# The Test of Simplicity

## Simplicity is a test of the effectiveness of scientific theories; but what is the test of simplicity?

Nelson Goodman

All scientific activity amounts to the invention of and the choice among systems of hypotheses. One of the primary considerations guiding this process is that of simplicity. Nothing could be much more mistaken than the traditional idea that we first seek a true system and then, for the sake of elegance alone, seek a simple one. We are inevitably concerned with simplicity as soon as we are concerned with system at all; for system is achieved just to the extent that the basic vocabulary and set of first principles used in dealing with the given subject matter are simplified. When simplicity of basis vanishes to zero—that is, when no term or principle is derived from any of the others—system also vanishes to zero. Systematization is the same thing as simplification of basis.

Furthermore, in the choice among alternative systems, truth and simplicity are not always clearly distinguishable factors. More often, simplicity is one test of truth. The martyr-bestrewn debate over what bodies in the universe are fixed has turned into the question: What choice of points of reference will give us the simplest description of their relative motions? And by now, after Duhem and Poincaré (*1*), it is almost a commonplace that the refutation of a mode of scientific explanation such as the Ptolemaic or the Newtonian consists not in showing that it is inapplicable but in showing that its application would be intolerably complex.

The case can be put even more strongly. We want to select a system or hypothesis that not only agrees with the established evidence but also predicts correctly the outcome of further observations and experiments. Thus selection of a theory must always be made in advance of the determination of some of the facts it covers; and, accordingly, some criterion other than conformity with such facts must be applied in making the selection. After as many points as we like have been plotted by experiment concerning the correlation of two functions (for example, of time and deterioration of radioactivity), we predict the remaining points by choosing one among all the infinitely many curves that cover the plotted points. Obviously, simplicity of some sort is a cardinal factor in making this choice (we pick the "smoothest" curve). The very validity of the choice depends upon whether the choice is properly made according to such criteria. Thus simplicity here is not a consideration applicable after truth is determined but is one of the standards of validity that are applied in the effort to discover truth.

### Nature of the Problem

But if simplicity is a test of truth and systematization, what is the test of simplicity? Explication of the standards of simplicity constitutes one of the most pressing current problems in the philosophy of science. The scientist, however, may be inclined to ask why there is any *problem* here. Given two alternative systems covering the same subject matter, isn't it always pretty clear which, if either, is the simpler? How does any puzzle arise? There are two good answers.

In the first place, the difficulty and significance of formulating precise general criteria of simplicity no more depend upon trouble encountered in making particular judgments of simplicity than the difficulty and significance of codifying deductive logic depend upon trouble encountered in ordinary reasoning. The systematic logic developed by Aristotle, Boole, Whitehead and Russell (*2*), and others is only incidentally a tool for drawing or correcting inferences needed in ordinary life or in the laboratory. Even the energetic investigation of inductive logic since Mill (*3*) aims much less at providing instructions for making predictions than at eliciting the laws of induction for their own sake. The utility that the results may have for the practicing natural scientist is as much a by-product as the utility that the scientist's results may have for the technologist. Investigation of the canons of deduction or induction or simplicity no more derives its main interest from the help it may give to physicists or biologists than investigation of the laws relating mass and energy derives its main interest from the help it may give to munitions-makers or surgeons.

But in the second place, comparative simplicity is often not very readily and surely judged. Of course if we succeed in deriving one of a set of hypotheses or concepts from the others, the saving is obvious. But comparison of theories incorporating different hypotheses or concepts can offer great difficulty. For example, just when does introduction of the concept of a new fundamental particle simplify and when does it complicate physics? Sometimes different aspects of over-all simplicity may set up competing claims. Is a genuine simplification achieved by deriving mathematics from a few logical concepts at the cost of three big volumes of complicated formulae? We may smugly reply "yes," on the ground that what counts is the simplicity of the basic notions and postulates, and that the derivation is bound to be complicated just to the extent that the basis is simplified. But we seem to take the opposite view when we rate description of the motions of astronomical bodies in terms of ellipses as simpler than that in terms of circles; for the advocate of epicycles might argue that the more elaborate constructions and computations required by his system are the symptomatic result of the greater simplicity of his elementary concepts. If we are tempted to dismiss the idea that circles are simpler than ellipses as a mere superstition, we shall be embarrassed by the fact that, as remarked above, we employ some such notion of the relative simplicity of different curves when we choose one to fit to plotted points in order to extrapolate from determined data to untested cases. Plainly, simplicity is not a single easily estimated characteristic of systems but several different interrelated characteristics, few of them easy to estimate. Thus simplicity is a problem for the scientist as well as for the philosopher. Canons of simplicity need to be formulated not only

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for their intrinsic interest but also as means for making needed judgments in actual scientific investigation.

