SCIENCE

Lifetimes of Orbiting Dipoles

Sunlight pressure should limit the average orbital lifetime of West Ford dipoles to about 7 years.

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The West Ford experiment (1) is concerned with creating a belt of orbiting dipoles for study by radio-wave propagation and communication scientists. Some 75 pounds of tiny hairlike copper dipoles are to be injected into a circular polar orbit about 3800 km above the earth's surface; the resonant frequency of the dipoles will be near 8000 Mcy/sec. Radio-frequency equipment located near San Francisco and Boston will be used to conduct communications tests and to study the physical and electrical characteristics of the belt by means of monostatic and bistatic radar experiments.

At various times, apprehension has been expressed concerning several possible deleterious effects which might result from such a dipole belt. Published quantitative investigation (2, 3) of these possible effects for this experimental belt show such concern to be groundless, since the individual dipoles are small and light, and since the total amount of material is modest. Because of the dispersion of the belt, even these small effects will be reduced by about an order of magnitude in a few years.

However, to allay concern over possible interference with scientific measurements which might be made perhaps decades in the future, the experimental dipole belt is to be formed in an orbit with a relatively short lifetime.

Since the belt must be placed at a high altitude, the cumulative effect of atmospheric drag cannot be counted upon to remove the belt from orbit in a reasonably short time. However, by using dipoles with a high area-to-mass ratio and by choosing a resonant orbit, solar radiation pressure can be utilized to drive the belt downward into the dense regions of the earth's atmosphere. The influence of sunlight pressure on orbiting objects of such high area-tomass ratio has been the subject of considerable analytical study (4, 5), and detailed quantitative verification has been obtained from observations of Echo I (5).

When producing a desirably short belt lifetime, sunlight pressure also has the undesirable effect of causing the belt to disperse because of the random tumbling patterns of the dipoles. Generally speaking, the smaller the dispersion of the belt after a given period of time, the longer will be its ultimate lifetime. Since the propagation and communication studies require an adequate period during which the belt will remain relatively undispersed, a compromise in lifetime must be made. The values chosen for the West Ford experiment appear to meet both lifetime criteria reasonably well: dipoles with maximum cross-sectional area-to-mass ratios of about 55 cm²/g injected into circular polar orbits at 3800-km altitude near the time of the winter solstice can be expected to have an average orbital lifetime of about 7 years and a useful experimental lifetime of about 2 years.

In the remainder of this article we give a general description of sunlight pressure perturbations with particular emphasis on resonant orbits, and we explain in some detail the methods used and the results obtained in calculations for the West Ford dipole belt.

Sunlight Pressure Perturbations

The effects of sunlight pressure will be of paramount importance on the orbits of earth satellites with high area-tomass ratios (like the West Ford dipoles). Because of widespread unfamiliarity with this subject, we present a qualitative description of these effects and then explain the concept of resonant and nonresonant orbits. For most of the discussion we assume that the sun lies in the plane of the satellite's orbit; however, our arguments will apply equally well for the general case. The only difference is that our statements should then be interpreted as referring to the component of the radiation force lying in the orbit plane. (The component of the force perpendicular to this plane has a much smaller effect on the orbit.)

Figure 1 illustrates the action of the sun's rays on an initially circular orbit. While the radiation exerts pressure continuously (except when the satellite passes through the earth's shadow), we consider its effects only at the two points A and B. At point A, the satellite is moving away from the sun; hence the radiation pressure causes a slight increase in the velocity of the satellite which, at the same time, increases the orbital energy of the satellite and therefore its major axis. The result would be an orbit like the dotted one passing through A. On the other hand, when the satellite is near point B, the sun's rays push against the direction of motion of the satellite, thus slowing it somewhat. This causes a decrease in the major axis, and tends to produce an orbit like the dotted one passing through B. Similarly, one can calculate the effect

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of sunlight pressure on all points in the orbit; the result is simply a displacement of the entire orbit in a direction perpendicular to that of the radiation force (as indicated in Fig. 1). The magnitude of this displacement is directly proportional to the area-to-mass ratio of the satellite.

