Reports

Effects of Solar Radiation Pressure on Earth Satellite Orbits

Abstract. Calculations show that, at a mean altitude of 1000 miles, radiation pressure can displace the orbit of the 100-foot Echo balloon at rates up to 3.7 miles per day, the orbit of the 12-foot Beacon satellite at 0.7 mile per day. For certain resonant conditions this effect accumulates, drastically affecting the satellite's lifetime.

We have integrated the perturbation equations (*1*), assuming constant orbital elements over 1 cycle, for direct solar radiation pressure, including the effect of the earth's shadow. This pressure produces an important perturbation for satellites with area-to-mass ratios of approximately 25 cm^2/gm or greater. The results of the integration, together with those for air drag and the earth's quadrupole moment, have been incorporated into a digital computer program which permits prediction of the motion of any spherical earth satellite when these perturbations are the dominant ones. This motion is quite complex; for example, for certain ("resonant") combinations of orbital altitudes and inclinations the effects of solar radiation essentially build up monotonically, seriously affecting the lifetime. For the Beacon satellite (area-to-mass ratio of 23.2 cm^2/gm) at mean altitude of 950 miles, an initial eccentricity of 0.106, and an inclination angle of 40°, the lifetime can vary by a factor of 10, depending on the hour of launch. For an inclination angle of 48°, these conditions are no longer resonant, and the variation in lifetime is reduced to a factor of 2. (See Fig. 1.)

The force on a sphere due to solar radiation pressure ($P = 4.5 \times 10^{-5}$ dyne/cm^2 near the earth) is the same for complete absorption and for specular reflection of the radiation. Using Kallmann's data (*2*) for air density, one finds that for circular orbits at about 500 miles altitude, air drag exerts a force comparable to that of radiation pressure. However, the effects of these two forces on the orbit are quite different.

In general, during a complete orbital period, solar pressure causes a first-order perturbation of all six orbital parameters. However the most conspicuous effect for a nearly circular orbit is a displacement of its geometric center. This displacement is perpendicular to the earth-sun line in the orbit plane and in a direction such as to decrease the altitude of that part of the orbit in which the satellite moves away from the sun. The mean radius is almost unaffected for orbits of small eccentricity. For such orbits the magnitude of the time rate of this displacement is

$$|\dot{\mathbf{c}}(\Theta)| \cong P - \frac{A}{m}\left[\frac{3(2\pi - \alpha) + \sin\alpha}{4\pi n}\right]\cos\Theta, \quad (1)$$

where α is the angle subtended at the earth's center by the portion of the orbit within the earth's shadow, Θ is the angle between the earth-sun line and the orbit plane, $n = (GM)^{1/2}/a^{3/2}$ is the mean motion, m is the mass, and A is the effective cross-sectional area of the satellite.

Combining the displacement rate $\dot{\mathbf{c}}$ with the perturbations due to earth oblateness (*3*), one finds that for nearly equatorial orbits, perigee height oscillates with period $2\pi/|\dot{\beta} - \dot{\Omega}_b - \dot{\omega}_b|$. Here $\dot{\beta}$ is the angular rate of the earth-sun line,

$$\dot{\Omega}_b \cong -4.98\ \frac{2\cos i}{a^{7/2}(1-e^2)^2}\ \text{deg/day},$$

and

$$\dot{\omega}_b \cong 4.98\ \frac{(5\cos^2 i - 1)}{a^{7/2}(1-e^2)^2}\ \text{deg/day},$$

are the angular rates of the nodal line and perigee argument, respectively, due only to the earth's equatorial bulge. In these equations, a is the major axis measured in earth radii, i is the inclination, and e is the eccentricity of the orbit. For nearly equatorial and nearly circular initial orbits (except those near the "stable" orbit defined below), the amplitude of the perigee height oscillation is

$$\Delta h \approx |\dot{\mathbf{c}}(\Theta = 0)|/|\dot{\beta} - \dot{\Omega}_b - \dot{\omega}_b|. \quad (2)$$

This equation agrees within 10 percent with accurate computer calculations for $e_0 < 0.2$ and $i_0 < 30°$. Cases A and B of Fig. 1 agree within 25 percent with Eq. 2, despite the effects of air drag at this low altitude. These curves have approximately the planned initial orbit of the unsuccessful Beacon satellite of August 1959. They differ only in launch time and were selected to show approximate maximum and minimum lifetimes attainable by varying only Ω_0. Since $e_0 \neq 0$, perigee height can oscillate either above its initial value (long lifetime) or below it (short lifetime).

