. ———, *Am. J. Anat.*, in press.
5. ———, *Anat. Record* **136**, 200 (1960); ———, unpublished.
6. W. W. Umbreit *et al.*, *Manometric Techniques* (Burgess, Minneapolis, ed. 3, 1957), p. 275.
7. T. H. Ingalls and C. G. Tedeschi, *Am. J. Obstet. Gynecol.* **71**, 16 (1956).
8. A. Giroud *et al.*, *J. Embryol. Exptl. Morphol.* **3**, 1 (1955); M. Martinet, *Ann. med.* (Paris) **53**, 286 (1952); G. M. Plagens, *J. Morphol.* **55**, 151 (1933); C. H. Waddington and T. C. Carter, *J. Embryol. Exptl. Morphol.* **1**, 167 (1953).

26 July 1961

Conformal Transformations and Space Travel

Abstract. Conformal transformations are applied to the motion of a space ship experiencing a constant acceleration. The role of proper time is interpreted in terms of atomic periods, and the relationship between the conformal transformations and the general theory of relativity is clarified.

Conformal transformations, connecting an inertial observer to a non-inertial observer experiencing a constant acceleration, were developed by Bateman (*1*), Cunningham (*2*), and Page (*3*). Further interesting work on these transformations has been carried out by Engstrom and Zorn (*4*), Robertson (*5*), Hill (*6*), and others.

This report applies the conformal transformations to the problem of space travel and establishes the equivalence of the resulting formulas with those obtained by standard relativistic methods. It also discusses the physical significance of proper time in space travel and clarifies the relationship between the conformal transformations and the general theory of relativity.

An observer in an arbitrary frame of reference in general experiences two types of gravitational fields, which I shall call the *real* and the *apparent* gravitational fields. A real gravitational field is produced by bodies consisting of all types of elementary particles, while an apparent gravitational field is produced by the acceleration of the observer's frame of reference. According to Einstein's ideas, there is a certain equivalence between the two types of gravitational fields, but this does not mean that they are indistinguishable. In fact, in nature it is always possible to distinguish between the real and the apparent gravitational fields. For instance, the gravitational field around the earth can possibly be produced either by the matter contained in the earth or by an outward acceleration of the surface of the earth. Since we know from observation that the surface of the earth is not exploding, we must conclude that the gravitational field around the earth is a real gravitational field. It is also evident that a real gravitational field must vanish at an infinite distance from all gravitating bodies, while an apparent gravitational field can exist even at infinity. A frame of reference in which the apparent gravitational field vanishes everywhere is an inertial frame of reference.

We can easily pass over from one inertial frame of reference to another by means of the Lorentz transformations, but a passage from an inertial frame of reference to a non-inertial frame of reference is not so simple. A particularly simple example of a non-inertial frame of reference is one in which the observer experiences a constant acceleration (*7*). The relationship between such a non-inertial frame of reference and an inertial frame of reference can be stated as follows:

Let there be an inertial frame of reference S and a non-inertial frame of reference S' which is moving with respect to S along the x-axis in such a way that the acceleration experienced by an observer at the origin of S' has the constant value α. Further, let the coordinates of any event be denoted by (x, y, z, t) in S and by $(x', y', z' t')$ in S', and let the origins of the two frames of reference meet at rest at $t = t' = 0$. Then, the conformal transformations connecting the two frames of reference are (*3*) as follows (Eq. 1):

$$X' = \frac{X}{X^2 + Y^2 + Z^2 - c^2T^2},$$
$$X = \frac{X'}{X'^2 + Y'^2 + Z'^2 - c^2T'^2},$$
$$Y' = \frac{Y}{X^2 + Y^2 + Z^2 - c^2T^2},$$
$$Y = \frac{Y'}{X'^2 + Y'^2 + Z'^2 - c^2T'^2},$$
$$Z' = \frac{Z}{X^2 + Y^2 + Z^2 - c^2T^2},$$
$$Z = \frac{Z'}{X'^2 + Y'^2 + Z'^2 - c^2T'^2},$$
$$T' = \frac{T}{X^2 + Y^2 + Z^2 - c^2T^2},$$
$$T = \frac{T'}{X'^2 + Y'^2 + Z'^2 - c^2T'^2},$$

where (Eq. 2)

$$X = 1 + \frac{\alpha x}{2c^2}, \qquad Y = \frac{\alpha y}{2c^2},$$
$$Z = \frac{\alpha z}{2c^2}, \qquad T = \frac{\alpha t}{2c^2},$$
$$X' = 1 - \frac{\alpha x'}{2c^2}, \qquad Y' = \frac{\alpha y'}{2c^2},$$
$$Z' = \frac{\alpha z'}{2c^2}, \qquad T' = \frac{\alpha t'}{2c^2}.$$