If the earth and sun were stationary with respect to one another, and if no other perturbing forces acted on the system, then the radiation pressure would cause a monotonic decrease in the perigee height of an initially circular orbit, until, finally, the satellite collided with the surface of the earth. This simple conclusion is invalidated mainly by the two following facts:

1) The earth rotates about the sun, causing the direction of sunlight pressure on the satellite to change, and the sunlight therefore to push the orbit in a new direction. Instead of a monotonic decrease in perigee height, one now finds that perigee height oscillates. After 6 months, for example, the sun's rays will push in the opposite direction, causing a displacement of the orbit opposite to the original one.

2) The earth's equatorial bulge causes the orbit to rotate in its plane. Again, since sunlight pressure displaces the orbit perpendicular to the earth-sun line, perigee height will oscillate rather than decrease monotonically. For example, when (because of the rotation of the orbit induced by the bulge) the position of perigee has rotated 180° , the displacement of the orbit due to solar pressure will increase the perigee height instead of decreasing it as originally.

For certain orbits (which we call resonant orbits of the first type) the effects mentioned separately in the two preceding paragraphs will cancel each other. That is, the rotation of the orbit in its plane (due to the earth's bulge) will just compensate for the change in direction of the sun's rays (because of the earth's rotation about the sun). In such cases the simple situation described



Fig. 1. Displacement of a satellite orbit by sunlight pressure.

earlier will again obtain: The perigee height of initially circular orbits will decrease monotonically, thus enabling us to predict lifetimes with reasonable accuracy irrespective of our inadequate knowledge of air densities. For noncircular initial orbits the perigee height may first increase and then decrease monotonically, depending on the initial orientation of the apsidal line with respect to the sun's rays (6). The lifetimes of satellites in these resonant orbits are inversely proportional to their area-to-mass ratios.

For polar and near-polar orbits it is also possible to have a somewhat different type of resonance. For this (second) type, the rotation of the orbit in its plane (due to the earth's bulge) is equal in magnitude but opposite in direction to the rotation of the component of the earth-sun line in the orbit plane. Figure 2 illustrates such a resonance; it is characterized by seasonal, small amplitude reversals in direction of the changes in perigee height (7). The second type is obtained from the first by a reversal in the direction of satellite motion. The most pronounced difference between the two occurs for the pair of ascending node (Ω) values 0° (type 2) and 180° (type 1). As Ω increases by 90° the distinction between the types gradually disappears.

What orbits, one might ask, will be resonant? Since the rate of rotation of the earth about the sun is fixed, we can only adjust the rotation rate of the satellite orbit in its plane by suitably choosing the initial orbit. This latter rate depends on the satellite inclination angle, on its major axis, and (weakly) on its eccentricity. The explicit formulas for the mean rotation rate of the orbit in its plane due to the bulge (ω_b) , and for the mean rotation rate of the plane itself (Ω_b) were given in a previous publication (4). For a polar orbit we find

$$\dot{\Omega}_b \approx 0; \ \dot{\omega}_b \approx - \frac{5}{a^{7/2}(1-e^2)^2} \ \mathrm{deg/day}, \ \ (1)$$

where a (the semimajor axis) is expressed in earth radii.

If we restrict ourselves to initially circular polar orbits, there is only one value of a for which $\dot{\omega}_b$ will equal the rate of rotation of the earth about the sun ($\dot{\beta}$). The altitude corresponding to this a is 3700 km. Actually, there exists a small band of initial orbits near this mean altitude which behave like resonant orbits. The bandwidth depends on the area-to-mass ratio (A/M) of the satellite; in general, it increases with in-

SCIENCE, VOL. 134

creasing A/M. Thus, for an A/M value of 50 cm²/g, it is approximately 300 km, whereas with A/M = 250 cm²/g the width is about 5000 km.