A more plausible excuse for neglecting the study of simplicity has been that the problem, far from being too easy, is hopeless. Simplicity, the argument runs, is vague, ambiguous, variable, and subjective, and therefore too elusive for measurement. But exactly the same argument might have been urged in primitive times against the possibility of measuring temperature or size. Ordinary judgments of size vary with distance, perspective, atmosphere, color, eyesight, and even with interest. Size may mean total bulk, or it may mean maximum diameter, or it may mean height and length and breadth or any of many other quantities. And size changes with temperature, pressure, growth, and wear. The arguments against the measurability of simplicity would, indeed, have been equally strong against the measurability of almost anything. Precision, fixity of meaning, verifiability, and objectivity are the results of measurement, not preconditions of it.

What the problem of simplicity needs is a lot of hard work. So far, just a little has been accomplished; the entire bibliography of contributions to the subject hardly lists more than a dozen items (*4*, *5*), most of them published during the past 15 years. Thus the problem is not only one of the most important in the philosophy of science but also one of the newest to be tackled seriously. We are still seeking proper formulation of some aspects of the problem, still exploring avenues of approach to others. Yet this gives the whole matter added interest; for here we can observe philosophy, and therefore science, in the early, formative stages of a typical development from a nebulous cluster of difficulties into articulated questions and on towards an organized discipline.

## Simplicity of Basic Terms

We must begin by staking out a very small part of the problem for concentrated attention (*5*). A theory is a system of statements. I shall be concerned here solely with the simplicity of the set of concepts, or the vocabulary of terms, employed in these statements. Furthermore, since some words and symbols—like "and," "or," "not," "if . . . . then," "all," "some," "=," or translations of these—are logical apparatus common to all the systems in question, we need consider only the remaining, extralogical terms. Such of these as are not defined in the system are called "primitive" and constitute the extralogical basis of the vocabulary of the system. It is the simplicity of such extralogical bases that I want to examine here.

Among the extralogical terms of a system may be property-terms like ". . . . is acid," and relation-terms of various degrees, like ". . . . is larger than _____" and ". . . . lies halfway between _____ and ..........." The examples given are respectively a one-place, a two-place, and a three-place predicate and may be abbreviated, in standard symbolic notation, as "$A(x)$," "$L(x,y)$," and "$B(x,y,z)$." In general, an $n$-place predicate has $n$ blanks or variables and stands for an $n$-adic relation.

Many of the most familiar terms occurring in scientific theories are not predicates but rather nonassertive function-terms, like "the father of . . . .," "the temperature of . . . .," "the distance between . . . . and _____." However, for every function of $n$ arguments there is a corresponding $n+1$-adic relation; and for uniformity, let us suppose that in the extralogical bases under consideration all function-terms have been eliminated in favor of corresponding predicates. Thus the function-term "the father of . . . .," or "$f_x$," gives way to the predicate "_____ is the father of . . . .," or "$F(x,y)$"; and the function-term "the distance between . . . . and _____," or "$d_{x,y}$" gives way to the predicate "........... is the distance between . . . . and _____," or "$D_{(x,y,z)}$."

Some predicates, like ". . . . is wooden" and ". . . . is a more ancient fossil than _____," are predicates of things or individuals. Others, like ". . . . are rare" and ". . . . is a subset of _____," are predicates of classes of things. Still others are predicates of classes of classes, numbers, and so on. Some predicates, like ". . . . is the temperature of _____" and ". . . . is a member of _____," are heterogeneous, relating numbers to things, or things to classes, and so forth. To avoid having to deal at the start with differences in simplicity arising from such differences in type, let us require that in every case under consideration all the predicates be of a single homogeneous type—that is, that all apply solely to things, or all apply solely to classes of things, and so on. Our results can afterwards be extended to cover cases where predicates of differing types are involved.

