almost impossible to deceive the expert. No forgery can be undertaken with the blood groups, unless we accept the interference of bacteria as criminal action! The use of statistics on the ABO groups throws light on many anthropological problems. One of the most interesting, touched on by the Boyds, is the date of introduction of these groups into America by the successive waves of migrants. Candela's data on Aleut mummies (*32*) have been used by Laughlin (*33*) in discussing the origins and racial affinities of the present population of the islands. It is possible that further studies would show relative dates for the introduction of the different ABO groups into the American continent. Group O is predominant there today, notably in South America, where work on ancient mummies and skeletons could contribute to the solution of such problems as the origins and racial affinities of Inca and pre-Inca peoples. But North America shows areas of high incidence of group A—an incidence that increases as one moves northwards. When did this gene arrive, and where did it come from? Is it possible to demonstrate the absence of group B in old Indian populations, or will its presence suggest an Asian origin, as has been postulated for the Eskimo? It is equally possible that it could be demonstrated that groups A and B had been lost in New World populations of the present day. Many fascinating problems can be posed, on the origins and the migrations of peoples all over the world. As our techniques improve and our knowledge extends, we shall add many more. It is the function and the adventure of paleoserology to solve them (*34*).

References and Notes

1. Landsteiner and Richter, *Z. Medizinalbeamte* **16**, 85 (1903).
2. L. Lattes, *Arch. antropol, criminale, psichiat. e med. legale* **37**, 3 (1916).
3. L. Hirszfeld and H. Hirszfeld, *Lancet* **1919**, 675 (1919).
4. W. C. Boyd and L. G. Boyd, *Science* **78**, 578 (1933).
5. P. B. Candela, *Am. J. Phys. Anthropol.* **27**, No. 3 (1940).
6. G. A. Matson, *J. Immunol.* **30**, 6 (1936).
7. Thieme, Otten, Sutton, *Am. J. Phys. Anthropol.* **14**, 3 (1956).
8. Laughlin, in "Tepexpan Man," *Viking Fund Publ. No. 11* (1948); *Proc. Am. Phil. Soc.* **94**, 4 (1950).
9. M. Smith, *Nature* **184**, 867 (1959).
10. W. C. Boyd and L. G. Boyd, *J. Immunol.* **33**, 159 (1937).
11. Morgan, *Chemistry and Biology of Mucopolysaccharides* (Churchill, London, 1958).
12. Dodd, *Brit. J. Exptl. Pathol.* **33**, 1 (1952).
13. D. Harley, *Medicolegal Blood Group Determination* (Grune and Stratton, rev. ed., New York, 1943).
14. Boorman and Dodd, *Blood Group Serology* (Churchill, London, 1957).
15. W. C. Boyd and L. G. Boyd, *J. Immunol.* **32**, 307 (1937).
16. Annison and Morgan, *Biochem. J.* **52**, 247 (1952).
17. M. Gray, thesis, University of Oregon (1952).
18. Allison and Morton, *J. Clin. Pathol.* **6**, 314 (1953).
19. Ruffié and Ducos, *Ann. méd. légale et criminol. police sci. et toxicol.* **36**, 1 (1956).
20. Ducos, *Vox Sanguinis* **3**, 385 (1958).
21. Thieme, Otten, Wheeler, *Report to QM Research and Development Command Project No. 7-64-12-004C.*
22. Iseki and Okada, *Proc. Japan Acad.* **28**, 370 (1952).
23. Iseki and Ikeda, *ibid.* **32**, 201 (1956).
24. Watkins, *Biochem. J.* **64**, 21P (1956).
25. Wyman and Boyd, *Am. Anthropologist* **39**, 4 (1937).
26. W. T. J. Morgan and W. M. Watkins, *Brit. J. Exptl. Pathol.* **29**, 159 (1948).
27. W. C. Boyd and L. G. Boyd, *Am. J. Phys. Anthropol.* **25**, 3 (1939).
28. G. Hartmann, *Group Antigens in Human Organs* (Munksgaard, Copenhagen, 1941).
29. Grubb, *Acta Pathol. Microbiol. Scand. Suppl. No. 84* (1949).
30. Gray, *Am. J. Phys. Anthropol.* **16**, 135 (1958).
31. A. E. Mourant, *The Distribution of the Human Blood Groups* (Blackwell, Oxford, England, 1954).
32. P. B. Candela, *Am. J. Phys. Anthropol.* **24**, 361 (1939).
33. Laughlin, "Physical Anthropology of the American Indian," *Viking Fund Publ.* (1949).

34. I wish to thank K. P. Oakley and A. E. Mourant for giving me the benefit of their comments on this article, and the Wellcome Trust for making my work possible.

Optics of Light Sources Moving in Refractive Media

Vavilov-Cherenkov radiation, though interesting, is but an experimental instance of a more general problem.

I. M. Frank

For a number of years the Vavilov-Cherenkov effect appeared as but a peculiar optical phenomenon difficult to observe. Light emission was induced by using radioactive preparations, and the glow was observed visually (*1*). The weakness of the glow seemed to preclude any application of the phenomenon in physics, and this was even more true in engineering.

Peculiarities of Radiation in a Medium

Since the theory of the Vavilov-Cherenkov effect appeared (*2–4*), the phenomenon could be regarded as an instance of superlight-velocity optics. This was a singular example in this field, which seemingly was isolated from any other known physical phenomena. It was evident that in principle other manifestations of superlight-velocity optics were also possible, but their observation appeared very complicated. For example, even the first calculations indicated that if the Vavilov-Cherenkov radiation were induced not by an electric charge but, say, by the magnetic moment of an electron, it should be so weak that its experimental detection would not be feasible (*5*). It was likewise evident that it would be difficult to create conditions for observation of atoms moving at superlight velocities (*6*).

Theoretical analysis of all these problems was for a number of years of interest chiefly from the viewpoint of principle.

Progress in nuclear physics and the improvement of experimental techniques in recent years has resulted in the fact that the Vavilov-Cherenkov effect has found numerous applications in the physics of high-energy particles. A connection between this phenomenon and many other problems has also been found, as, for example, the physics of plasma, astrophysics, the problem of radio wave generation, the problem of acceleration of particles, and so on.

A broader approach to the treatment

of the phenomena related to the Vavilov-Cherenkov effect has now become not only justified but essentially necessary.

The question naturally arises as to the peculiarities of a radiation which may be set up not only by an electric charge, but by any source of light, moving in a refractive medium (*7*). Such a general approach to the problem, involving, notably, the Vavilov-Cherenkov effect, is of interest now not only from the viewpoint of principle. It may be hoped that some phenomena of this range will become, in the immediate future, a subject of experimental study, too.

Since the discovery of the Vavilov-Cherenkov effect, our ideas of the mechanism of interaction between a rapidly moving particle and a medium have undergone a considerable change.

Formerly it appeared unquestionable that radiation arising during an electromagnetic interaction between high-energy particles and a medium is always some kind of bremsstrahlung. Most of the energy of such radiation is carried by high-energy photons. The optical properties of the medium should not be of significance for the emission and propagation of such photons. It was also assumed that the processes of ionization and excitation by fast particles might be regarded as a sum of independent interactions of such particles with individual atoms and molecules. This led to the deduction that generally for interaction between high-energy particles and a substance, macroscopic properties of the medium are likewise of no importance.

The discovery and interpretation of the Vavilov-Cherenkov effect, and then the connection between this phenomenon and ionization losses, found by Fermi (*8*), have led to a revision of this viewpoint. It has now become evident that the macroscopic properties of the medium play an important part in the processes of radiation of light by rapidly moving particles.

The ratio between the velocity of the emitter and that of light is a highly important factor, on which radiation depends. In a vacuum, the velocity of light is constant and always exceeds that of the emitter. It enters the formulas determining the radiation, as a universal constant. Radiation in a vacuum is therefore determined solely by the nature of the emitter and the law of its motion. The case is different in a refractive medium. The phase and group velocities of light differ from those in a vacuum. They depend on the properties of the medium and on the frequency of the light. In optically anisotropic media, they are a function of the direction of propagation and polarization of the waves. In media of limited dimensions, changes in the velocity of light during transition through the boundary of the media are also of importance. Hence, in a refractive medium, the ratio between the velocity of the emitter and that of wave propagation depends considerably on the velocity of light in a medium and on its changes. Unlike the ratio in a vacuum, the ratio may, notably, exceed unity. As a result, not only the properties of the radiation but sometimes even the radiation phenomenon itself depends on the peculiarities of light propagation in a medium. The Vavilov-Cherenkov effect is a case in point.

Radiation in a medium naturally also depends to a very great extent on the nature of the emitter. The theory makes it possible to foretell the properties of the Vavilov-Cherenkov radiation not only for a moving electric charge but also for other cases. For instance, similar to an electric charge, the Vavilov-Cherenkov radiation should have also been produced by a magnetic charge, had it been proved to exist (*9*).