This band is, however, not centered at a 3700-km altitude: Since e increases monotonically in a resonant orbit, we see from Eq. 1 that the magnitude of $\dot{\omega}_b$ will also increase monotonically (8). Hence, the magnitude of $\dot{\omega}_b$ will not remain equal to $\dot{\beta}$ and the resonance condition will not continue to hold precisely. If an initial altitude somewhat greater than 3700 km is chosen, then the value of $\dot{\omega}_b$ (averaged over the lifetime of the satellite) will be closer in magnitude to $\dot{\beta}$. For this reason, the band of initial polar orbits, each of which behaves most like a truly resonant orbit, is centered at an altitude above 3700 km.

For all mean altitudes that lie outside the narrow resonance band, we see that $\dot{\omega}_b$ will be noticeably different from $\dot{\beta}$. Hence, the effects 1 and 2, mentioned above, will not cancel, and perigee height will oscillate. The amplitude of this oscillation is (roughly) inversely proportional to the difference in the magnitudes of $\dot{\beta}$ and $\dot{\omega}_b$. It is, of course, also proportional to the value of A/M.

Effective Area of a Dipole

From the above discussion, we see that the lifetime of a West Ford dipole (that is, the time span during which it remains in orbit) is critically dependent on the initial orbital altitude and inclination angle. This lifetime also depends on the value of A/M appropriate for the dipoles. More specifically, the area of a dipole when projected on a plane perpendicular to the earth-sun line is the relevant value of A, the calculation of which is complicated by the tumbling of the dipoles: Theoretical and experimental studies conducted under conditions closely simulating those expected to be encountered in space indicate that about 95 percent of the dipoles will be dispensed with tumbling rates exceeding 2 rev/sec (9). The tumbling periods of

the dipoles, compared with their orbital periods, will obviously be quite small. In calculating an effective projected area, we therefore average over a tumbling period. Simple geometrical considerations show that the maximum A (A_{\max}) results when the angular momentum vector (10) of the dipole is oriented along the earth-sun line; parallel reasoning indicates that the minimum value of A (A_{min}) occurs when this vector has a perpendicular orientation. The value A_{max} is given by the product of the dipole diameter and length; $A_{\min} = (2/\pi) \quad A_{\max}$. (For simplicity, we are restricting this discussion to dipoles which absorb sunlight and reradiate thermally. The lifetime results for specularly reflecting dipoles are not appreciably different.)

The effective projected area (A_{eff}) of a given dipole will not remain constant during its lifetime: First, A_{eff} will change because of the rotation of the earth-sun line. Second, there are various torques (such as that due to the inhomogeniety of the earth's gravita-



Fig. 2. Perigee altitude and mean altitude versus time for a resonant orbit.



Fig. 3. Comparison of experimental and theoretical values for orbital elements of Echo I.

tional field and that due to the action of solar radiation on irregularities in the shape and/or surface conditions of the dipoles) which cause the dipole angular momentum vector to precess. Third, there are damping torques caused by interaction with the atmosphere and with the earth's magnetic field (that is, eddy current torques and torques due to magnetic impurities in the dipoles). At altitudes of several thousand kilometers these damping torques are so small that the tumbling rates of the dipoles would be virtually unaffected.

In addition, we find that impulse torques due to collisions with micrometeoroids are significant in changing (actually, statistically increasing) the angular momentum of the dipoles (11). Considering all of these factors in combination, we conclude that the dipoles will continue to tumble rapidly compared with an orbital period until they penetrate the denser portions of the earth's atmosphere. For any individual dipole, $A_{\rm eff}$ will vary between $A_{\rm max}$ and A_{\min} throughout its life; and, since the lifetime is determined mainly by sunlight pressure, we can determine a lower bound on it by assuming that A_{eff} maintains the value A_{\max} . Similarly, an upper bound on the lifetime for a given initial orbit can be obtained by fixing A_{eff} at the value A_{\min} . (Strictly speaking, these bounds are rigorously valid only for resonant and near-resonant orbits of the first type; the situation is more complicated for a general orbit.)

