In Musen's results the solar and lunar potentials appear as sums of trigonometric terms, whose arguments are combinations of the angles λ, ω, Ω, defined as follows: λ = mean longitude in orbit; ω = mean argument of perigee, measured from the line of the ascending node; Ω = mean longitude of the ascending node. When these symbols appear without subscripts they refer to the satellite; otherwise they refer to the sun or moon, as indicated by the subscripts *e* and *m*.

From the perturbing potentials we find the rate of change of perigee height by the method of variation of constants.

The results of the calculation are expressed most simply, in their dependence on the orbit elements, when the latter are defined with respect to the plane of the ecliptic. In terms of these elements, we can recognize five principal effects on the perigee height. Of these, four are resonances associated with the following conditions:

$$2\dot{\omega} - 2(\dot{\lambda}_e - \dot{\Omega}) = 0 \qquad (1a)$$

$$2\dot{\omega} + 2(\dot{\lambda}_e - \dot{\Omega}) = 0 \qquad (1b)$$

$$2\dot{\omega} - (\dot{\Omega}_m - \dot{\Omega}) = 0 \qquad (1c)$$

$$2\dot{\omega} + (\dot{\Omega}_m - \dot{\Omega}) = 0 \qquad (1d)$$

where $\dot{\omega}$, $\dot{\lambda}$, $\dot{\Omega}$, are the average angular velocities of ω, λ, Ω, respectively.

These resonance conditions have a simple interpretation. For example, in case 1a ($\lambda_e - \Omega$) represents the longitude of the sun relative to the line of nodes, and ω, the position of the perigee in the orbital plane, is also defined relative to the lines of nodes. Therefore, in a system in which the line of nodes is fixed the satisfaction of the resonance condition (case 1a) signifies that the mean angular velocities of sun and perigee are equal—that is, the line of apsides follows the sun. In this circumstance the orbital perturbation produced by the sun is clearly maximized.

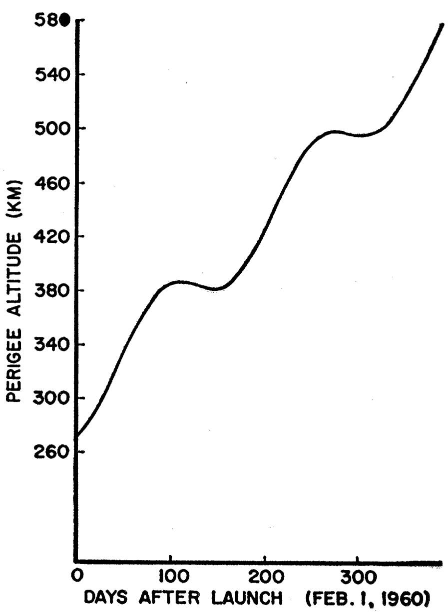

Fig. 1. Rate of change of perigee.

In case 1b the sun and the line of apsides have the same period of revolution, but opposite directions. Again it is clear that the solar perturbation will be maximized. Cases 1c and 1d represent similar resonances keyed to the motion of the moon's orbital plane. The fifth effect is produced by the perturbations of the sun and moon averaged over many periods of revolution of these bodies. This term has a period of $2\pi/2\dot{\omega}$, or approximately 800 days for the case of Explorer VI.

We return now to the quantitative treatment of the perturbations. Let q be the perigee distance from the center of the earth. The rate of change of q is then found to have the following form:

$$\begin{aligned}\frac{dq}{dt} = &+ A_1 \sin(2\omega + 2\Omega - 2\lambda_e)\\ &+ A_2 \sin(2\omega - 2\Omega + 2\lambda_e)\\ &+ A_3 \sin(2\omega + \Omega - \Omega_m)\\ &+ A_4 \sin(2\omega - \Omega + \Omega_m)\\ &+ A_5 \sin 2\omega\end{aligned}$$

The coefficients A_i depend on the size and shape of the orbit and on its inclination to the plane of the ecliptic. As this inclination varies, the relative importance of each term changes.

The effects on perigee height may be maximized or minimized by choosing suitable values of the orbital inclination and the time of launch. Long period effects occur when the inclination to the equator is near 63.4°—the critical angle at which there is no motion of the argument of perigee. At this inclination the 2ω term increases steadily with time. For an orbit with an apogee of 46,550 km and perigee of 6650 km the rate of change of perigee is approximately 1 km/day, as shown in Fig. 1. The sign and precise magnitude of the rate of change depend on the initial argument of perigee. The hour of launch does not affect this result.

At angles of inclination other than 63.4°, a variety of effects may be obtained by a suitable choice of the hour of launch. Selecting the hour of launch is equivalent to selecting Ω, the longitude of the ascending node, with any value available once in 24 sidereal hours. In Fig. 2, using the same apogee and perigee as in Fig. 1, an argument of perigee equal to 135°, and an equatorial inclination of 28° for 1 Feb. 1960, we show the results of three different choices of launch time. Curve *A* corresponds to a launch time of 7 hours U.T. on 1 Feb., curve *B* to 23 hours U.T., and curve *C* to 13 hours U.T. Cases *A* and *C* demonstrate rapid initial variations of perigee height. Case *B* represents a relatively stable orbit.