Whereas the question of radiation of a magnetic charge should now, too, be considered as being only theoretically possible, the question of the Vavilov-Cherenkov effect for magnetic and electric dipoles and multipoles is quite real at present.

As a matter of fact, analysis of the radiation of a moving system of particles may prove necessary in resolving the numerous tasks related to processes in plasma and to problems of acceleration of particles. It is evident that a system of particles may, notably, be quasineutral, but it may possess an electric and, particularly, a magnetic moment due to moving ring currents.

Not only may a system of particles move as a whole, it may also have natural frequencies of oscillations. This is true to an even greater extent of such systems as a moving atom, ion, or atomic nucleus. An electron moving in a magnetic field may likewise possess natural frequency (Larmor frequency of revolution about the lines of a field). Therefore, apart from generalization of the theory of the Vavilov-Cherenkov effect, analysis of the general case of the radiation of systems possessing natural frequencies of oscillations is also required (*7*).

Such a general analysis also includes the Vavilov-Cherenkov effect. The latter corresponds to the limiting case when the natural frequency is zero.

The fact that the theory of radiation of a charge moving with a velocity exceeding that of light has not been revised in the past 20 years does not mean at all that the theory of this effect has been fully consummated. This can be seen from the following example. L. I. Mandelshtam was the first to point out that it is not necessary for a charge to move in a continuous medium in order to radiate during superlight velocity (*10*). The radiation remains the same if the charge moves along the axis of a hollow cylindrical channel inside the medium, provided the diameter of the channel is small in comparison with the length of the emitted wave. For practical purposes this is very important, since it makes it possible to obtain radiation in a medium under conditions when the emitter does not collide directly with the atoms of the medium, which may deform or destroy it. It seemed that this applies also to the radiation of a dipole in a medium.

As was recently shown, however, by V. L. Ginzburg and his associates, this question is not so simple as it appeared before (*11*). The properties of a medium directly adjacent to the dipole may play an important part, and the presence of a channel of any, even the smallest, diameter cannot, therefore, be ignored.

This important factor has called for a critical analysis of the formerly obtained data as well. Thus, two contradictory results were obtained by two different methods for the radiation of a magnetic dipole (*6, 9*). It may now be assumed that this was not due to the erroneousness of one of the methods used, but to the fact that the methods differed in taking into account the effect of the medium adjacent to the moving dipole. Possibly both results are correct, but they apply to different physical cases. The matter requires, however, further consideration.

The series of problems dealt with in this article, despite their diversity, comprises but the simplest case of radiation in a medium—namely, radiation during

Dr. Frank is affiliated with the P. N. Lebedev Institute of Physics of the Academy of Sciences of the U.S.S.R., Moscow. This article is the lecture he gave in Stockholm, Sweden, on 11 Dec. 1958, when he was awarded the Nobel prize in physics for 1958, a prize which he shared with P. A. Cherenkov and I. E. Tamm. It is published here with the permission of the Nobel Foundation.

which the translational motion of the system may be regarded as uniform and rectilinear.

Transition Radiation

A typical example of radiation in a medium and, notably, during the uniform motion of an electric charge is provided by the so-called transition radiation. The assertion that there is no radiation during a rectilinear and uniform motion of an electric charge at a velocity smaller than the phase velocity of light is correct only under the condition that the velocity of light along the path of the particle remains unchanged. For example, if a uniformly moving charged particle crosses the boundary of two media with different indices of refraction, transition radiation appears. Radiation appears because the jump in the magnitude of the phase velocity of light at the boundary of two media is to some extent equivalent to the jump in the magnitude of the velocity of a particle. The equivalence to bremsstrahlung becomes complete in an extreme case, when the particle moves from a vacuum to a metal in which light is absorbed over a length smaller than the wavelength of the light. The intensity of the transition radiation is at its maximum in this case. In the optical range of the spectrum, the spectrum and magnitude of the radiated energy are then exactly identical to those of the radiation which would have been produced by an electric charge and a charge of the opposite sign moving toward it (its electric image in the metal) which stop instantaneously at the point of encounter.

The spectral density of energy of transition radiation at low velocities is proportional to the kinetic energy of the particle, and it rises in the region of relativistic velocities as the logarithm of the total energy. Like bremsstrahlung, it becomes sharply directed in this case. It has been suggested that transition radiation might be useful in determining the energy of ultrarelativistic particles. This is important because it is very difficult to use for this purpose the Vavilov-Cherenkov effect for ultrarelativistic particles. As is well known, the angle at which the Vavilov-Cherenkov radiation is directed and the intensity of the radiation attain in this case a practically constant value.

The use of transition radiation is, however, impeded by the fact that its intensity is very low. The probability of emission of a photon is of the order of the fine-structure constant—that is, of the order of a hundredth. If it is not possible to sum up transition radiation from many plates, observation of an individual particle by transition radiation may be carried out with but little efficiency. In this connection I should like to note the peculiarities of transition radiation at ultrarelativistic velocities. Unlike particles with a low velocity, transition radiation is almost the same during the incidence of such a particle from vacuum on a transparent dielectric as during the incidence on a metal. This is easy to understand by analogy with bremsstrahlung. Indeed, a change in the velocity of light is equivalent to a slight change in the velocity of the particle. But even a small change in the velocity of an ultrarelativistic particle means a great change in its energy—that is, a great deceleration of the particle. This peculiarity may permit us to sum up transition radiation from the surfaces of many parallel transparent plates in a vacuum.

The second peculiarity consists in the fact that at ultrarelativistic velocities, the equilibrium field entrained by the particle in a vacuum is formed along a considerable path length. Consequently, to prevent the intensity of radiation from being reduced, the vacuum layers between the plates should not be less than some preset magnitude. For instance, for the radiation of the visible light of a proton with energy of 10^{11} electron volts, this minimum distance is of the order of 1 millimeter, which is reasonable; but for a proton with energy of 10^{14} electron volts it rises to the unreasonable magnitude of 1 kilometer.

I have dwelt on the subject of transition radiation in order to emphasize the peculiarity of the optical phenomena for radiation sources moving in refractive media, which so greatly depends on the peculiarities of propagation of light in a substance.

It should be noted that although the theory of transition radiation was developed by Ginzburg and me (*12*) more than ten years ago and has since been analyzed in a number of works (as *13*, *14*), it has not yet been studied experimentally. The situation in this case is almost the same as in the case of the Vavilov-Cherenkov radiation before the papers of these workers were published. There is no doubt that transition radiation has also been observed on numerous occasions by various physicists, since the glow of the surfaces of electrodes under the impact of bombarding particles is well known. But even today the part played in this glow by luminescence, bremsstrahlung, and transition radiation has not been elucidated. The most reliable data on transition radiation have recently been obtained by Chudakov (*15*). Using the coincidence method, Chudakov observed photons emitted from the surface of a metal foil during the incidence on it of fast electrons from radiophosphorus. The intensity of radiation thus found proved to coincide with the estimated intensity for transition radiation, at least in order of magnitude (*16*).

It is also worth mentioning that transition radiation is practically always an intrinsic part of the Vavilov-Cherenkov radiation, due to the limited thickness of the radiator. As shown by V. E. Pafomov for a radiator of very small thickness, this factor should be taken into account (*17*).

Radiation Spectrum and Quantum Interpretation of the Phenomenon

The radiation of a charged particle uniformly moving at a velocity exceeding that of light may, as is well known, be fully described by the methods of classical electrodynamics. The quantum theory of this phenomenon was first developed by Ginzburg (*5*) and then by many other investigators (see, for example, *4*). Ginzburg has shown that the classical formula for the cosine of the angle at which radiation occurs is correct up to a very small correction of the order of magnitude of the ratio between the energy of the radiated photon and the total energy of the moving emitter. (Even for an electron the ratio is less than 10^{-5}.) If this slight quantum correction contained in the exact formula is disregarded, identical relations between the frequency of the radiated light and the direction of its emission are obtained by both the classical and the quantum methods. Let us write them down in a quantum form for a system possessing a natural frequency ω_0 (*7*, *18*), which is the frequency in the laboratory system of coordinates—that is,

$$\omega_0 = \omega'_0 \, (1 - \beta^2)^{1/2}$$

There is no need to assume in this case that ω_0 is the only natural frequency possessed by the system. It may be regarded as a component of a complex spectrum of frequencies, and it should be sufficient to study the radiation related to this frequency.

If the momentum of the photon, which in a medium should be assumed to equal $n\hbar\omega/c$, is very small in comparison with that of the emitter, then the law of conservation of momentum during radiation may be expressed as follows:

$$(n\hbar\omega/c)\cos\theta = \Delta E/v \qquad (1)$$

where ΔE is the change in the kinetic energy of the emitter, and v is its velocity. From this ratio we obtain the magnitude of the change in the momentum of the system.