Prediction of Dipole Orbits

To predict the long-term behavior of a dipole orbit we proceed as follows: Using the standard differential equations relating the rates of change of the orbital elements to the perturbing accelerations (12), we integrate analytically over an orbital period, keeping the elements constant during the integration. The new values of the elements are then used to determine the changes in the elements during the next orbital period, and so forth. This iteration procedure, carried out by an I.B.M. 7090 digital computer, yields good approximations for the long-period and secular contributions to the element changes. We have included in the program perturbing forces due to the second through the fifth harmonic of the earth's gravitational field, the action of direct solar radiation pressure (including the effects of the earth's shadow), atmospheric drag, lunar and solar gravitational effects, and the pressure of sunlight reflected from the earth.

Two types of error are, unfortunately, unavoidable in these theoretical predictions. First, errors are introduced by deficiencies in our theoretical model: For example, the physical constants associated with any perturbation can never be known precisely. Second, our solution to the equations of motion is inexact: The mathematical approximations used will cause inaccuracies in prediction which will be greater the longer the time interval over which the predictions are made.

Comparison with Orbit of Echo I

To check the adequacy of our theoretical method, we compared our results for the orbit of Echo I (area-to-mass ratio approximately 100 cm²/g, mean altitude 1600 km, inclination 47°) with those determined by the Smithsonian Astrophysical Observatory from Baker-Nunn photographic data. Using the average air density as determined experimentally, and allowing the area-tomass ratio to increase (to take into account in a "best fit" manner the change in reflection properties and the 20-percent loss of mass due to the gradual escape of sublimating powders through punctures in the balloon), we can match the experimental values of all the orbital elements quite well from the time of launch to the present. As an illustration of the good agreement obtained between our theoretical determination and the observed orbit, we reproduce in Fig. 3 a graph from an earlier paper (5) which shows a comparison for the time variations of eccentricity and of geocentric perigee distance for the Echo satellite from time of launch through 1 March 1961. (To enlarge the scale, both the experimental and theoretical values were plotted as residuals from a polynomial expression obtained by the method of least squares.) It should be emphasized that in our calculations we made use only of the initial (observed) values of the orbital elements; all subsequent values were calculated theoretically in the above-described manner. These results give us confidence that our theoretical procedure is reasonably accurate over long time intervals and that we have not neglected any important perturbations.

Influence of Air Density

It might be thought that an accurate knowledge of air densities is essential to an accurate prediction of the dipole belt's lifetime. And, in general, this is true. However, if the belt is in a resonant (or near-resonant) orbit, its lifetime is almost independent of air density since in these cases radiation pressure alone is responsible for driving the orbit perigee toward the earth's surface. In Fig. 2 this insensitivity is illustrated for a resonant polar orbit of the second type. Here we have chosen three different air density models. In the first, the density (ρ) is consistent with the data determined from the orbit of Echo I and with the 1959 ARDC Model Atmosphere; in the second, ρ is everywhere 10 times greater than it is in the first model; and in the third, the density is set equal to 0. We see that these extreme models yield lifetimes differing by less than 10 percent.

For a belt in an orbit sufficiently far removed from a resonance, radiation pressure causes long-period oscillations in perigee height with an amplitude proportional to the area-to-mass ratio of the satellite. For such orbits, the lifetime does depend significantly on air





density. We can make estimates of the effect of density on lifetime for these nonresonant high-altitude orbits by using the following formula:

$$\Delta a = -2\pi C_D (A/M) a^2 \rho \qquad (2)$$

Here, Δa is the decrease in mean altitude per revolution, a is the semimajor axis of the orbit, C_{P} is the drag coefficient of the satellite, A/M is an average cross-sectional area-to-mass ratio appropriate for air drag, and ρ is the air density. As an example of the use of this formula, consider a dipole orbit whose perigee altitude is in the vicinity of 1600 km. From the Echo data, it has been established that a representative air density at 1600 km is approximately 10^{-18} g/cm³ (5, 13). Therefore, by assuming C_{P} to be 5 (to take into account in an approximate manner both

charge and neutral drag), we see that the mean altitude will decrease at a rate of less than 50 km/yr. When the oscillations caused by sunlight pressure bring the orbit perigee to altitudes lower than 1600 km, similar estimates can be made for the rate of decrease of mean altitude in these regions. Crude estimates of the lifetime of dipoles in clearly nonresonant orbits can be constructed by this method. The minimum air density likely to be found at altitudes of several thousand kilometers is that of interplanetary space, which is estimated to be several orders of magnitude less than 10^{-18} g/cm³. Using this smallest possible air density, we can obtain estimates which are rigorous upper bounds on belt lifetimes. (These will be quite far above least upper bounds, even for nonresonant orbits.)