I shall apply the conformal transformations to the motion of a space ship whose occupants experience a constant acceleration α. Let the space ship be located at the origin of the frame of reference S'. Then, putting $x' = y' = z' = 0$ in Eq. 1, we have

$$1 + \frac{\alpha x}{2c^2} = \left(1 - \frac{\alpha^2 t'^2}{4c^2}\right)^{-1}, \quad (3)$$
$$t = t' \left(1 - \frac{\alpha^2 t'^2}{4c^2}\right)^{-1}, \quad (4)$$

which give us

$$x = \frac{\alpha t'^2}{2}\left(1 - \frac{\alpha^2 t'^2}{4c^2}\right)^{-1}, \quad (5)$$

or

$$x = \frac{c^2}{\alpha}\left[\left(1 + \frac{\alpha^2 t^2}{c^2}\right)^{\frac{1}{2}} - 1\right], \quad (6)$$

where x is the distance traveled by the space ship with respect to S, and t and t' are the times of travel according to the observers in S and S' respectively.

Differentiating Eq. 6 with respect to t, we obtain for the velocity of the space ship relative to the frame of reference S

$$v = \alpha t \left(1 + \frac{\alpha^2 t^2}{c^2}\right)^{-\frac{1}{2}}, \quad (7)$$

which can also be expressed as

$$v = \alpha t' \left(1 + \frac{\alpha^2 t'^2}{4c^2}\right)^{-1} \qquad (8)$$

or

$$v = (2\alpha x)^{\frac{1}{2}} \left(1 + \frac{\alpha x}{2c^2}\right)^{\frac{1}{2}} \left(1 + \frac{\alpha x}{c^2}\right)^{-1} \qquad (9)$$

It follows from Eq. 9 that the kinetic energy of the space ship relative to S is

$$E = m_0 c^2 \left[\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} - 1\right]$$
$$= m_0 \alpha x, \qquad (10)$$

where m_0 is the rest mass of the space ship.

We shall denote the proper time of the observer in the space ship as τ, and find the relationship between t' and τ without involving the arguments of the general theory of relativity, which will also throw new light on the physical significance of the proper time.

It seems reasonable to assume that the most significant units of time for us are the periods of atoms we are made of. For, if the periods of atoms were to change, the rates of chemical reactions would also change, and the process of aging within our bodies would proceed at a different rate. Thus, in order to measure how much different observers have aged between two given events, each observer must assign the same value to the periods of atoms in his own frame of reference. For simplicity, let us confine our attention to the period of a hydrogen atom, which according to the semiclassical Bohr model is given in an inertial frame of reference by

$$T_0 = \frac{2\pi\hbar}{mc^2} \left(\frac{4\pi c\hbar}{e^2}\right)^2. \qquad (11)$$

Now the basic equations of electrodynamics are invariant under the conformal transformations, but this invariance is obtained by assuming that the rest mass m of a particle transforms into a different rest mass m'. Therefore, to the occupant of the space ship the period of a hydrogen atom in the space ship is given by

$$T'_0 = \frac{2\pi\hbar}{m'c^2} \left(\frac{4\pi c\hbar}{e^2}\right)^2. \qquad (12)$$

We must replace t' by a new time variable τ such that in terms of τ the period of the hydrogen atom in the space ship acquires the value shown in Eq. 11 for the occupant of the space ship. This can be achieved by defining τ by the relation

$$d\tau = \frac{m'}{m}\, dt'. \qquad (13)$$

Since under the conformal transformations the rest mass transforms as (*3*)

$$m = m' (X'^2 + Y'^2 + Z'^2 - c^2T'^2), \qquad (14)$$

we obtain

$$d\tau = \frac{dt'}{X'^2 + Y'^2 + Z'^2 - c^2T'^2} \qquad (15)$$

or, putting $x' = y' = z' = 0$ for the space ship,

$$d\tau = \frac{dt'}{1 - \dfrac{\alpha^2 t'^2}{4c^2}}. \qquad (16)$$

We can obtain a similar relation for the proper distance for the occupant of the space ship by requiring that each observer assign the same value to the Bohr radius of the hydrogen atom in his own frame of reference.