The change in kinetic energy is apparently determined by the energy of the radiated photon $\hbar\omega$ and the change in the internal energy of system $\hbar\omega_0$.

$$\Delta E = \hbar\omega \pm \hbar\omega_0 \qquad (2)$$

The term $\hbar\omega_0$ should be taken with a minus sign if, when emitting the photon, the system passes from a higher energy level to a lower one—that is, if the energy of the emitted photon is supplied, partly at least, from excitation energy. The plus sign should be used if the system becomes excited in the process of emission that is, if the kinetic energy is spent both on radiation and excitation.

By combining Eqs. 1 and 2, we obtain

$$(n\omega/c)\cos\theta = (\omega \pm \omega_0)/v \qquad (3)$$

Factor $\hbar$ has been canceled out, and the equation does not, indeed, contain anything of a specifically quantum nature. The same result is also obtained from classical wave analysis.

In Eq. 3 we can distinguish three cases.

1) Let us assume that

$$(nv/c)\cos\theta = 1 \qquad (4)$$

Then Eq. 3 is satisfied only if $\omega_0 = 0$. This is precisely a case of Vavilov-Cherenkov radiation, while Eq. 4 is a well-known condition determining the direction of emission of light for this radiation. The natural frequency $\omega_0 = 0$ required for bringing Eq. 4 into effect means that the moving system should contain a source of time-independent electromagnetic field (an electric charge, a constant dipole moment, and so on). Consequently, for the Vavilov-Cherenkov radiation to take place it is necessary that the constant component of the field should differ from zero. In this case, Eq. 4 yields the relation between angle θ and the radiated frequency, inasmuch as the index of refraction $n(\omega)$ is a function of frequency.

2) Suppose now that the left-hand member of Eq. 4 is less than unity. Then Eq. 3 may be satisfied only if ω_0 has a minus sign—that is,

$$\frac{n\omega}{c}\cos\theta = \frac{\omega - \omega_0}{v}; \quad \frac{nv}{c}\cos\theta < 1 \qquad (5)$$

This is nothing else but the Doppler condition for a source of light in a moving medium. It was obtained by Lorenz when he was studying the optics of moving media.

Equation 5 may evidently be expressed in the following ordinary way:

$$\omega = \frac{\omega_0}{1 - (nv/c)\cos\theta} \qquad (5a)$$

It determines the frequency when the component of the velocity along a ray, $v\cos\theta$, is less than the phase velocity of light c/n for frequency ω.

Equations 5 and 5a differ from the usual Doppler condition for a source of light moving in a vacuum only in that the velocity of light in a vacuum has been replaced by the phase velocity c/n. If v is small in comparison with the phase velocity of light, and if the dispersion of light is not great in the range of frequencies close to ω_0, this does not lead to anything fundamentally new. There is only a change in the absolute magnitude of the Doppler shift. It is obtained as if for light moving in a vacuum at a velocity equal to nv, or n times greater.

If the dispersion of light in the medium is great, important peculiarities arise. The presence of dispersion should not be ignored in any medium when the velocities of motion are comparable to the phase velocity of light. Indeed, with n constant, and for $\theta = 0$, the quantity $(nv/c)\cos\theta$ would tend toward unity with an increase in v, while ω, as can be seen from Eq. 5a, would tend toward infinity. At still greater velocities, the inequality sign in Eq. 5 would not be valid, and consequently there would be no solution. As a matter of fact, the refractive index of any medium becomes practically equal to unity at sufficiently large values of ω. Hence the Doppler frequency in this case is the same as it would have been in a vacuum—that is, it is certainly finite. In other words, at any velocity v and any value of θ, Eq. 5 will have a solution. Moreover, as is shown below, there may not be one but several solutions (*6, 7*) ("complex" Doppler effect).

3) The third case takes place when the left-hand member of Eq. 4 is greater than unity. Then a plus sign should appear before ω_0 in Eq. 3, and thus

$$\frac{n\omega}{c}\cos\theta = \frac{\omega + \omega_0}{v}; \quad \frac{nv}{c}\cos\theta > 1 \qquad (6)$$

This is a generalization of Doppler's formula for the case when the velocity of the emitter exceeds the phase velocity of light for a radiated frequency (*6, 18, 19*). It determines the "superlight" Doppler frequencies. Like the Vavilov-Cherenkov effect, the superlight Doppler frequencies appear when the velocity exceeds some threshold velocity. They are radiated simultaneously with ordinary frequencies, but only at sufficiently high velocities and within some range of acute angles.

It can be seen from the above quantum analysis that the plus sign at ω_0 in Eqs. 2 and 6, respectively, means excitation of the system. Hence radiation of superlight photons occurs not during the transition from the higher—that is, excited—state into the lower state, as in a general case, but quite the contrary, from the lower into the higher state, the energy being supplied from the kinetic energy of the translational motion of the system (*18*). Such a radiation, accompanied by excitation of the system, should take place spontaneously if the system is in the lower energy state. This is likewise possible as a spontaneous transition of the system from the higher energy state into the lower, accompanied by emission of photons with a frequency satisfying Eq. 5. As a matter of fact, the transition occurs in either case between the same energy states, and the question as to which of them takes place spontaneously is wholly determined by the initial state and the requirements of the conservation laws. In this case Eqs. 5 and 6 are, in equal degree, consequences of these laws.

The question regarding the Doppler effect in a refractive medium may also be considered within the framework of classical physics. From the viewpoint of classical physics, the results are interpreted as follows. Oscillations with natural frequency ω_0 bring about the appearance of radiation with frequencies which depend on the direction of propagation. It forms a spectrum of Doppler frequencies, which may be of two types. There is always a spectrum of radiation with frequencies satisfying Eq. 5, whose reaction on the emitter causes its damping. Under certain conditions, another spectrum with frequencies satisfying Eq. 6 appears, in addition to the first. The reaction of radiation of these frequencies promotes the building up of oscillations. If damping prevails over building-up, oscillations will not arise by themselves in a system for which the classical formulas are correct, and if oscillations existed in the beginning, they will be attenuated.

In a quantum system the situation is fundamentally different. The processes of quantum radiation should be considered separately for spectra of both types. Therefore, if a process corresponding to Eq. 6 is possible, it is certain to take place—that is, the system will become excited owing to its own kinetic energy, will radiate light, and will pass in the usual way to the lower state. In principle, a two-photon mechanism is also possible, photons of both types being radiated simultaneously. Hence, as in the Vavilov-Cherenkov effect, a system possessing a natural frequency of oscillations will spend its kinetic energy on radiation at superlight velocity (*18, 20*).

This can be formulated in the following way. As is well known, motion at a velocity greater than that of light is impossible in a vacuum. It is possible in a medium, but nature does not lift its ban completely. Any system capable of interacting with radiation will slow down at a superlight velocity by radiating light.

Radiation Thresholds

It is evident from the above analysis that the radiation spectrum is determined by the velocity of motion of the system, v, its natural frequency, ω_0, and the phase velocity of light, c/n, in a medium in which the radiation is emitted. Both the Vavilov-Cherenkov effect and the Doppler superlight effect are possible, as can be seen from Eqs. 4 and 6, if $vn(\omega)/c > 1$. This obvious condition for the threshold of their appearance means that the velocity of motion should exceed the phase velocity of light.

This statement, correct for an isotropic medium, determines the threshold of emission of light of a given frequency ω for which the refraction index equals $n(\omega)$. As the refraction index depends on frequency, the threshold is different for another ω. This justifies raising the question in another way: Under what condition do the Vavilov-Cherenkov effect and Doppler superlight effect generally become possible in a given medium (*21*)?

During radiation in a medium there is yet another peculiarity which likewise appears under certain threshold conditions. It consists in the following. Equation 3 and, naturally, its sequels Eqs. 4, 5, and 6, are not linear with respect to ω. As a matter fact, they contain the refraction index $n(\omega)$, which is a function of the radiated frequency. As a result, not one but several values of ω, satisfying Eq. 3, are possible in some cases for given values of θ, v, and ω_0. This means that several components of different frequency may be radiated simultaneously in a given direction. The appearance of such additional frequencies—that is, of the so-called complex effects of radiation—is possible only under certain conditions. They may arise not only in the superlight Doppler effect and Vavilov-Cherenkov radiation but also in the ordinary Doppler effect subordinated to Eq. 5.