One more temporary assumption is that all predicates in a basis are applicable—that is, that none are like ". . . . is a centaur" or ". . . . is a square circle" in applying to nothing. Although this may seem a common-sense restriction, we may sometimes want to determine the simplicity of a predicate when we are not sure whether it applies to anything. However, the restriction can later be easily removed.

Still further specification of our immediate problem is needed. We are not concerned with the purely grammatical construction of predicates or with their length. Nor are we concerned with their relative familiarity or the ease of comprehending them. These aspects of simplicity and many others may be philosophically and scientifically significant, but our present concern is rather with that logical or structural simplicity of bases which pertains directly to the degree of systematization of theories founded upon them. Just what constitutes such structural simplicity is indeed part of our problem and will have to be made clearer as we proceed. But we may note here that predicates in different languages, or in entirely different words of the same language, or quite unlike in grammatical composition, may be equally simple in this sense, and that for two predicates to apply in exactly the same instances—that is, to have the same denotation or extension—is a sufficient though not necessary condition for their having the same degree of structural simplicity.

In summary, then, we want to find a way of measuring the structural simplicity of the set of undefined extralogical terms of a theory or system. That is, we want to be able to assign to any such set of terms a number that will indicate the complexity of that set and, accordingly, one significant aspect of the complexity of the theory. We begin by assuming that all the terms under consideration are applicable predicates belonging to some one homogeneous type. But they may have any number of places, and of course the several predicates in a given basis need not have the same number of places.

## A Clue to an Answer

The first step toward measuring the size of objects must have been to fix upon a single elementary clue: the application of a yardstick of some sort directly to the object in a certain way. This at once abstracted from apparent differences resulting from variations in distance and perspective, picked out as standard certain among the innumerable dimensions of familiar objects, and provided a unit for the numerical expression of size. We seek some comparable clue to the problem of measuring the simplicity of extralogical bases.

From the primitive predicates constituting the extralogical basis of a system, with the help of the specified logical apparatus, all other extralogical terms of the system are generated by definition. One might first think of measuring complexity by definitional yield, or defining-power, on the principle that if one set of predicates is definable from a second but the second is not definable from the first, then the second is more complex, and that interdefinable sets of predicates are equally complex. For example, we can define ". . . . differs by one from ____," among the natural numbers, from ". . . . is the immediate successor of ____" (as follows: "$x$ differs by one from $y$ if and only if $x$ is the immediate successor of $y$, or $y$ is the immediate successor of $x$") but not vice versa; hence a basis consisting solely of the former predicate would be rated simpler, according to this proposal, than a basis consisting solely of the latter predicate. Yet, plausible as this idea may be in a few such cases, it is quite mistaken as a general principle, for it would have the consequence that no simpler basis could be found for a system than that arrived at by merely taking all the extralogical terms of the system as primitive! For, since any adequate basis for a system must yield, through definition, all the terms of the system, the set of all these terms would, by the proposed criterion, be as simple as any adequate basis of fewer of these predicates. What is actually maximal complexity would be accounted maximal simplicity.

Counting the predicates in a basis seems, offhand, a better test of simplicity, but this likewise fails. For if the number of predicates in a basis were the sole measure of complexity, ultimate simplicity would always be achieved in a purely trivial way. Any number of predicates can be readily combined, so to speak, into a single predicate. A basis consisting of a one-place predicate (say ". . . . is copper") and a two-place predicate (say, "____ is more durable than ............") can always be replaced by a single three-place predicate (say, ". . . . is copper, and ____ is more durable than ............"). And, taking all the predicates in a system, we can immediately construct a single many-place predicate from which all are definable by a routine procedure. If the total number of places in those original predicates is $m$, the single primitive thus arrived at will have $m$ places. Obviously, no genuine increase in simplicity is effected in this way; many simpler predicates are replaced by a single correspondingly more complex one. The proposal to measure the complexity of bases by a mere counting of the predicates they contain fatally ignores all differences of complexity among predicates.