Orbital Lifetime

As stated earlier, the West Ford test belt will be placed in a circular polar orbit at an altitude close to 3800 km; the dipoles to be used have a value of $A_{\rm max}/M$ of about 55 cm²/g and hence a value of A_{\min}/M of about 35 cm²/g. To illustrate the lifetimes of dipole belts in this region we have constructed the contour curves shown in Fig. 4. Before discussing these results in detail, we note that it is impossible to present succinctly the detailed dependence of lifetime on all the relevant parameters. Therefore we are concerned mainly with showing the behavior of lifetime with respect to the most critical parameters, altitude and inclination angle. Thus all the data presented refer to initially circular orbits, for the test belt is expected to be nearly circular and our results for lifetime are insensitive to small changes in eccentricity.

For the planned West Ford belt (and hence for all of Fig. 4) we have chosen the winter solstice as a representative initial day and have assumed that the launch will occur at dawn in a southerly direction. The initial polar resonance is then of the second type, and yields the *greatest* dipole lifetimes. Were launch to occur at dusk, for example, the resonance would initially be of the first type, resulting in lifetimes several years less (14).

In particular, we show lifetime contours for dipoles which maintain an $A_{\rm eff}/M$ value of 50 cm²/g. The numbers accompanying the initial orbits lying inside the contours give the corresponding lifetimes in years; outside the outer lifetime contour, the initial orbit conditions are effectively nonresonant. In determining lifetimes for these latter conditions, air density becomes increasingly important; in fact, lifetimes considerably greater than those explicitly indicated in Fig. 4 can sometimes result for initial orbits lying just outside these contours. The numbers accompanying these latter orbits give the minimum perigee altitudes encountered during the first 8 years after launch.

For launch at the time of the winter solstice, the orbit of the planned test belt will lie well within the estimated 8-year lifetime contour. The predicted lifetime, which approximates the average to be expected for dipoles in the test belt, is seen to be about 7 years. For a dipole which maintains an $A_{\rm eff}/M$

SCIENCE, VOL. 134

value of about 35 cm^2/g , we obtain the maximum lifetime. Since, within the resonance region, dipole lifetime is inversely proportional to the average value of $A_{\rm eff}/M$, a maximum lifetime of about 10 years can be expected (15). We emphasize, however, that statistically it is overwhelmingly unlikely for a dipole to spend more than a small fraction of its life with the necessary orientation to yield a value of $A_{\text{eff}} = A_{\min}$. Hence only a completely negligible number of dipoles could be expected to have lifetimes corresponding to $A_{\rm eff}/M$ remaining at 35 cm²/g over a period of years.

From our earlier discussion on prediction, we can infer that the exact positions of the contours shown in Fig. 4 are not to be taken too literally (16). Lifetimes for dipoles whose orbits are near the center of the resonance region are probably quite reliable; but for orbits near the border of this region the reliability of predictions is markedly reduced.

Experimental Lifetime

Communication and propagation studies can be made only while the West Ford belt remains relatively undispersed. To estimate a useful experimental lifetime, we calculate an outer bound on the cross-sectional expansion of the belt.

The action of most disturbing forces is to perturb the orbits of different dipoles differently. These perturbations will thus cause the belt to spread (disperse) far beyond the original size determined by the small differential velocities imparted to the dipoles during dispensing (2).