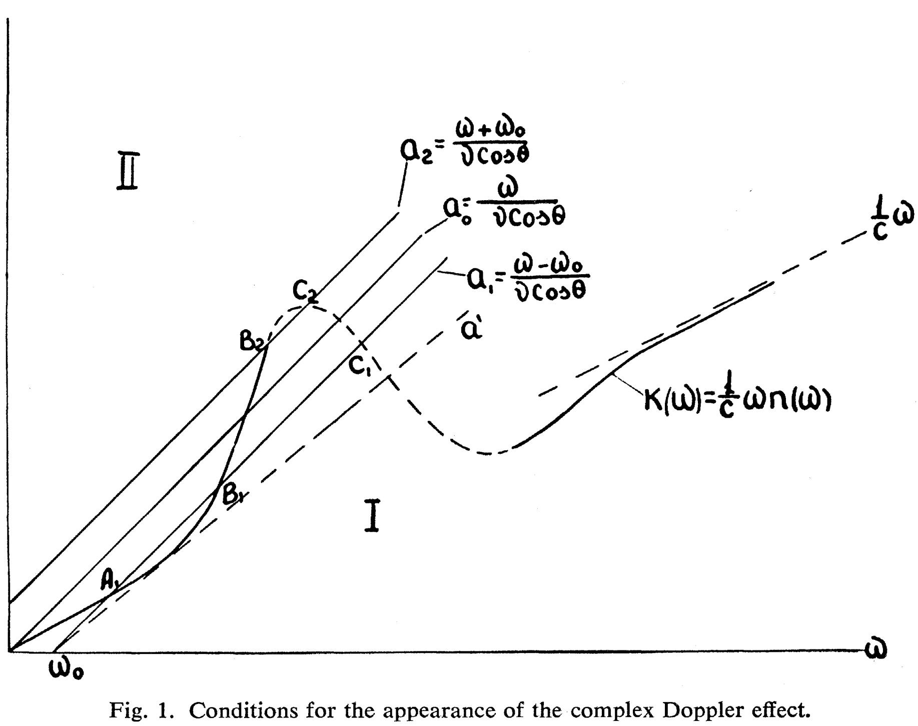


Fig. 1. Conditions for the appearance of the complex Doppler effect.

L. I. Mandelshtam was the first to draw attention to the fact that the condition under which the complex Doppler effect appeared (*6*) was related to the magnitude of the group velocity of light. The statement proved to be of a general nature (*7*).

If we consider radiation in the direction of motion, then in all the enumerated cases the condition for appearance of the radiation or of its new components is that the velocity of the emitter should equal the group velocity of light for a frequency which may radiate (that is, which satisfies condition 3). This threshold frequency should evidently satisfy Eqs. 4, 5, or 6, depending on the kind of radiation under consideration.

It is well known that in a refractive medium the transfer of radiation energy occurs not with the phase but precisely with the group velocity. Thus, it is not surprising that the group velocity of light is of importance for the processes of radiation in a medium.

The fact that the radiation threshold is connected precisely with the group velocity can be explained by some simple qualitative considerations. Let us assume that the conditions for appearance of the radiation have been fulfilled. Radiation arises and carries energy away from the emitter. Suppose, furthermore, that the velocity of motion changes and approaches the threshold velocity. When the threshold is attained, the radiation should disappear—that is, removal of energy from the emitter ceases. When the velocity of motion equals the group velocity of light, this will actually take place, since there occurs simply a transfer of energy together with the emitter.

The condition of appearance of the complex effect may be easily determined by analyzing the chart in Fig. 1. The curve in Fig. 1 represents dependence of the magnitude of wave vector

$$\kappa(\omega) = \omega n(\omega)/c$$

on the frequency for some imaginable medium. In addition to curve $\kappa(\omega)$,

Fig. 1 contains three straight lines whose equations are

$$a_0 = \omega/v \cos\theta \qquad (7)$$
$$a_1 = (\omega - \omega_0)/v \cos\theta \qquad (8)$$
$$a_2 = (\omega + \omega_0)/v \cos\theta \qquad (9)$$

The points where the straight lines cross the curve seem to determine at once the frequencies satisfying Eqs. 4, 5, and 6, respectively.

The tangent of the angle of incline of the straight lines a_0, a_1, a_2 to axis ω apparently equals $1/v \cos\theta$. Let us assume, in accordance with Fig. 1, that $\cos\theta > 0$—that is, $\theta < \pi/2$.

The nature of the intersections of the straight lines a with curve $\kappa(\omega)$ may differ. If we move along the straight line in the direction of increased ω, the straight line may go over at the point of intersection from the region underlying the curve (region I) into the region above the curve (region II). This takes place if the slope of the tangent to curve $\kappa(\omega)$—that is, $d\kappa/d\omega$—is less than $\gamma = 1/v \cos\theta$ (see, for example, point A_1 on the straight line a_1). On the contrary, if $d\kappa/d\omega > 1/v \cos\theta$, then there is a transfer from region II into region I at the point of intersection. Finally, $d\kappa/d\omega = 1/v \cos\theta$ takes place at the point of tangency.

As can be easily proved, the slope of the tangent to curve $\kappa(\omega)$ is equal to the reciprocal of the group velocity of light. Indeed, when there is no absorption, the group velocity W, as is well known, satisfies the relationship

$$\frac{1}{W}=\frac{d\kappa}{d\omega}=\frac{1}{c}\frac{d}{d\omega}(\omega n)=\frac{1}{c}\left(n+\omega\frac{dn}{d\omega}\right) \qquad (10)$$

Hence, the group velocity of light for frequencies that can be radiated is related to the velocity of motion v and $\cos\theta$ by the relationships (*22*):

$(v \cos\theta)/W < 1$
transition from I into II (11)
$(v \cos\theta)/W > 1$
transition from II into I (12)
$(v \cos\theta)/W = 1$
tangency (13)

At a sufficiently high value of ω the quantity W becomes equal to c. Indeed, the refractive index tends toward unity, and hence curve $\kappa(\omega) = \omega n/c$ approaches a straight line with a slope of $1/c$.

The straight lines a rise more abruptly since $v < c$, and consequently $1/v \cos\theta > 1/c$. Hence, all three straight lines a are, at great ω, in region II.

This entails a number of consequences. First of all, it is evident that the straight line a_1 will necessarily cross curve $\kappa(\omega)$; that is, Eq. 5, as has already been noted, must always have a solution. As a matter of fact, the straight line a_1 passes through point $\omega = \omega_0$ lying on the abscissa, which means that the straight line must go over somewhere from region I into region II. Moreover, it means that at any rate a frequency is radiated for which inequality 11, corresponding to a transition from region I into region II, is applicable.

The straight lines a_0 and a_2, as might have been expected, do not always cross curve $\kappa(\omega)$. For them not to cross requires that their incline to the abscissa should be sufficiently small. This means that the velocity should be high and that angle θ should not be large.

At great ω both these straight lines also prove to be in region II. It follows from this that if there are crossings, then, at any rate, the last of them which determines the highest of the radiated frequencies corresponds to a transition from region I into region II. The result is then, again, that there is a frequency in the radiation for which inequality 11 is valid. For forward radiation, that is, $\theta = 0$, this means that there is a component for which $v < W$, and, consequently, that for at least a part of the radiation, energy is propagated at a higher velocity than that of the source of light (Eq. 5).

It also follows from the above discussion that if there is a frequency satisfying condition 12 (for instance, corresponding to point B_1 on the straight line a_1, the composition of the radiation will infallibly be complex, since there must be a frequency or frequencies satisfying condition 11. (In the general case the number of possible crossings for the straight line a_1 is always odd, and for the straight line a_2 always even.)

The boundary of the appearance of radiation or of new components of radiation is evidently represented by a case where the corresponding straight line a begins to touch curve $\kappa(\omega)$. This means the fulfillment of Eq. 13. With $\theta = 0$ we obtain, in agreement with the above, $v = W$ for the threshold frequency.

The dotted line in Fig. 1 corresponds to the threshold of appearance of the complex effect for the ordinary Doppler effect. As shown in the figure, the frequency begins to split when the slope of the straight line a_1 increases in comparison with that of the dotted line. This means that the complex Doppler effect arises in this case not when the velocity increases in comparison with the threshold velocity but, quite the contrary, when it decreases or when the angle

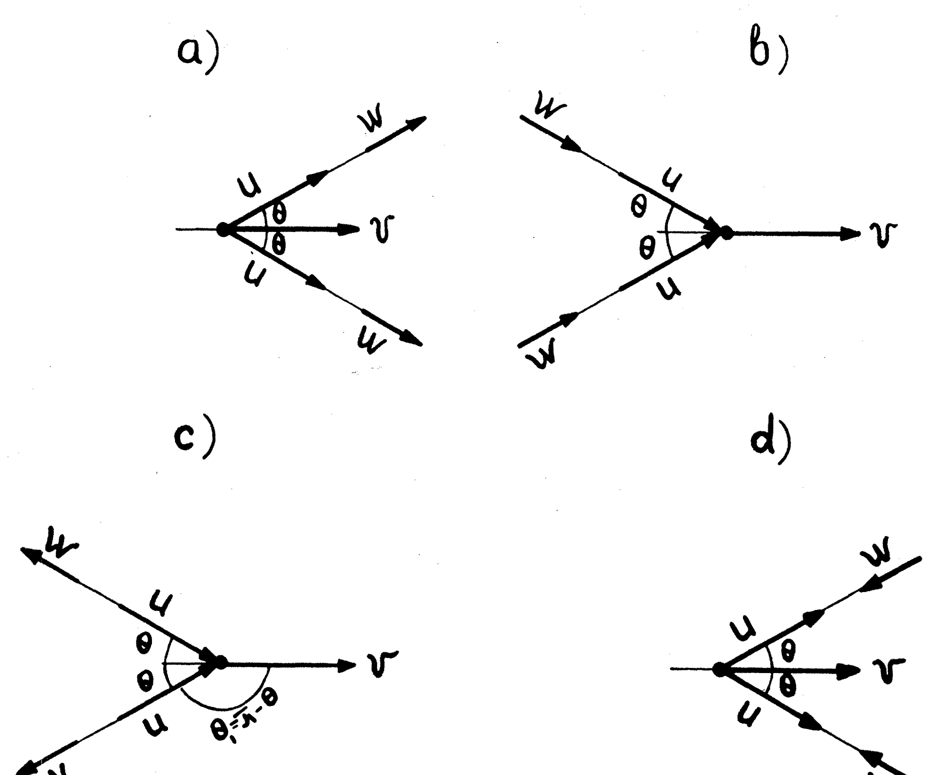


Fig. 2. Vavilov-Cherenkov effect in a medium with a positive group velocity: (*a*) radiation of energy, (*b*) absorption of energy; at negative group velocity: (*c*) radiation of energy, (*d*) absorption of energy.