The idea that inevitably suggests itself next is to measure complexity by the total number of places in all the predicates in a basis. The spurious simplicity effected by artificially combining several predicates into one will thus be properly discounted. But the new proposal is again too hasty. While we have clear grounds for not regarding an $m$-place predicate as simpler than a set of several predicates with a total of $m$ places, we have no such grounds for not regarding the set of predicates as simpler than the single predicate. Replacement of set by single predicate is always possible, but not replacement of single predicate by set. For example, the two one-place predicates ". . . . is a parent" and ". . . . has a parent" will not serve instead of the two-place predicate ". . . . is a parent of . . . ."; to say that $x$ is a parent and $y$ has a parent is not to say that $x$ is a parent *of* $y$. The complexity of bases seems to vary not only with the number of places but also in some manner with the way this number is distributed among predicates in the basis. Several plausible formulas for this variation can easily be devised, but there is no immediately obvious method for choosing among them.

Not only have our efforts been unsuccessful so far, but trying out one rule of thumb after another begins to look like an unpromising method of attacking our problem. Yet, all the time, the clue we want has lain ready in our hands. In rejecting the mere counting of primitives as a measure of complexity, we argued that any set of predicates can always be replaced by a single predicate. We tacitly appealed to the principle that replacement of a basis by another effects no genuine simplification where such a replacement can always be made by a purely routine procedure. Other applications of the same principle come quickly to mind. Predicates can always be added to a basis without destroying its adequacy, and a predicate can always be replaced by another having more places; obviously, in neither case is simplicity increased. In other words, an elementary principle applied in judging proposed measures of complexity is this: If every basis like a given one can always be replaced by some basis like a second, then the first is not more complex than the second. This may seem too meagre and negative a principle to carry us very far, but it is the key to our problem. More carefully formulated, clarified, and supplemented, it will constitute the fundamental axiom of a calculus of simplicity.

## First Axioms of Simplicity

Let us, then, adopt as our first postulate:

> **P1.** *If every basis of a relevant kind K is always replaceable by some basis of a relevant kind L, then K is not more complex than L* (that is, $K$ does not have a higher complexity value than $L$—or, briefly, when "$v$" stands for "the complexity-value of," $vK \leqq vL$).

Now a good many points here call for explanation. In the first place, to say that every basis of kind $K$ is *always replaceable* by some basis of kind $L$ is to say not only that there always is some equivalent basis of kind $L$ but also that we can always find one; that is, that given any basis $B$ of kind $K$, with no further information than that $B$ is of kind $K$, we can define in terms of $B$ and the stated logical apparatus alone some basis $B'$ of kind $L$ such that we can redefine $B$ from $B'$ and the logic alone.

In the second place, our postulate speaks of the complexity of kinds rather than of bases. A kind has the highest complexity-value possessed by any basis of that kind, and a basis has the complexity-value of the narrowest relevant kind to which the basis belongs.

In the third place, what are the *relevant kinds*? Structural kinds, certainly, since we are concerned with structural complexity. But not every structural difference constitutes a difference in relevant kind; for if that were the case, then, since definition from a basis always depends upon structural features, our postulate would in effect reduce to the test in terms of defining power that we have already rejected. What will constitute a relevant kind depends upon the fact that our postulate is intended to express the principle that *purely routine* replacement effects no genuine simplification.

Now, replacement of a basis $B$ by another, $B'$, is purely routine if every basis like $B$, or of the same broad sort or general kind as $B$, is always replaceable by some basis of the same general kind as $B'$. The ordinary notion of a "broad sort" or "general kind" is vague and has to be supplanted by something much more clear-cut. As a first approximation, we may define such a relevant kind as any class of bases delimited by specifying the number of predicates in a basis and the number of places in each predicate, together with any or no information concerning the three most commonplace properties of predicates: reflexivity, transitivity, and symmetry. (At the moment, the reader need not understand what these properties are; they will be explained presently, and our tentative definition of relevant kinds will be somewhat revised). Thus, for example, the class of all bases consisting of a two-place and a one-place predicate is a relevant kind; so also is the narrower class of bases consisting of a symmetric two-place predicate and a one-place predicate. Our tentative definition of relevant kinds will presently be further explained and somewhat revised.