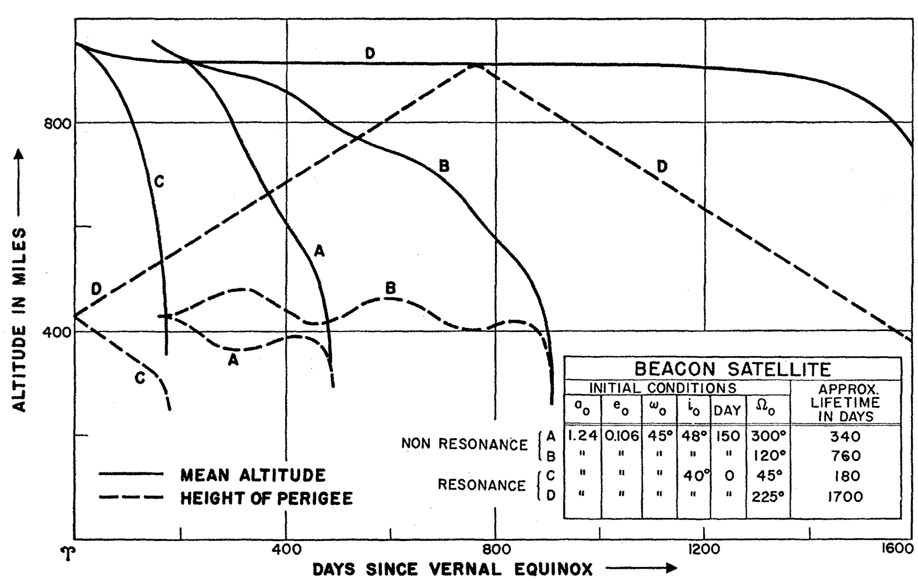

Fig. 1. Time variation of perigee height and mean altitude for the Beacon satellite.

When the denominator of Eq. 2 vanishes, we have a resonance. The earth's oblateness then keeps approximately constant the orientation between the perigee position and the projection of the earth-sun line onto the orbit plane. Hence, radiation pressure can increase or decrease the eccentricity monotonically. This occurs in cases C and D of Fig. 1, which were chosen so that $\dot{\mathbf{c}}$ is directed along the orbit's major axis. The average slope of each nearly straight segment of the curves for perigee height is about 10 percent less than the value given by $|\dot{\mathbf{c}}\,(\Theta = 0)|$. Actually, Θ varies over a wide range, and the detailed changes in slope correlate very well with the corresponding changes of $\cos \Theta$. The lifetime for this process is not sensitive to the model used for air density, and is sensitive to the launch time only for noncircular initial orbits.

For the Echo balloon (area-to-mass ratio 125 cm^2/gm) placed in a 1000-mile-altitude circular orbit, we find $|\dot{\mathbf{c}}| = 3.7$ mi/day with the sun in the orbit plane. Furthermore, if i_0 has the near resonance value of 35°, the lifetime is 240 days. Inclinations about the resonant value of 40° must therefore be avoided if longer lifetimes at this altitude are desired. For an initially circular equatorial orbit, the resonance altitude of 4000 miles leads to a 1.3 year lifetime, while the same orbit at 1000 mile altitude has an extremely long lifetime. The perigee height oscillations of the latter orbit have amplitudes of 60 miles and exhibit cusps at their highest points.

Smaller amplitudes of perigee height oscillation than those given by Eq. 2 result from near-equatorial initial orbits which lie close to a "stable" orbit. A stable orbit has approximately constant perigee height, and its geometric center lies on the projection of the earth-sun line at a distance Δh from the earth's center. For other orbits the oscillation amplitude is approximately twice the distance from their initial centers to that of the stable orbit, provided that this distance is smaller than Δh. The perigee height of the equatorial stable orbit of the 1000-mile mean altitude Echo balloon lies 60 miles below this altitude.