becomes larger. [It is worth recalling that the tangent of the incline of the straight line a_1 equals $1/(v \cos \theta)$.] This is explained by the fact that the complex Doppler effect takes place here only within some range of velocities or angles, and the dotted line corresponds to the upper, and not the lower, threshold of the effect.

It has been assumed up till now that angle θ is acute—that is, that the product $v \cos \theta$ is positive. The statements made above regarding the complex Doppler effect may also be applied to the case of obtuse angles θ, but in this case negative group velocity will have to be taken into consideration. It appears that the threshold for the appearance of the complex Doppler effect, with $\theta > \pi/2$, is determined by Eq. 13. The quantity $\cos \theta$ is negative in this case; therefore Eq. 13 is valid only when the quantity W is less than zero. The import of negative group velocity for the Vavilov-Cherenkov effect was first investigated by Pafomov (*14, 17*) who pointed out that such a case should be real in anisotropic media (*23*). This is a very interesting case. We are accustomed to the idea that the Vavilov-Cherenkov radiation is directed forward at an acute angle. This is, however, correct only if the group velocity is positive. If it is negative, the picture is quite different.

Figure 2*a* shows schematically the ordinary case of Vavilov-Cherenkov radiation. The phase velocity for radiated light, $u = c/n$, forms in this case an acute angle θ with the direction of velocity v. The equation of electrodynamics also permits of the solution schematically represented in Fig. 2*b*. The direction of phase velocity—that is, the direction of wave propagation—forms in this case, too, the same acute angle θ with a velocity vector. The waves do not, however, come from the emitter, but towards it. The first case is interpreted as a radiation of waves, and the second, as their absorption. If there is no source of energy feeding the waves, flowing to the emitter, then the case of Fig. 2*b* is not realizable and the corresponding solution is rejected. But this is correct only if the group velocity is positive—that is, if its direction coincides with that of phase velocity (see vector W in Figs. 2*a* and 2*b*). The direction of the energy flux coincides in this case with the direction of phase velocity and, consequently, Fig. 2*a* really corresponds to the radiation of the waves, and Fig. 2*b*, to their absorption. In a medium with a negative group velocity, vector W is so directed as to meet vector u (the medium is considered optically isotropic, and hence vectors u and W may be only parallel or antiparallel). Therefore, with $W < 0$, Fig. 2*c* corresponds to radiation of energy, and Fig. 2*d*, to absorption of energy. Hence, if the group velocity is negative, the direction of the energy flux of the Vavilov-Cherenkov radiation forms an obtuse angle $\theta_1 = \pi - \theta$ with the direction of the velocity, and the motion of the waves is directed not from the particle but, quite the contrary, toward it (*24*). A similar analysis can also be made of an emitter with a natural frequency ω_0, moving in a medium with a negative group velocity (*7, 14*).

It can be seen from the above discussion that many substantial peculiarities of radiation in a refractive medium are actually related not only to the magnitude of the phase velocity of light but also to the group velocity of light. It may be expected that the role of the group velocity of light will reveal itself most distinctly in anisotropic media in which the directions u and W form some angle with one another.

Radiation in Optically Anisotropic Media

Radiation of a light source moving in a crystal should possess a number of features as compared with that in isotropic media. Interest in this range of problems has increased recently, in connection with studies of the processes in plasma (*25*). As to propagation of waves, a plasma placed in a magnetic field is similar to a uniaxial gyrotropic crystal.

The Vavilov-Cherenkov effect in crystals was first investigated theoretically by V. L. Ginzburg (*26*) and then by other investigators (see, for example, *4*). It has not, however, been studied experimentally to this day.

The equation determining the radiation frequency ω remains the same as in an isotropic medium—that is, ω is determined by Eq. 4. The magnitude of the index of refraction n in the case of an anisotropic medium depends, however, not only on the frequency of light but also on the angle and polarization. The result is that for the Vavilov-Cherenkov radiation the cone of normals to the wave surfaces is not circular in this case, as in an isotropic medium, but may have quite an odd shape. Thus, the direction of velocity does not necessarily coincide with the axis of the cone and in some cases may even lie beyond the cone (*17*).

Another peculiarity is related to polarization of the light. The Vavilov-Cherenkov radiation is always polarized. As a rule, polarization of light in this phenomenon does not attract attention, since it has not been used so far in present-day practical application of the radiation. However, from the viewpoint of the mechanism of the phenomenon, polarization is highly important. It is worth mentioning, for example, that the radiation of a magnetic charge, if it exists at all, could be distinguished at once from the radiation of an electric charge, since in this case the magnetic and electric vectors change places. The question of polarization of light is also of importance for the quite real case of radiation of dipoles and multipoles, though it has not yet been studied experimentally.

The role of polarization is manifested most distinctly in an anisotropic medium. First of all, one can obtain here, depending on the polarization of the radiated light, not one but two cones of wave normals corresponding to so-called ordinary and extraordinary rays in a uniaxial crystal. Moreover, the distribution of the radiation intensity is a complex function of the angles and is related to polarization of the light. The fulfillment of condition 4 does not suffice to bring about radiation, since the intensity of the waves of a given polarization may prove to equal zero. For example, if a particle moves in the direction of the axis of a uniaxial crystal, the cone of ordinary rays must disappear in the radiation (*4*).

The third peculiarity is related to the fact that in an anisotropic medium the direction of the ray—that is, the direction of a narrow beam of light—does not, generally speaking, coincide with the normal to the wave surface. There exist such directions of rays in a crystal, for which the normal to the wave surface forms some angle α with the ray (see Fig. 3).

The velocity at which the phase of the wave propagates in the direction of the ray, as can be seen from Fig. 3, is $1/\cos \alpha$ greater than the phase velocity; that is, $u' = u/\cos \alpha = c/n \cos \alpha$. We shall call u' the velocity of the waves along the ray. It should not be confused with the group velocity of light—that is, with the velocity of transfer of light energy which, naturally enough, is also directed along the ray. The group velocity equals velocity u' only under the condition that there is no dispersion of light in the medium. Indeed, the velocity of the waves along the ray does not depend in this case on frequency, and hence the group of waves moves with the same velocity.

The velocity of the waves along the ray is important for radiation in anisotropic media. Let us consider in this connection the threshold velocity for the appearance of the Vavilov-Cherenkov effect. The assertion that the Vavilov-Cherenkov radiation for a light of frequency ω arises at a velocity greater than the phase velocity of light with the given frequency implies that the medium is isotropic. If this statement be considered applicable to anisotropic media (as will be seen below, it is not always applicable), it is necessary, at least, to indicate with which direction of the phase velocity the velocity of motion is to be compared.

Equation 4 $[(nv/c) \cos \theta = 1]$ is also valid for anisotropic media, and in this case $c/n = u$ is the phase velocity for the given direction of the normal to the wave, forming angle θ with vector $\mathbf{v}$. As is well known, when the velocity approaches threshold velocity in an isotropic medium, θ decreases to zero—that is, the cone of wave normals is compressed in the direction $\mathbf{v}$. In a crystal, the cone of wave normals is likewise compressed, in this case toward some axis which as a rule does not, however, coincide with $\mathbf{v}$. If this axis is represented by the direction of the velocity, the threshold $\theta = 0$, and then we obtain from Eq. 4 that $v = c/n$ where c/n is assumed for the direction $\mathbf{u} = (\mathbf{c}/\mathbf{n})$ coinciding with $\mathbf{v}$. Hence, $\mathbf{v} = \mathbf{u}$. This relationship actually proves to be correct for boundary velocity in the usual cases of motion in a uniaxial crystal parallel or perpendicular to the optical axis. It has not, however, been stressed that it cannot always be applied.