One may ask what justifies this particular interpretation of "broad sort" or "general kind," this particular decision in spelling out the imprecise notion of purely routine replacement. Quite plainly, no precise interpretation can claim to be uniquely indicated. Developing any method of measurement is a process of forging a sharp and effective tool from rough practice. We must look to the practice where it offers us guidance and, at the same time, remove obscurities, resolve conflicts, and fill in gaps, by rulings designed to yield the most significant results. A method of measuring anything must meet taxing but somewhat elusive demands of faithfulness and serviceability, but no method is exclusively correct. Thus our chosen definition of relevant kinds must find its justification in the combination of its plausibility as a translation of the rough notion of "general kinds" with the overall acceptability of the calculus of measurement to which this interpretation is a contributing factor.

From postulate 1, we can show that two kinds are equally complex if every basis of each kind is always replaceable by some basis of the other kind, and we can derive complexity-value equations for many kinds of bases. But in order to derive inequalities—to show that certain kinds are more complex than certain others—we need something more. This is provided by a prosaic postulate to the effect that a basis consisting of some predicates is more complex than a basis consisting of none, and that the value of a basis can be computed by adding the complexity-values of the predicates in it.

**P2.** *Every predicate in an extralogical basis has a positive complexity-value, and the value of the basis is the sum of the values of the predicates in it.*

The second clause operates to exclude from consideration, in determining complexity-values, any interconnections among the predicates in a basis. But such interconnections will be taken into account later, after a primary scale of complexity-values has been established.

## Development of a Calculus

Easily proved from these postulates are certain elementary theorems, such as that *where m is less than n, the complexity-value of the class of m-place predicates* (that is, of the kind of basis that consists of a single *m*-place predicate) (6) *is less than the complexity-value of the class of n-place predicates.* But the further development of the calculus requires treatment of more specific properties of predicates and becomes highly complicated. I shall describe it here very sketchily, merely to suggest its general character and the results obtained.

1) Two-place predicates are *irreflexive* if they never relate anything to itself. Thus, ". . . . is a parent of _____" is irreflexive. On the other hand, the predicate ". . . . has the same blood type as _____" is reflexive, since everyone has the same blood type as himself. Some predicates, like ". . . . has a brother in common with _____," are neither reflexive nor irreflexive; every person with a brother, but no person without, has a brother in common with himself. One-place predicates are, degenerately, both reflexive and irreflexive.

Reflexive predicates are interreplaceable with irreflexive predicates and hence they are equal in complexity. A two-place predicate "P" that is neither reflexive nor irreflexive has the same complexity-value as a set of two irreflexive predicates: a two-place predicate that relates every two distinct elements, $x$ and $y$, related by "P," and a one-place predicate applying to every $x$ that "P" relates to $x$ itself.

With predicates of more than two places, the varieties of reflexivity-properties multiply rapidly, for a many-place predicate may be reflexive or irreflexive or nonreflexive with respect to all or to any given selection of its places. But fortunately, *the complexity of any basis can be proved to be equal to that of a certain basis consisting solely of thoroughly irreflexive predicates* (that is, irreflexive with respect to all their places). Every other predicate of $n$ places in the basis gives way to a set of one or more thoroughly irreflexive predicates, each having not more than $n$ places. Thus we can confine our attention to thoroughly irreflexive predicates.

2) A two-place predicate, like ". . . . is greater than _____," is said to be *transitive,* since if $x$ is greater than $y$ and $y$ is greater than $z$, then $x$ is greater than $z$. However, transitivity proves to be less pertinent to complexity measurement than (and is now to be supplanted as a defining property of relevant kinds by) a stronger property of predicates that may be called *self-completeness.* If two one-place predicates—say, ". . . . is red" and "_____ is white"—are compounded into one two-place predicate—say, ". . . . is red, and _____ is white"—the latter is self-complete. In general, a self-complete two-place predicate is such that if it joins $x$ to $y$ and also $z$ to $w$ (and if all these except possibly $y$ and $z$ are different), then it joins $x$ to $w$. Predicates of more places may likewise be self-complete with respect to all their places: for example, the predicate ". . . . is red, and _____ is white, and .......... is square." Such predicates (whatever their number of places) are, figuratively, rather unstable; they break down easily into one-place predicates. For that reason, thoroughly self-complete predicates do not often occur in actual systems; but for the same reason, their consideration is important for our present purposes. Their resolubility enables us to show that *the complexity-value of an n-place thoroughly self-complete predicate is equal to the complexity-value of n one-place predicates.* This is a crucial step towards determination of the general relationship between the complexity-values of predicates differing in number of places.

