Mass of the Moon from Satellite Observations

Abstract. The four classical methods of determining the mass of the moon are noted, and a new use of an artificial earth satellite is proposed. The procedure, based on Kepler's law, is outlined, but at present the uncertainties in the observed data preclude an improved estimate of the lunar mass.

One of the more difficult astronometric problems is the calculation of the mass of the moon (*1*). This particular measurement has been performed by making geocentric observations of the position of the sun, planets, or the asteroid Eros. The monthly to-and-fro oscillations in the precise position of these objects are caused by the motion of the earth around the center of gravity of the earth-moon system. This displacement and the known distance of the moon are easily related to their relative masses.

A second method is based on a determination of the mass at the distance of the moon sufficient to cause the lunar component of the tides. A third method is based on a term in the expression for the motion of the equinoxes. The moon's attraction on the equatorial bulge causes a torque, or couple, about an equatorial axis, which exhibits itself in a monthly noding of the poles (the nutation). A similar torque, due to the sun, produces the equinoctial precession. The known distance of the sun and moon allows a comparison of the mass of the moon to that of the sun.

A fourth method, based on the exact form of Kepler's law (Eq. 1) requires the precise measurement of the lunar parallax and the period and gives the sum of the masses of the earth and moon.

With a satellite of very small mass we may take the ratio of the conditions for dynamic stability (Kepler's law),

$$G(M_e + M_m) = 4\pi^2 \frac{a_m^3}{T_m^2}$$

and (1)

$$G(M_e + M_s) = 4\pi^2 \frac{a_s^3}{T_s^2}$$

where M_e, M_m, and M_s are the masses of the earth, moon, and satellite, respectively, and a_m, a_s, T_m, and T_s are the mean distances and periods of the moon and satellite, and rearrange, neglecting the mass of the satellite,

$$\frac{M_m}{M_e} = \left(\frac{a_m}{a_s}\right)^3 \left(\frac{T_s}{T_m}\right)^2 - 1 \qquad (2)$$

Obviously, the data used to compute the mass ratio must be observational. Note that, astronomically, the term *mean distance* does not mean the average distance, but is the semimajor axis of the undisturbed elliptic motion. Furthermore, the period is not the observed time of circumscription, since the period of a close body about an ellipsoid depends upon its inclination and orbital eccentricity as well as its major axis dimension (*2*).

The effect of the accuracy of the observed artificial satellite data on the value of the lunar mass obtained must be examined. Differentiating Eq. 2, we obtain

$$\frac{\delta(M_m/M_e)}{(M_m/M_e)+1} = \frac{2\delta T_s}{T_s} - \frac{3\delta a_s}{a_s} \qquad (3)$$

From Eq. 3 it is evident that the error depends upon the absolute magnitude of the ratio and therefore a small uncertainty in the measurement produces a large uncertainty in the ratio. The present accuracy of artificial satellite measurements is insufficient to warrant the presentation of such calculations. However, the observations are capable of such precision, and the value of the lunar mass determination may be improved over the present 5 percent uncertainty.

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References and Notes

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A pH Calculator Based on Linear Transformations of the Henderson-Hasselbalch Equation

Titration curves of weak acids and bases are usually represented by symmetrical sigmoid curves based on the Henderson-Hasselbalch equation. The equilibrium of hydrogen ions, neutral molecules, and the ionized form in buffer solutions is governed by the law of mass action. The logarithmic form of the relation is

$$p\mathrm{H} = pK + \log\,(B/A)$$

where pK is the negative logarithm of K, the equilibrium constant; B refers to the base, defined as the proton-acceptor form of the weak electrolyte; and A refers to the acid, the proton-donor form.

The equilibrium of the two forms with hydrogen ions is expressed by the relation

$$A = B + \mathrm{H}^+$$

For example, in the buffer system acetic acid–sodium acetate, acetate ion is the conjugate base, while the undissociated molecule is the acid. In the system ammonium chloride–ammonium hydroxide, ammonium ion is the acid and ammonium hydroxide the base.