It may be shown that the general condition for the appearance of the Vavilov-Cherenkov radiation of frequency ω should be formulated in the following way. The threshold velocity of the source of light should equal the velocity of waves along the ray in the direction of motion. In other words, the threshold velocity $\mathbf{v} = \mathbf{u}'$. For the threshold velocity, the direction of the ray coincides with $\mathbf{v}$ and not the normal to the wave, which forms an angle α with v. Hence, in the general case, the threshold value is $\theta = \alpha$.

In a special case, when the direction of the ray coincides with the wave normal in an anisotropic medium—that is, $\alpha = 0$, $\mathbf{u}' = \mathbf{u}$. Then we have $\mathbf{v} = \mathbf{u}$ for the threshold velocity. Finally, in an isotropic medium, where the phase velocity of light c/n is the same in all directions, it is possible to go over from vectors to scalar quantities, which means that $v = u$. Hence, the well-known statement that the velocity equal to the phase velocity of light is the threshold velocity has a limited field of application. It is a special case of a more general condition.

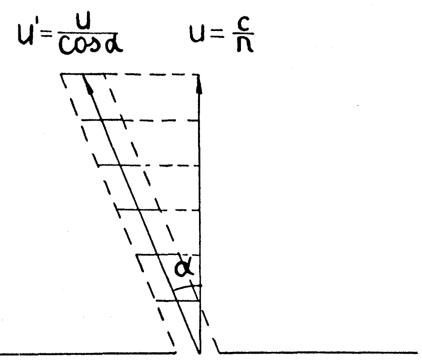


Fig. 3. Direction of a ray in crystals forms an angle α with the wave normal.

It is easy to explain this by using the Huygens principle for plotting the wave surface of radiation. This procedure is still generally used at present to describe the Vavilov-Cherenkov effect in an elementary way, and at the time it was one of the guiding ideas in the creation of the theory. This method can easily be applied to the case of an anisotropic medium.

The Huygens principle is frequently used in crystalloptics to explain the peculiarities of behavior of the so-called extraordinary ray during the refraction of light. The wave surface is found, by the Huygens principle, as an envelope of the waves emitted from separate points. Whereas, however, for an isotropic medium a sphere of radius $\zeta = (c/n)t$ is plotted around every point, where t is the time of movement of the waves, a crystal calls for a different approach. Of importance is the distance covered by the wave from a given point in the given direction of the ray. The distance equals the velocity of the waves along the ray, multiplied by time t—that is, $\mathbf{u}'t$. Therefore, the unknown quantity is represented by the envelope of the so-called surfaces of the rays plotted around every source of waves and determined by the equation $\vec{\zeta} = \mathbf{u}'t$.

Let us apply the Huygens principle to the case of Vavilov-Cherenkov radiation in a uniaxial crystal. The velocity of the ordinary and extraordinary rays is not the same here, and therefore, generally speaking, two cones of waves are obtained. In order not to encumber the drawing, they are shown separately in Figs. 4 and 5. We have to consider each point of the particle trajectory as a source of waves. In this case the wave phase is determined by the instant of passage of the particle through a given point. Let us assume that at moment $t = -t_3$ the emitter was at point A_3; at moment $t = -t_2$, at point A_2; at moment $t = -t_1$, at A_1; and finally, at the moment of observation $t = 0$, at point A_0.

For ordinary rays, the velocity of the waves along the ray, as in an isotropic medium, is equal to the phase velocity of light c/n and does not depend on the direction. The surfaces of the rays are simply spheres whose radii for points A_3, A_2, A_1, and A_0 are $(c/n)t_3$, $(c/n)t_2$, $(c/n)t_1$, and 0, respectively (see Fig. 4). The envelope of these spheres evidently represents a cone of circular cross section with the apex at A_0 (*27*). Its generatrices lying in the plane of the drawing are A_0B and A_0B'.

According to the Huygens principle, the directions of the rays are defined by the radius vectors drawn from some center of the waves to the point of tangency with the envelope. For example, in Fig. 4 (*left*) it is A_3B or A_3B', coinciding with the generatrices of the wave-normal cone for ordinary rays. Thus, the radiation cone is obtained for ordinary rays in the same way as in the Vavilov-Cherenkov effect in an isotropic medium. The substantial difference from an isotropic medium is related to the polarization of light and the distribution of intensity, depending on it. This was not taken into account in the construction.

From Fig. 4 it is not difficult to determine the magnitude of the threshold velocity. When the velocity diminishes, the distance between points A decrease. The threshold case arises when point A_0 occupies the position of A'_0 on the surface of the sphere. [This case is depicted separately in Fig. 4 (*right*).] At lower velocities, one of the spheres lies completely within the other and they do not have a common envelope. In the threshold case, they have only a common point of tangency A'_0. Thus, evidently, $(c/n)t_3 = v_0t_3$—that is, $v_0 = c/n$. The cone of wave normals is compressed in the direction of velocity v, and the wave cone transforms into a plane perpendicular to the axis of motion at point A'_0 [Fig. 4 (*right*)].

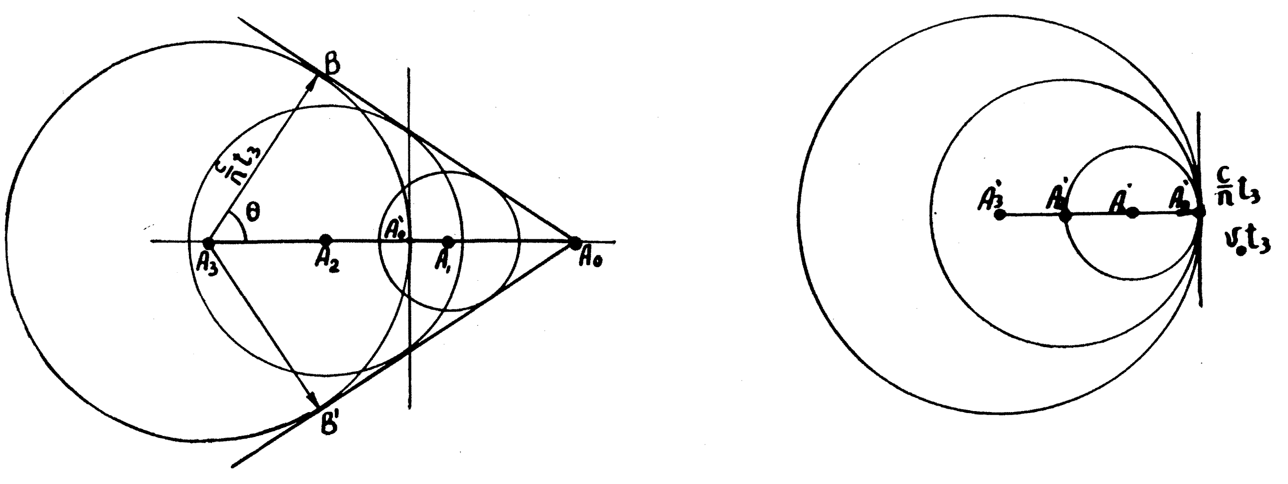
The Huygens principle can also be applied in a similar way to obtain a wave cone for the extraordinary rays (Fig. 5). The difference lies in the fact that surfaces of rays $\mathbf{u}'t_3$, $\mathbf{u}'t_2$, and $\mathbf{u}'t_1$, instead of spheres, are plotted around points A_3, A_2, and A_1. The cone enveloping the surfaces with an apex at A_0 is not circular in the case shown in Fig. Fig. 5 (*left*). The generatrices of this wave cone, A_0C and A_0C', lie in the plane of the drawing. The lines perpendicular to them, for instance A_3D and A_3D', determine the wave normals, and their length is proportional to the phase velocities. The vectors drawn from A_3 to the points of tangency A_3F and A_3F' indicate the corresponding directions of rays, which, as seen from Fig. 5 (*left*), do not coincide with the wave normals. It can also be seen from the drawing that the direction of an extraordinary ray for the Vavilov-Cherenkov radiation in a crystal may even constitute an obtuse angle with the direction of velocity [direction A_3F' in Fig. 5 (*left*)].

It is not difficult to determine the magnitude of the threshold velocity for the appearance of extraordinary rays in the Vavilov-Cherenkov radiation, which, generally speaking, differs from threshold velocity for ordinary rays. The threshold case occurs when the velocity diminishes to such an extent that point A_0 coincides with point A_0''. In this case all the surfaces of the rays lie within one another and have a common point of tangency A''_0. It can be seen from Fig. 5, which shows a threshold case, that the threshold value is $\mathbf{v} = \mathbf{v}_0 = \mathbf{u}'$. The wave cone then transforms into plane $A''_0\, D''$, and the wave normal forms an angle α with direction v. By tracing what happens to the cone of wave normals [its generatrices are A_3D and A_3D' in Fig. 5 (*left*)] during a decrease in velocity—that is, when point A_0 approaches A''_0—it is not difficult to prove that it is compressed not in direction v but in direction AD''. Hence, in a threshold case in Eq. 4, it may be assumed not that $\theta = 0$ but that $\theta = \alpha$. Then Eq. 4 produces $(nv/c)\cos\alpha = 1$—that is, actually, $v = c/(n \cos \alpha) = u'$.