The major dispersive influence on the polar test belt is that of radiation pressure. Since the dipoles tumble randomly, their effective projected areas vary from $A_{\rm max}$ to $A_{\rm min}$, resulting in a corresponding distribution of dipole orbits. Hence, we have used our computer program to calculate the time variations of the orbital elements for the two cases $A_{\rm eff} = A_{\rm max}$ and $A_{\rm eff} = A_{\rm min}$, the differences giving an outer bound on the expansion of the belt. To obtain rigorous outer bounds we consider dipoles placed in a resonant polar orbit of the first type; the results are shown in Fig. 5, where the bounds on the separations in the various orbital parameters are plotted as functions of time (17). (These separations are not sensitive to small changes in the initial orbit.) As can be seen, the largest difference occurs in the perigee height (Δq) . During the early stages of the belt this difference grows approximately linearly with time. The curve labeled $\Delta \Omega$ is an upper bound on the lateral width attained by the belt over the equator, while curve Δi is a similar measure of the belt width over the poles. Curve Δp represents an upper bound on the in-plane separation of the dipoles at the semi-latus rectum points of the orbit. While at perigee and apogee the in-plane spreading is rather large, we see that the separation at the semi-latus rectum points is much smaller, comparable to the out-of-plane separations of the dipoles (18).

Considering the original formation of the belt, we can show that its cross section will be everywhere more or less elliptical. The curves in Fig. 5, then, represent outer bounds for the diameters through these ellipses in various directions and at different points along the orbit. (For example, Δi is an upper bound to the lateral diameter of the elliptical cross section over the poles.)

Since near the semi-latus rectum points of the orbit the belt will remain relatively undispersed, this portion can be used the longest for experimental purposes. Scattering cross-section studies indicate a useful lifetime for communications of about 2 years (19). Near apogee and perigee the expansion is much more rapid, and effective use can be made in these regions only for about 8 months.

Conclusions

It is expected that sunlight pressure will limit the average orbital lifetime of the Project West Ford dipoles to about 7 years, the useful experimental lifetime of the belt being about 2 years. These conclusions depend on the plan to launch the dipoles into a resonant polar orbit. For such an orbit, uncertainties in air density have little effect on lifetime predictions whose reliability is supported by comparison with the observed orbit of Echo I (20).

References and Notes

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- 6. The magnitude of the component of sunlight pressure in the orbit plane varies as the sea-sons progress. Therefore, even when changing monotonically perigee height will change with varying rate.
- 7. We present a more complete discussion of this figure in a later section.
- For the purposes of this discussion, the changes in a and i can be neglected.
- 9. M. C. Crocker, II, private communication. 10. In the context of this paper, the term *angular momentum* refers only to the motion of a dipole with respect to its center of mass.
- 11. Some collisions with micrometeoroids will result in the severing of dipoles. Eventually, they will be broken into small segments, perhaps of comparable size to the micrometeoroids which are present in the vicinity of the earth in considerably greater numbers.
- 12. See, for example, F. R. Moulton, An Intro-duction to Celestial Mechanics (Macmillan, New York, 1914), p. 404.
- 13. Note that since upper atmosphere densities are strongly dependent on solar activity, they will probably decrease in the next few years during the "quiet sun" period.
- 14. Because of the complicated interplay between the perturbations and the elements, the resonance will actually change with time from the initial type: In long lifetime cases there will be a continuous drift between extreme conditions which approximate first one and then the other of the two resonance types.
- Since the resonance region is narrower for lower values of A/M, our lifetime predic-tions become less reliable. 15.
- 16. Note, in addition, that some small perturba-tions were omitted in the computations for Fig. 4. Spot calculations made in the vicinity of the planned test orbit (and including these perturbations) indicated no large changes in the results. The non-zero value of the ordinate at the
- 17. time of launch denotes the maximum separation due to the maximum difference in disensing velocity.
- 18. The separations of the arguments of perigee $(\Delta \omega)$, being less than 2° in neglected in this discussion. in 2 years, can be
- 19. D. C. MacLellan, private communication. 20. This work was performed at the M.I.T. Lincoln Laboratory, which is operated with sup-port from the U.S. Air Force. We thank T. F. Rogers for his very helpful comments on the communications aspects of the West Ford belt.