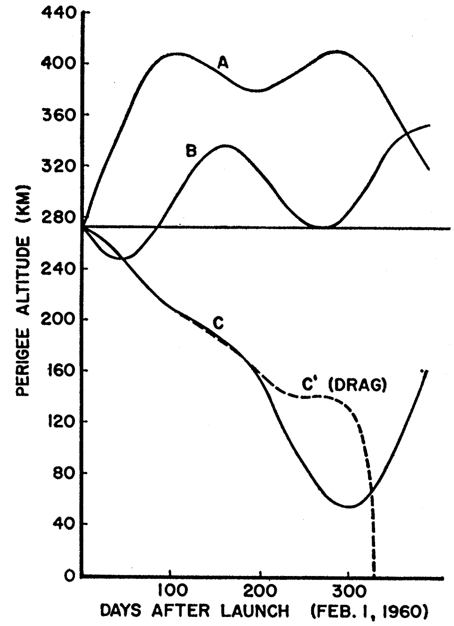

Fig. 2. Results of three different choices of launch time.

Curve *C'* represents the addition of drag to case *C*. It rises initially above the solar and lunar perturbation curve because the drag decreases the period and the eccentricity and these changes in turn decrease the solar and lunar perturbations. It is interesting to note that for a satellite with the parameters of Fig. 2, the lifetime is 25 years in the absence of lunar and solar perturbations, and approximately 1 year when they are included.

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Fat and Growth during Childhood

Abstract. Fat thickness, size, and maturity status are significantly correlated from 1.5 through 11.5 years in both sexes. Children who are fatter than their contemporaries at 8.5 to 9.5 years reach menarche earlier and complete epiphysial union sooner.

On the basis of available evidence an association would be expected between the amount of stored fat and size and maturity status in children during the growing period. Boys and girls from well-nourished populations are of greater

Table 1. Correlations between fat thickness and developmental status in boys and girls.

Lower-thoracic fat: At age (yr)	Lower-thoracic fat: Correlated with	Sex	Observations (No.)	Correlation coefficient (r)
1.5–11.5	Length*	F	471	0.41
1.5–11.5	Length*	M	685	0.32
12.5–17.5	Length	F	227	0.25
12.5–17.5	Length	M	284	0.04
8.5	Bone age	F	60	0.43
9.5	Bone age	M	61	0.35
12.5	Bone age	F	45	0.45
13.5	Bone age	M	63	0.26
8.5	Menarcheal age	F	59	0.34†
8.5	Tibial union	F	53	0.34†
9.5	Tibial union	M	56	0.47†

* Crown-heel length, 1.5 to 8.5 years; standing height, 9.5 to 17.5 years. †Reflected values of r (7).

stature and reach maturity earlier than children from less favored groups (*1*). Size during childhood is clearly related to economic level (*2*). Obese children have been reported to be taller than average and advanced in physiologic age (*3*).

In the study described here, the thickness of the fat-plus-skin shadow measured uniformly at the lower-thoracic site (*4*) on radiographs of children from 1.5 through 17.5 years was compared with length at the same ages. Fat thickness was further related to bone age (*5*) at a prepubertal and a pubertal developmental horizon, and fat thickness at 8.5 to 9.5 years was correlated with age at menarche in girls and with the age of completion of epiphysial union in the tibia in both sexes. Normalized *T*-scores obtained according to McCall's method were utilized throughout (*6, 7*), both to eliminate skewness and to provide age- and sex-specific measures for the correlations.

In 1667 separate observations on 259 clinically healthy white children, fat and size were found to be unquestionably related during the growing period. For children aged 1.5 through 11.5 years, age-specific correlations ranged up to 0.6, and over-all fat-length correlations (on the basis of sex- and age-specific *T*-scores) were 0.41 in girls and 0.32 in boys (Table 1). Thereafter, correlations between fat and length, though still positive, generally failed to attain the 5-percent level of confidence.

In similar fashion, fat thickness and bone age were positively correlated at the two developmental horizons considered. Further, girls who were fatter at 8.5 years of age attained menarche earlier, the correlation coefficient being 0.34. The long-term concomitants of fat were further demonstrated by similar correlations between fat thickness at 8.5 to 9.5 years and the time of complete union of the tibial epiphyses (see Table 1).

To translate these findings into developmental equivalents, increases of 1 standard deviation (10 *T*) above the average in fat thickness were associated with increases in stature of up to 0.45 of a year's growth at 11.5 years of age. Children with fat thickness of 1 standard deviation above the average were advanced in bone development at the particular ages investigated by an average of 0.25 to 0.54 year. Girls comparably fatter than the average at 8.5 years of age reached menarche 0.48 year earlier. A comparable relationship was observed between fat thickness at 8.5 to 9.5 years of age and the time of tibial union: the fatter boys and girls were advanced by 0.6 and 0.4 year, respectively.

Clearly, fatter children are both advanced in maturity and taller during the growing period (*8*). They reach menarche earlier, on the average, and complete tibial growth sooner. The data reported here, however, do not indicate whether the size advantage persists after completion of epiphysial union, and whether there is an asymptotic point beyond which greater-than-average fatness is no longer associated with accelerated development (*9*).

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