It is worth recalling that with the aid of Figs. 4 and 5 we have determined the threshold of appearance of light of some given frequency ω. The velocity at which radiation generally appears is determined by a minimal magnitude of wave velocity of waves along the ray—namely, $u' = u'_{min}$ in a given medium for a ray directed along motion. For frequency ω' for which $u' = u'_{min}$, the velocity of the waves along the ray does not depend on frequency and is thus equal to the group velocity. Hence, we again come to the conclusion that the threshold is related to the group velocity.

The analysis of radiation of a system possessing a natural frequency of oscillations ω_0 may also be applied to the case of an optically anisotropic medium. The same peculiarities are manifested here as were referred to in connection with Vavilov-Cherenkov radiation. The connection between ω, θ, v, and ω_0 is determined, as before, by the same Eqs. 5 and 6 as in an isotropic medium, but now quantity n refers to the direction of a wave normal at an angle θ to the velocity.

The dependence of n on the direction



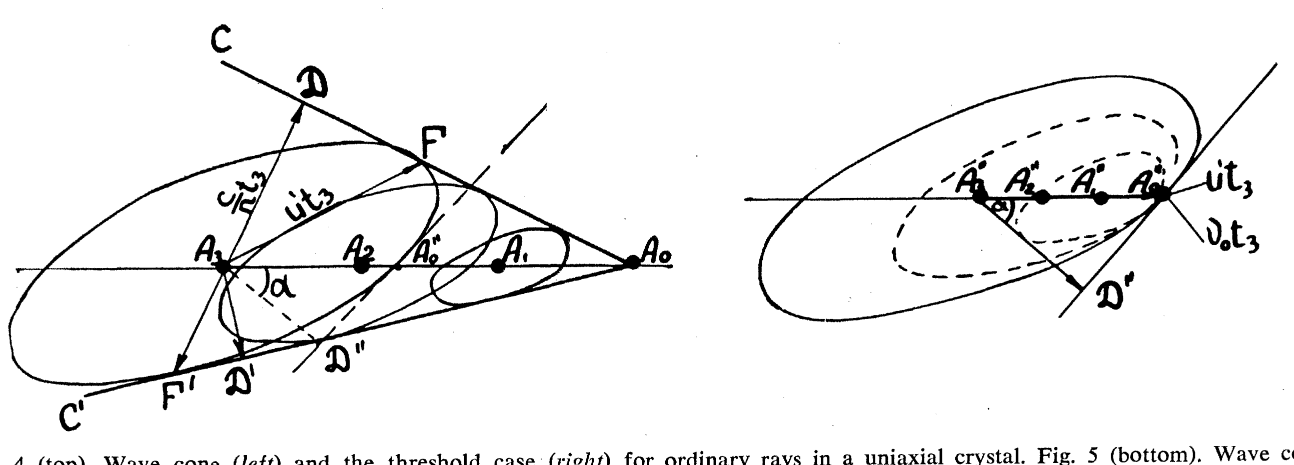


Fig. 4 (top). Wave cone (*left*) and the threshold case (*right*) for ordinary rays in a uniaxial crystal. Fig. 5 (bottom). Wave cone (*left*) and the threshold case (*right*) for extraordinary rays in a uniaxial crystal.

leads to the fact that the connection between θ and the frequency of radiation ω at preset natural frequency ω_0 and velocity v is not elementary. To find θ, use can be made of the graphic method suggested by V. E. Pafomov (*17*) for analyzing the Vavilov-Cherenkov effect in crystals, by applying it to the case of an arbitrary ω_0 (see Fig. 6). The figure shows a section of a surface of wave vectors $\overrightarrow{\kappa(\omega)} = \overrightarrow{\omega n/c}$ for the given ω in the case of extraordinary rays in a uniaxial crystal. The surface indicating dependence on the direction of vectors $\vec{\kappa}$ (they are oriented along the normal to the wave) differs from that of refraction indices only by a constant factor ω/c (we consider magnitude ω as prescribed). Thus, for a uniaxial crystal, the surface represents an ellipsoid of rotation. Let us assume that axis **v** is the direction of motion of the emitter. Let us plot on axis **v** segment OA of length b, which equals b_0, b_1, or b_2, depending on whether the analysis deals with the Vavilov-Cherenkov effect, the Doppler ordinary effect, or the Doppler superlight effect. Then

$$b_0 = \omega/v \tag{14}$$

$$b_1 = (\omega - \omega_0)/v \tag{15}$$

$$b_2 = (\omega + \omega_0)/v \tag{16}$$

At point A, which is the end of b, we shall plot plane a perpendicular to axis v. Let us consider the curve where the plane crosses surface $\kappa(\omega)$ as a section of some cone with the apex at O. The generatrices of this cone, OC and OC', lie in the plane of the figure. The cone defines the magnitude and direction of vectors $\vec{\kappa}$ for light of frequency ω appearing in the case under consideration—that is, for the given kind of radiation with preset ω_0 and v.

Indeed, as can be seen from Fig. 6, $OA = b$ is a projection of vector OC or OC'—that is, of vector $\kappa = \omega n(\omega_1\theta)/c$. Hence,

$$\frac{\omega n(\omega, \theta)}{c} \cos \theta = b \tag{17}$$

By substituting the values of b from Eqs. 14, 15, or 16, we obtain identical Eqs. 4, 5, or 6.

It can be seen from Fig. 6 that not only may the cone of wave normals be actually asymmetric but, as has already been mentioned, axis v may even lie outside the cone.

Plane a does not always cross the surface of $\kappa(\omega)$. This corresponds to the evident fact that not every frequency is radiated for given v and ω_0. If $b = b' = OA'$ (see Fig. 6), the plane touches the surface and, consequently, $b' = OA'$ is a boundary for the appearance of the given frequency ω in the spectrum. Vector $\vec{\kappa}$—that is, the wave normal, coincides in this case with OB. It can be easily proved that it forms angle α with the direction of velocity, the direction of the ray coinciding with that of motion. If, in accordance with this, angle $\theta = \alpha$ is inserted in Eq. 3, we obtain the following general condition for velocity v_0 required for the appearance of frequency ω:

$$\frac{\omega}{u'} = \frac{\omega \pm \omega_0}{v_0} \tag{18}$$

where u' is the velocity of the waves along axis v (positive or negative—that is directed along v or opposite it). In a special case of the Vavilov-Cherenkov radiation, $\omega_0 = 0$.

Radiation of a system possessing a natural frequency of oscillations and moving in an optically anisotropic medium was first studied by K. A. Barsukov and A. A. Kolomensky (*28*). They elucidated a number of peculiarities of radiation related to the presence of ordinary and extraordinary rays and the significant role of wave polarization.

It is highly interesting that this seemingly more complex case appears to present even now some interest from an experimental point of view. Barsukov and Kolomensky made a special study of radiation of radio waves in the ionosphere, which behaves like an optically anisotropic medium under the action of the earth's magnetic field. It is important that this medium possesses strong dispersion at some range of frequencies and that the complex Doppler effect is possible in it. Kolomensky and Barsukov have pointed out that this phenomenon may take place in the case of radio waves of appropriate frequency, transmitted by an artificial earth satellite moving in the ionosphere. They found that the Doppler shift of frequency of the order of 10 to 100 cycles per second should be accompanied in this case by splitting of the radiation frequency into components of several hundredths of a cycle per second apart. Apparently, with a well-stabilized frequency of the transmitter, such splitting could be detected.

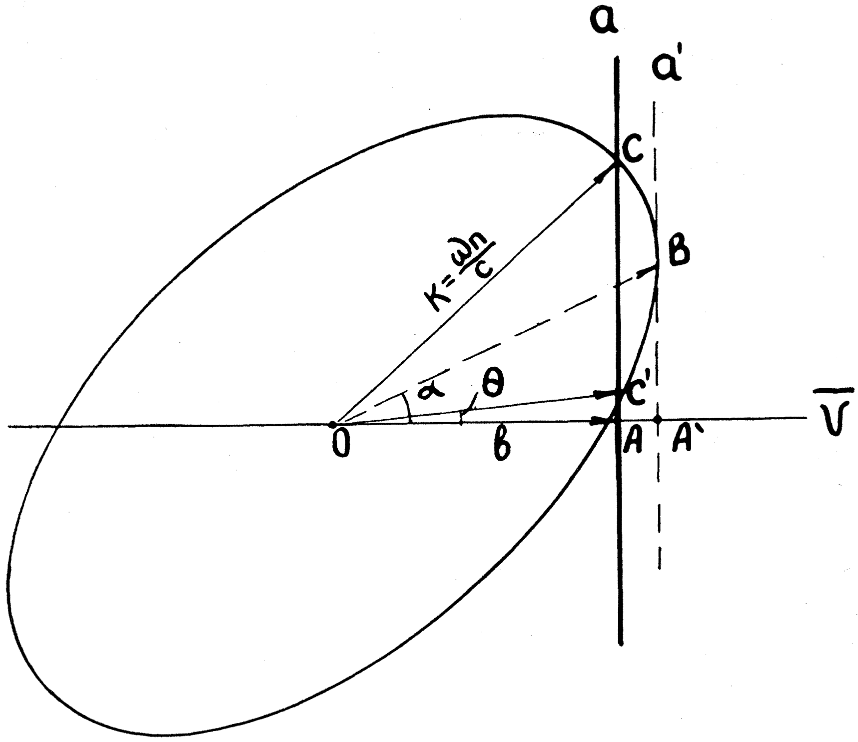


Fig. 6. Graphic plotting of a cone of wave vectors for radiation in crystals.