3) A two-place predicate is *symmetric* if it pairs elements in both directions whenever it pairs them in either; for example, ". . . . is a sibling of _____" is symmetric, since everyone is a sibling of anyone he or she has as a sibling. The three-place predicate ". . . . and _____ and .............. are triplets" is also symmetric with respect to all its places, and a predicate of more places that is simi-

larly entirely order-indifferent is likewise thoroughly symmetric.

We saw that a self-complete two-place predicate has the same complexity-value as two one-place predicates; but a *symmetric* self-complete two-place predicate has the same value as a *single* one-place predicate. Such a predicate merely pairs in both directions every two elements of a set. Likewise, *an n-place thoroughly symmetric and thoroughly self-complete predicate* merely combines in all directions (or applies to all permutations of) every $n$ elements of a set, and such an $n$-place predicate *has the same complexity-value as a single one-place predicate.*

4) Complications pile up fast when we consider predicates that are only partially, rather than thoroughly, self-complete or symmetric. For instance, many-place predicates may be symmetric with respect to some rather than all of their places. If $x$ lies on a straight line between $y$ and $z$, it follows that $x$ lies on a straight line between $z$ and $y$, but not that $y$ lies on a straight line between $x$ and $z$. Thus the three-place predicate ". . . . lies on a straight line between ——— and ..........." is symmetric with respect to the last two, but not with respect to all, of its places.

Again, a predicate may be symmetric with respect to sequences of its places rather than with respect to its places severally. For example, if $x$ is exactly as much greater than $y$ as $w$ is greater than $z$, then $w$ is also exactly as much greater than $z$ as $x$ is greater than $y$. The four-place predicate in question here is not symmetric with respect to any two or more of its places severally but is symmetric with respect to pairs of its places: the pair of its first two and the pair of its last two places.

Similarly, self-completeness may occur with respect to sequences of places rather than with respect to places severally. And a single predicate may exhibit many varieties of symmetry and self-completeness at once.

All this makes the full treatment of our problem very intricate. Briefly, what we do is define the *symmetry index* of a predicate as a certain function of all the symmetries the predicate has, and also define the *self-completeness index* of a predicate in a comparable way. We then examine how complexity varies in relation to these indices.

5) This examination, carried out with the help of two supplementary postulates, yields the means for determining the complexity-value of any relevant kind of basis as a function of two constants (either but not both of which may occur vacuously): the complexity-value of one-place predicates and the complexity-value of two-place irreflexive predicates. In order to achieve a fully quantitative measure, a final postulate is needed to fix the numerical value of these constants. This postulate stipulates that all, and only, those kinds of bases that can be shown by preceding postulates to have the same value as one-place predicates shall have the value 1, and that all other kinds shall have the lowest integral value consistent with this requirement and with preceding postulates. Assignment of the value 1 is a mere convenience; we might have used some arbitrary constant $c$. The choice of integral values is always indicated in any scheme of measurement where, as here, use of nonintegral values can be avoided.

## Resultant Simplicity Formulae

Some of the resulting complexity-values are given by the following formulae:

1) The class of $n$-place thoroughly irreflexive predicates has value $2n-1$.

2) The class of $n$-place thoroughly irreflexive and thoroughly self-complete predicates has the value $n$.

3) The class of $n$-place thoroughly irreflexive and thoroughly symmetric predicates has the value $n$.

4) The class of $n$-place predicates that are thoroughly irreflexive, self-complete, and symmetric has the value 1.

More generally, *the value of any kind of n-place irreflexive predicate is $2n-1$ minus the sum of its symmetry and self-completeness indices.*

When the limitation to irreflexive predicates is dropped, the values of relevant kinds are obtained, as suggested earlier, by computing the value of corresponding kinds of bases consisting of irreflexive predicates. These values rise very rapidly with the number of places; the class of all two-place predicates has the complexity-value 4, and that of all three-place predicates, the value 15.