In either of these systems, if the acid is titrated with sodium hydroxide, the titration curve is calculated from the formula

$$p\mathrm{H} = pK + \log\,[b/(a-b)]$$

where a denotes the equivalents of acid initially present and b is the number of equivalents of added base. If, on the other hand, a strong acid, HCl, is used in the titration, the curve is represented by

$$p\mathrm{H} = pK + \log\,[(b-a)/a]$$

where b is constant. In either case the curve is symmetrically sigmoid when pH is plotted against added base, b, or acid, a. The point of inflection is at the midpoint, where pH is equal to pK, and B is equal to A. For any series of univalent weak acids and bases, a family of titration curves exists in which all the curves are similar. The position of each curve with respect to the pH axis is determined by pK.

If the negative logarithms of A and B are denoted respectively by pA and pB, the Henderson-Hasselbalch equation may be expressed in the form

$$p\mathrm{H} - pK = pA - pB$$

In this form of the expression the titration curves for any series of weak univalent acids and bases become linear and identical. At the midpoint of the line, pA and pB are equal, and pH is equal to pK. Transformation to the linear form has the advantage of eliminating the calculations necessary to construct the sigmoid form. Accurate construction of the sigmoid curve requires the calculation of four or more points on either side of the midpoint, a total of eight or ten points. For the linear form it is necessary only to draw a straight line on semilogarith-

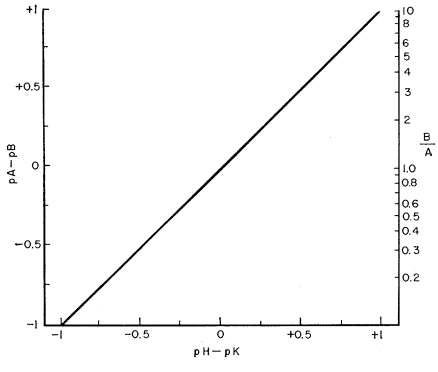

Fig. 1. Linear transformation of Henderson-Hasselbalch equation. Ordinate: $(pA - pB)$; abscissa: $(p\mathrm{H} - pK)$. On parallel ordinate values of the ratio B/A are given on logarithmic scale.

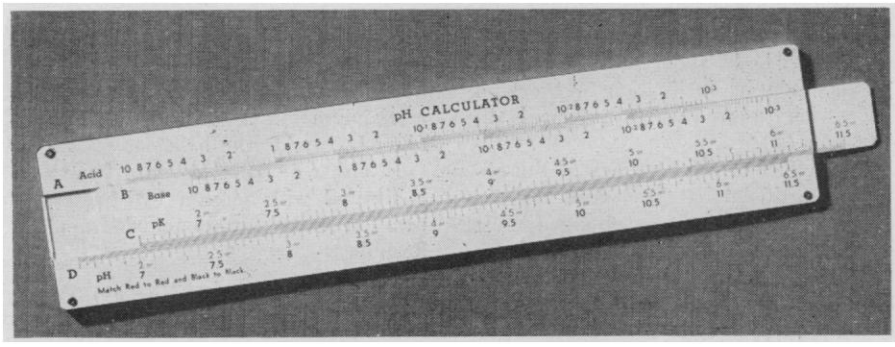

Fig. 2. A pH calculator constructed from linear transformations of the Henderson-Hasselbalch equation. The ratio B/A at this position is 8/3 or 2.67. For any univalent acid the value of $(p\mathrm{H} - pK)$ is then 0.43.

mic paper. The use of two-cycle paper permits the curve to be plotted over a range of two pH units, with the midpoint at pK (Fig. 1). In this diagram $(pA - pB)$ is represented on a linear scale parallel to the logarithmic scale of B/A.

By a further transformation, a semilogarithmic slide rule has been constructed (Fig. 2) (*1*). In this construction, A and B are identical logarithmic scales of acid and base concentrations. These scales are accurately calibrated in four logarithmic cycles at concentrations from $10^{-3}M$ to $10M$. These limits represent values of pA and pB of 3 and -1, respectively. Scales C and D are identical linear scales of pK and pH, respectively. A distance on these scales of 1 pH or pK unit is equal to 5 cm. The scales are calibrated to 0.01 pH, corresponding to 0.5 mm per scale division. The length of one logarithmic cycle on scales A and B is likewise 5 cm; this spacing permits concentrations to be read to two or more significant figures. On the slide rule, scales A and D are fixed, while B and C are movable. The standard position, where acid and base concentrations are equal, corresponds to the midpoint of any titration curve. In this position pH and pK are also equal.