For inclinations between 55° and 125°, even an approximate description of orbital evolution becomes considerably more complicated. Along with shorter periods, we find $2\pi/|\beta - \dot{\Omega}_0 + \dot{\omega}_0|$. When the denominator vanishes we again have a resonant condition. To understand this qualitatively, consider, for example, a satellite launched north in a resonant circular orbit, at dawn on 21 December. Since the orbit is polar, $\dot{\Omega}_0$ vanishes and $\dot{\omega}_0$ is negative. The component of radiation force in the orbit plane causes perigee to appear at the launch position. As the year progresses, this component rotates in the orbit plane in the same direction and at the same rate as the perigee position, but perigee leads by 90°. At the spring and fall equinoxes the radiation force vector lies in the orbit plane and perigee height changes at maximum rate. Perigee height thus monotonically decreases, but with a varying rate. For the same satellite launched at dusk, the radiation force component and perigee position rotate in opposite directions, resulting in a large increase in lifetime. Thus even for $e_0 = 0$ the launch time affects lifetime. The qualitative behavior of perigee for other polar orbits can be similarly analyzed. We quote one quantitative result: the Echo balloon in a resonant 2400-mile circular polar orbit has a lifetime varying from 1.3 to 3.1 years, depending on launch time (*4, 5*).

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References and Notes

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2. H. K. Kallmann, *J. Geophys. Research* **46**, 615 (1959).

3. We are indebted to Dr. Eugene Levin of the Rand Corporation for discussions about this point.
4. This work was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology with the joint support of the U.S. Army, Navy, and Air Force, and at Ramo-Wooldridge, a division of Thompson Ramo Wooldridge, Inc. under a subcontract with Lincoln Laboratory.

5. A more thorough treatment of these effects, including the effect of solar radiation reflected from the earth, is in preparation.

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Post-Bomb Rise in Radiocarbon Activity in Denmark

Abstract. During the summers of 1958 and 1959 the increase in concentration of bomb-produced radiocarbon in Denmark was several percent higher than the average increase for the hemisphere. This additional increase is probably a carbon-14 equivalent to the spring peaks in strontium-90 fallout in the North Temperate Zone in the same years, and suggests latitudinal variations in carbon-14 contamination.

Measurements of bomb-produced radiocarbon have been reported by several investigators (*1–4*). Estimates of a linear yearly increase in the C^{14} concentration of atmospheric carbon dioxide from the Northern Hemisphere up to 1958 have ranged from 3.2 percent [Münnich and Vogel (*2*)] to 4.3 percent [de Vries (*3*)] and 5 percent [Broecker and Walton (*4*)].

Since 1956 samples have been collected in Denmark for the purpose of monitoring the rise in C^{14} in the atmosphere. The results from a series of very uniform samples are reported here.

All the samples were derived from cereals which grew in locations about 10 miles north of Copenhagen (latitude 55°50′N, longitude 12°30′E). Except for one case, only the ears of the cereals were used. These had developed and had grown during the months of June and July of the year in question. With this material, uncertainties due to unknown time lags between assimilation and deposition of carbon compounds in plant tissues of perennial plants were avoided. In June the house-heating season in Denmark has ended, and, since the area is not very heavily industrialized, there is no local Suess effect to influence the results. This is indicated by the fact that the measure for the pre-bomb decrease in C^{14} in Denmark as compared to findings for 19th-century wood is 2.5 ± 0.5 percent, while the average decrease throughout the world is calculated by Fergusson to have been 2.03 ± .15 percent (*5*). The samples, therefore, should closely reflect the mean concentration of C^{14} in CO_2 of the unchanged atmosphere at the sample locations in June and July of the growth year in question.

The sample materials are listed in Table 1, together with the measured C^{14} activities. The samples were assayed as CO_2 in a proportional gas counter (*6*). The C^{14} activities are given as

$$\Delta C^{14} = \delta C^{14} - 2\delta C^{13}\,(1 + \delta C^{14}/1000) - 50.0$$

In this expression the activity is normalized to a common C^{13}/C^{12} ratio (*8*) and stated as the permillage difference from 95 percent of the activity of the National Bureau of Standards oxalic acid standard, a value which falls close to the mean activity of 19th-century wood (*4, 7*).

The rise is plotted in Fig. 1. Up to the summer of 1957 the measurements show an approximate linear rise of about 5 percent per year, which is consistent with the afore-mentioned estimates. From 1958 on, the increase is definitely steeper than reported by other investigators. From June-July 1957 to June-July 1958 the increase was 8 percent, or 3 percent more than that measured by Broecker and Walton (*4*); from the summer of 1958 to the summer of 1959 the increase was 14 percent. The 1959 value is 12 percent higher than that predicted by extrapolation of the linear curve of Broecker and Walton. Broecker and Walton's prediction was based on a curve derived from samples collected in more southern latitudes, mainly from the Great Basin in the