

weeks. No analyses were made for the other two species beyond the second week. Work is now under way to isolate and characterize this material. As judged visually from the chromatograms, there were quantitative differences in the composition of the protein hydrolyzates. For instance, tyrosine and phenylalanine were present only in faint traces in *Aedes aegypti* as compared with higher concentrations in *Anopheles* and *Culex*.

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References and Notes

1. L. A. Terzian, F. Irreverre, N. Stahler, *J. Insect Physiol.* 1, 221 (1957).
2. H. M. Kalckar, *J. Biol. Chem.* 167, 429 (1947).
3. E. Praetorius, *J. Clin. and Lab. Invest.* 1, 222 (1949). The uricase used was obtained from the Worthington Biochemical Corporation.
4. D. Seligson and H. Seligson, *J. Lab. Clin. Med.* 38, 324 (1951). We are grateful to Dr. Sidney S. Chernick, National Institutes of Health, for the microanalyses of urea and ammonia.
5. S. Moore and W. H. Stein, *J. Biol. Chem.* 211, 907 (1954).
6. F. Irreverre and W. Martin, *Anal. Chem.* 26, 257 (1954); K. A. Piez, F. Irreverre, H. L. Wolff, *J. Biol. Chem.* 223, 687 (1956).

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The Clock Paradox

Abstract. In two-dimensional Minkowski space a geodesic arc is longer than every other admissible arc with the same end points. Thus a particle whose world-line is a geodesic—that is, an unaccelerated particle—requires more (local) time to travel between two events than an accelerated particle. Similar results in four-dimensional Minkowski space can be established.

The so-called "clock paradox" (1) in Special Relativity may be loosely stated by saying that if a space traveler were to take an extended trip into space, he would find upon returning to earth that he was younger than he would have been if he had remained at home. Since there is no such concept in Relativity as the same place at two different times, it is clear that a more precise formulation of the problem is required.

To simplify the problem, let us consider it in a space of one time dimension and one space dimension, and let the units be so chosen that the velocity of light is 1. In this two-dimensional Minkowski space, the element of local time is the element of arc length

$$ds = (dt^2 - dx^2)^{1/2}$$

The clock paradox is a simple problem in the calculus of variations.

Let $P_1(t_1, x_1)$ and $P_2(t_2, x_2)$ be two events, or points in Minkowski space, $t_2 > t_1$. The length of arc between P_1

and P_2 along a curve $x = f(t)$ connecting the two points is given by the line integral

$$s = \int_{t_1}^{t_2} (1 - f'^2)^{1/2} dt$$

The clock problem consists in comparing arc lengths between P_1 and P_2 along different paths.

The extremals of arc length, or geodesics, are given by the solutions of the well-known Euler equations

$$\frac{\partial \theta}{\partial f} - \frac{d}{dt} \left(\frac{\partial \theta}{\partial f'} \right) = 0$$

where $\theta = (1 - f'^2)^{1/2}$, and are readily found to be of the form

$$x = f(t) = at + b,$$

where $-1 < a < 1$. This is a straight line in Minkowski space and represents uniform motion in a straight line in ordinary space. Since Euler's equations are invariant under Lorentz transformations, the concept of geodesic is absolute.

Now comes the crucial difference between geodesics in Euclidean space and geodesics in Minkowski space, a difference which points up the danger of attempting to apply geometric intuition to this problem: In Minkowski space, the extremals (geodesics) represent relative maxima.

To show this, let

$$x = f(t) = at + b$$

be a geodesic connecting $P_1(t_1, x_1)$ and $P_2(t_2, x_2)$, $t_2 > t_1$, and let $\omega(t)$ be an admissible function vanishing at t_1 and t_2 , so that

$$x = f(t) + \omega(t)$$

is a neighboring arc to the geodesic. The arc lengths along the geodesic and the neighboring arc are, respectively,

$$s = \int_{t_1}^{t_2} (1 - f'^2)^{1/2} dt$$

$$s + \Delta s = \int_{t_1}^{t_2} [1 - (f' + \omega')^2]^{1/2} dt$$

By the Mean Value Theorem

$$[1 - (f' + \omega')^2]^{1/2} = (1 - f'^2)^{1/2} +$$

$$\frac{-f'}{(1 - f'^2)^{3/2}} \omega' + \frac{1}{2} \frac{-1}{[1 - (f' + \phi \omega')^2]^{3/2}} \omega'^2$$

where $0 < \phi < 1$. Now if $f(t) = at + b$ and $-1 < a < 1$,

$$\int_{t_1}^{t_2} \frac{-f'}{(1 - f'^2)^{3/2}} \omega' dt = \frac{-a}{(1 - a^2)^{3/2}} [\omega(t_2) - \omega(t_1)] = 0$$

so that

$$\Delta s = -\frac{1}{2} \int_{t_1}^{t_2} \frac{\omega'^2 dt}{[1 - (f' + \phi \omega')^2]^{3/2}}$$

The integrand is never negative, and is 0 only when $\omega(t)$ is a constant and therefore 0. Thus for $t_2 > t_1$, Δs is negative for every neighboring curve. That is, the geodesic $x = at + b$ is an arc along which time is a relative maximum.

Let us assume, then, that we are in a space where there is no gravitation and that the earth is moving with constant velocity in a straight line. Then its world line in Minkowski space is a geodesic. Let $P_1(t_1, x_1)$ and $P_2(t_2, x_2)$, $t_2 > t_1$, be two distinct points on this geodesic. Any other arc connecting P_1 and P_2 would represent the time of another traveler whose motion is, for at least part of the journey, accelerated. This arc will be shorter than the geodesic so that a space traveler leaving the earth at P_1 would indeed be younger when he again met the earth at P_2 than he would have been if he had remained at home.

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Note

1. For a bibliography, see E. M. McMillan, *Science* 126, 381 (1957).

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Carotenogenesis and Resistance of *Micrococcus pyogenes* to Tetracyclines

Abstract. Although reddish-yellow pigments, mainly δ -carotene and rubixanthine, were present in the original strain of *Micrococcus pyogenes* var. *aureus*, mutants highly resistant to tetracyclines were observed to become colorless. All strains lack lipoxidase activity. The colorless strains probably reflect blocking by tetracyclines during carotenogenesis.

When *Micrococcus pyogenes* var. *aureus* (*Staphylococcus aureus*), strain 209 P, was cultured successively in media containing gradually increased amounts of tetracyclines, such as oxytetracycline, chlortetracycline, and tetracycline, acquisition of high resistance to these antibiotics was observed. In contrast to the original strain, which was sensitive to these antibiotics—that is, was killed easily by the antibiotics at a concentration of 0.5 $\mu\text{g}/\text{ml}$ —and had a tinge of yellowish color, due to the presence of carotenoid pigments, the resistant strains, which withstood the addition of over 300 μg of tetracycline per milliliter of culture media, were observed to become colorless.

Both sensitive and resistant strains were grown on nutrient agar containing 2 percent glycerol and adjusted to pH 7.2 for mass cultivation. After incubation for 1 day at 37°C, and then for 6