When scales B and C are moved toward the right, the value of $(pA - pB)$ in any position becomes positive and equal to $(p\mathrm{H} - pK)$. Positive values are obtained for the upper half of the titration curve, where B is greater than A, negative values for the lower half of the curve. In all displacements, positive or negative, $(pA - pB)$ is equal to and of the same sign as $(p\mathrm{H} - pK)$. At all positions of scales B and C on scales A and D, the Henderson-Hasselbalch equation is satisfied. These properties are evident from the nature of the linear transformation (Fig. 1).

By the use of an accurate slide rule, any point of a titration curve may be determined in a few seconds. If pK of the buffer is known, the pH of a solution is determined by placing the concentration of base under the concentration of acid. The eight or ten points required for an accurate sigmoid curve may be computed within a minute or two. The converse problem is that of finding the concentrations of acid and base required to yield a buffer of given pH. In this case the value of pK is placed over pH; base concentration is read under acid concentration. In the illustration (Fig. 2), pK of acetic acid, 4.7, is placed on the desired pH, 5.13. At any part of the concentration scales the ratio B/A is 8/3, or 2.67. This number is approximately the antilog of 0.43, the difference between pH and pK. With the slide rule, tables of logarithms or graphs are not required, and calculations of this kind may be made in a few seconds.

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Property of Cerebrospinal Fluid Associated with Disturbed Metabolism of Central Nervous System

Abstract. Cerebrospinal fluid was assayed for the capacity to contract smooth muscle and for the capacity to develop such activity when incubated with globulin. Activity was observed in fluid collected from patients with inflammatory or degenerative disease of the central nervous system, sustained intracranial vasodilatation, sustained noxious stimulation, or chronic schizophrenia. Control specimens lacked measurable activity.

Cerebrospinal fluid was collected by lumbar puncture from subjects with (i) active disease of the central nervous system (CNS) (recent cerebrovascular accident, neoplasm of the central nervous system, cerebral atrophy, multiple sclerosis), (ii) inactive nonprogressive or no disease of the central nervous system (myasthenia gravis, progressive muscular dystrophy, "old" cerebrovascular accident), (iii) vascular headache of the migraine type, (iv) sustained noxious stimulation and pain arising from disorders of the legs or pelvic organs, (v) disease syndromes classified as chronic schizophrenia (all were hospitalized males between 20 and 50 years of age; they were free of other significant disease and were not overactive or assaultive; some had experienced hallucinations and most had delusions; all were ambulatory and none exhibited evidence of dietary insufficiency; none had had induced convulsions in the year previous to study). Specimens either were assayed at once or were immediately stored in solid carbon dioxide.

The spinal fluid so collected was assayed for its capacity to contract smooth muscle (isolated rat uterus or guinea pig ileum suspended in a 10-ml saline bath at 29°C). The sensitivity of the preparation was assessed by the response of the muscle to 0.2 ml of a polypeptide (bradykinin) standard prepared by the action of trypsin on globulin. (If the response to this material was inadequate, the preparation was discarded.) Freshly collected or thawed cerebrospinal fluid (0.2 ml) was added to the chamber. If contractions were observed the specimen was recorded as inducing a type I reaction. If no contractions were observed, 0.2 ml of the specimen was incubated with 0.2 ml of an 8 percent solution of bovine globulin (fraction II) for 3 minutes at 29°C. If the resultant mixture induced contractions, the specimen was recorded as inducing a type II reaction. If no contractions were observed, the specimen was recorded as negative. The results of this bioassay are shown in Table 1.

The contractions so induced by cerebrospinal fluid or by the incubated mixtures were not inhibited by atropine, antihistaminics (histamine contracts guinea pig ileum but not rat uterus), or dihydroergotamine in amounts large enough to inhibit contractions induced by acetylcholine, histamine, or serotonin, respectively. They were inhibited by slightly larger amounts of dihydroergotamine, by lysergic acid diethylamide, by salicylate, and by soluble adrenal steroids. Prolonged (1 hour) incubation with plasma diminished the response.

Cerebrospinal fluid or incubated mixtures that induced contractions of the smooth muscle also induced pain when applied to an exposed blister base, lowered the blood pressure of a cat when injected into the venous circulation, dilated minute vessels when applied to the bulbar conjunctiva, and increased capillary permeability as indicated by increased spreading of dye in the region surrounding an intradermal injection of the material in a guinea pig with dye injected into the venous circulation.

These properties are similar to those of vasodilator polypeptides derived from