By integrating Eq. 16 we get

$$\frac{\alpha t'}{2c} = \tanh \frac{\alpha\tau}{2c}, \qquad (17)$$

so that Eqs. 4, 5, and 8 can be expressed as

$$t = \frac{c}{\alpha} \sinh \frac{\alpha\tau}{c}, \qquad (18)$$

$$x = \frac{c^2}{\alpha}\left(\cosh \frac{\alpha\tau}{c} - 1\right), \qquad (19)$$

$$v = c \tanh \frac{\alpha\tau}{c}, \qquad (20)$$

which are identical with the usual relativistic results (*8*). Further, using Eqs. 4 and 8, we obtain from Eq. 16,

$$d\tau = \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} dt, \qquad (21)$$

which again is in agreement with the well-known result that the relationship between $d\tau$ and dt is the same as if the space ship were moving with a constant velocity v with respect to S.

It is also interesting to note that we can derive from Eq. 1 the relations

$$(X^2 + Y^2 + Z^2 - c^2T^2)(X'^2 + Y'^2 + Z'^2 - c^2T'^2) = 1 \qquad (22)$$

and

$$\frac{dX^2 + dY^2 + dZ^2 - c^2dT^2}{X^2 + Y^2 + Z^2 - c^2T^2} = \frac{dX'^2 + dY'^2 + dZ'^2 - c^2dT'^2}{X'^2 + Y'^2 + Z'^2 - c^2T'^2} \qquad (23)$$

whence it follows that

$$dx^2 + dy^2 + dz^2 - c^2dt^2 = \frac{dx'^2 + dy'^2 + dz'^2 - c^2dt'^2}{(X'^2 + Y'^2 + Z'^2 - c^2T'^2)^2}. \qquad (24)$$

According to the general theory of relativity, the above relation can be interpreted as meaning that while the metrical tensor has the Minkowskian values in the frame of reference S, it has quite different values in the frame of reference S'. In particular, the g_{44} component of the metrical tensor in S' is

$$g_{44} = \frac{1}{(X'^2 + Y'^2 + Z'^2 - c^2T'^2)^2}, \qquad (25)$$

so that the proper time in S' is given by Eq. 26:

$$d\tau = (g_{44})^{\frac{1}{2}}\, dt'$$
$$= \frac{dt'}{X'^2 + Y'^2 + Z'^2 - c^2T'^2}, \qquad (26)$$

which agrees with Eq. 15. Thus, the conformal transformations can be regarded as a special case of the general relativistic transformations of the space and time coordinates. We have also shown that the physical definition of proper time, in terms of atomic periods, is in complete agreement with the ideas of the general theory of relativity (*9*).

SURAJ N. GUPTA
Department of Physics, Wayne State University, Detroit, Michigan

References and Notes

- H. Bateman, *Proc. London Math. Soc.* **8**, 223, 469 (1910).
- E. Cunningham, *ibid.* **8**, 77 (1910).
- L. Page, *Phys. Rev.* **49**, 254 (193 L. Page and N. I. Adams, *ibid.* **49**, 466 (1936); ———, *Electrodynamics* (Van Nostrand, New York, 1940).
- H. T. Engstrom and M. Zorn, *Phys. Rev.* **49**, 701 (1936).
- H. P. Robertson, *ibid.* **49**, 755 (1936).
- E. L. Hill, *ibid.* **67**, 358 (1945).
7. The acceleration experienced by an observer means the acceleration of the observer in an inertial frame of reference in which he is instantaneously at rest.
8. C. Møller, *Theory of Relativity* (Oxford Univ. Press, New York, 1952).

9. This work was supported in part by the National Science Foundation.

8 May 1961

Persistence of a Photosynthetic Rhythm in Enucleated Acetabularia

Abstract. The unicellular alga *Acetabularia* was found to show a diurnal rhythm in photosynthesis. This rhythm continued for at least three cycles in constant light and temperature, and hence can be considered endogenous. Plants from which the nucleus had been removed by severing the basal rhizoids showed no modification in the photosynthetic rhythm over a number of cycles. The nucleus is, therefore, not immediately essential for the maintenance of rhythmicity in *Acetabularia.* Conversely, a mechanism for sustaining time-keeping must exist in the cytoplasm.

Endogenous diurnal rhythmicity, well known in a wide variety of multicellular plants and animals, has also been demonstrated in populations of unicellular organisms, for example, *Euglena* (*1*),