Conclusion

I have aimed to prove that there is a wide range of problems related to the radiation of sources of light, moving in refractive media. Radiation of an electric charge moving at superlight velocity in an isotropic medium—that is, the experimentally investigated case of the Vavilov-Cherenkov effect—is, in essence, but a special, though a highly interesting, instance in this realm of phenomena.

References and Notes

1. P. A. Cherenkov, *Doklady Akad. Nauk S.S.S.R.* **2**, 451 (1934); S. I. Vavilov, *ibid.* **2**, 457 (1934); P. A. Cherenkov, *Trudy Fiz. Inst. Akad. Nauk S.S.S.R., Fiz. Inst. im. P. N. Lebedeva* **2**, No. 4 (1944).
2. I. E. Tamm and I. M. Frank, *Doklady Akad. Nauk S.S.S.R.* **14**, 107 (1937).
3. A summary of the results of theoretical works, and a list of references, are given in the review by B. M. Bolotovsky (*4*).
4. B. M. Bolotovsky, *Usepekhi Fiz. Nauk* **62**, 201 (1957).
5. V. L. Ginzburg, *Zhur. Eksptl'. i Teoret. Fiz.* **10**, 589 (1940); *J. Phys. U.S.S.R.* **2**, 441 (1940).
6. I. M. Frank, *Izvest. Akad. Nauk S.S.S.R. Ser. Fiz.* **6**, 3 (1942); *J. Phys. U.S.S.R.* **7**, 49 (1943).
7. I. M. Frank, *Zhur. Eksptl'. i Teoret. Fiz.* **36**, 823 (1959).
8. E. Fermi, *Phys. Rev.* **57**, 485 (1940).
9. I. M. Frank, Vavilov Memorial Symposium (U.S.S.R. Academy of Sciences, Moscow, 1952).
10. V. L. Ginzburg and I. M. Frank, *Doklady Akad. Nauk S.S.S.R.* **56**, 699 (1947).
11. V. L. Ginzburg and V. Y. Eidman, *Zhur. Eksptl'. i Teoret. Fiz.* **35**, 1509 (1958); L. S. Bogdankevich, *Zhur. Tekh. Fiz.* **29**, 1086 (1959).
12. V. L. Ginzburg and I. M. Frank, *Zhur. Eksptl'. i Teoret. Fiz.* **16**, 15 (1946); *J. Phys. U.S.S.R.* **9**, 353 (1945).
13. G. M. Garibyan, *Zhur. Eksptl'. i Teoret. Fiz.* **33**, 1403 (1957).
14. V. E. Pafomov, *ibid.* **37**, 1853 (1959).
15. A. E. Chudakov, unpublished.
16. In the book by Jelley [*Cerenkov Radiation and Its Applications* (Pergamon, New York and London, 1958)], with which I had the opportunity of becoming acquainted after this article had been written, there is mention of the fact that in 1958 the author, together with Elliot and Goldsmith, observed a radiation emitted by 1.5-Mev protons incident on a polished aluminum target. On the basis of the data on the intensity and polarization, the investigators concluded that the glow was transition radiation. This observation has now been published as an article [P. Goldsmith and J. V. Jelley, *Phil. Mag.* **4**, 836 (1959)].
17. V. E. Pafomov, thesis, Physical Institute of the U.S.S.R. Academy of Sciences, Moscow (1957); *Zhur. Eksptl'. i Teoret. Fiz.* **32**, 366 (1957); *ibid.* **37**, 1853 (1959).
18. V. L. Ginzburg and I. M. Frank, *Doklady Akad. Nauk S.S.S.R.* **56**, 583 (1947).
19. Apparently Eq. 6 may be put down in a form similar to Eq. 5a. The difference consists only in that the sign in the denominator of the right-hand member of Eq. 5 should be changed.
20. I. M. Frank, *Usepekhi Fiz. Nauk* **30**, 149 (1946).
21. For the Vavilov-Cherenkov radiation in an isotropic medium, this point regarding the threshold is elementary, since the latter is determined simply by the maximum value assumed by the refraction index in the given medium. Of importance for further consideration is the fact that for a frequency corresponding to n_{max}, the phase and group velocities are equal (see Eq. 10), it being evident for n_{max} that $dn/dw = 0$. Hence, the fact that the threshold velocity of motion is equal to the phase velocity means that it is also equal to the group velocity of light.
22. The magnitude determined by Eq. 10 has the meaning of the group velocity of light only when there is no strong absorption—that is, in those regions of the spectrum for which the medium is transparent. The part of the curve $\kappa(\omega)$ corresponding to the region of anomalous dispersion, in which there is unquestionable dispersion, is shown in Fig. 1 by a dotted line. The peculiarities of radiation for frequencies getting into this region call for special consideration.
23. This is related to the fact that in an anisotropic medium the direction of the group velocity does not coincide with the direction of the phase velocity. This question is treated in the next section.
24. The analysis given in Fig. 2 is similar in many ways to the example given in L. I. Mandelshtam's lectures on the refraction of light by a medium with a negative group velocity [L. I. Mandelshtam, *Collected Works*, vol. 5, p. 463].
25. Some of the problems connected with plasma were dealt with in I. E. Tamm's Nobel lecture [*Science* **131**, 206 (1960)].
26. V. L. Ginzburg, *Zhur. Eksptl'. i Teoret. Fiz.* **10**, 608 (1940); *J. Phys. U.S.S.R.* **3**, 101 (1940).
27. Strictly speaking, such an analysis presupposes that there is a superposition of monochromatic waves. Each point of the trajectory should, therefore, be regarded as a source of such waves emitted for an infinitely long time. Actually, it is only the summation of waves of various frequency that produces a light impulse when the particle passes through a given point. Hence, there exists, of course, not one but an unlimited multitude of wave surfaces for a given frequency. The one that is generally plotted is singled out only by its passage through the instantaneous position of the particle (which we shall term the wave cone).
28. K. A. Barsukov and A. A. Kolomensky, *Zhur. Tekh. Fiz.* **29**, 954 (1959).

Significance of Carbon-14 Dates for Rancho La Brea

Tests analyzed in the light of early field notes emphasize the complexity of dating the several traps.

Hildegarde Howard

Carbon-14 datings are at last available on well-documented material from the Rancho La Brea fossil deposits of Los Angeles, California. These important deposits, first scientifically investigated in 1906, yielded tens of thousands of bones of extinct animals, as well as remains of insects and plants, and afforded a remarkable representation of the Pleistocene life of the Los Angeles Basin area. Although there has never been any doubt that these deposits were accumulated in Pleistocene time, there has been some change in thinking regarding the part of the Pleistocene represented (*1*) and its equivalent in terms of calendar years. Several years back, when I had occasion to conduct school groups through the exhibit of Rancho La Brea fossils at the Los Angeles County Museum, the expression "approximately 50,000 years old" was used in referring to the fossils. It has for some time been agreed that, geologically speaking, the deposits are of late Pleistocene age (*2*). Carbon-14 dating has revealed that some "late Pleistocene" glacial deposits are only 11,000 to 12,000 years old (*3*). Cave deposits containing remains of ground sloths identical with those found at Rancho La Brea have been given an age of 10,000 to 11,000 years (*4*). It has become a matter of considerable significance, therefore, to procure radiocarbon datings for the most prolific of all late Pleistocene deposits—Rancho La Brea.

In 1949, tests were made at California Institute of Technology by David L. Douglas (then a research fellow in chemistry) in the course of experimentation with the use of ionization chambers for measurement of low-level carbon-14. As Douglas did not consider his method to be perfected, and the pit source of the wood tested was unknown, his results were not noted in paleontological literature; they were, however, later recorded by Douglas (*5*) in an article explaining his method.

Tests have now been made on documented material, and by two laboratories: the Geochronometric Laboratory of Yale University, directed by Edward S. Deevey, and the Radiocarbon Laboratory developed by Hans E. Suess at the Scripps Institution of Oceanography of the University of California, La Jolla. Both laboratories tested sections from

The author is chief curator in the division of science, Los Angeles County Museum, Los Angeles, Calif.