Some temporary exclusions made earlier can now be removed. In the first place, the restriction to applicable predicates is easily eliminated without affecting any of our results; for since inapplicable predicates have zero complexity, adding them to a relevant kind cannot increase its complexity-value. In the second place, where the predicates in question are not all of one homogeneous type (but meet certain conditions of finitude), available methods of correlating each predicate with a set of predicates of the lowest type may be applied, and the complexity-values may then be readily calculated. Finally, our temporary exclusion from consideration of interconnections between predicates is compensated for by adoption of a secondary rule for choosing in certain cases between bases of different structure but of equal computed complexity-value.

## Present Status of Simplicity Study

The calculus I have outlined is virtually complete; proofs for all theorems are currently being checked. However, some possibilities for improvement still need consideration, and other writers have entered some objections and have proposed modifications and alternatives (*7*). Investigation of the measurement of the structural simplicity of the extralogical bases of theories is unlikely to reach its stage of ultimate stagnation for some time. But that can also be said of many an older scientific inquiry. I hope enough has been said to show that our problem has at least been carried some steps away from its stage of initial confusion.

We have been dealing, it must be remembered, with a very small corner of the big problem of simplicity. We have considered the simplicity of terms but not of postulates framed in these terms. We still have no measure of the over-all simplicity of theories. Nor does our calculus answer the crucial questions involved in the fitting of curves and in induction generally (*8*). But perhaps the progress made on one aspect of the problem will somewhat alleviate despair concerning the rest. If some of the remaining questions seem too vague to be amenable to precise formulation and treatment, we may well reflect that most scientific problems seemed that way once. The obscurities of problems are due less to the subject matter than to short-comings in its investigation.

### References and Notes


1. P. Duhem, *Système du Monde* (Paris, 1913); H. Poincaré, *Science et Methode* (Paris, 1908).
2. G. Boole, *An Investigation into the Laws of Thought* (London, 1854); A. N. Whitehead and B. Russell, *Principia Mathematica* (Cambridge, England, 1910–1913).
3. J. S. Mill, *A System of Logic* (London, 1843).
4. The articles mentioned in reference 5 together contain a fairly complete listing of relevant publications, from an article by Lindenbaum in 1935 to the present: Karl Popper's discussion in *Logik der Forschung* (Vienna, 1935) should be added to the list.
5. The treatment of simplicity to be outlined here

has been developed in the course of several of my publications, beginning with "On the simplicity of ideas" [*J. Symbolic Logic* 8, 107 (1943)] and including chapter 3, sections 3–7, of *The Structure of Appearance* [Harvard Univ. Press, Cambridge, Mass., 1951]. The most recent articles, and those most closely followed here, are "Axiomatic measurement of simplicity" [*J. Philosophy* 52, 709 (1955)] and "Recent developments in the theory of simplicity," in *Philosophy and Phenomenological Research*, in press.

6. I shall often abbreviate such a locution as "the complexity-value of the kind of basis that consists of one two-place predicate" to "the complexity-value of two-place predicates" or to "the complexity-value of a two-place predicate."
7. A discussion of these will appear in "Recent developments in the theory of simplicity," in *Philosophy and Phenomenological Research*, in press.
8. On this topic, see I. Scheffler, "Inductive inference: A new approach" [*Science* 127, 177 (1958)] and N. Goodman, *Fact, Fiction, and Forecast* (Harvard Univ. Press, Cambridge, Mass., 1955).

# Richard Goldschmidt, Biologist

On 12 April 1958, Richard Goldschmidt was 80 years old. He was recovering from a nearly fatal illness which had struck a few weeks earlier. Letters from friends and well-wishers arrived from many lands, praising his achievements as a biologist, a zoologist, and a geneticist, expressing amazed admiration for his continued scientific productivity, and professing warm personal feelings of affection. That birthday, quietly spent with his family, was a happy day in a period of years in which physical pain and fear that the body might force the mind into inactivity were ever-present. Less than two weeks later, on 24 April, the end came.

Richard Goldschmidt's ancestors belonged to respected families who had lived for centuries in Frankfurt am Main. He received an excellent education at the Gymnasium and at an early age decided to become a naturalist. In 1896 he entered the University of Heidelberg where, to use his own words, he "had such glorious teachers as Bütschli, Gegenbaur, Kühne, V. Meyer, Kossel, Rosenbusch." After a short period in Munich under Richard Hertwig, he returned to Heidelberg and obtained the Ph.D. degree in 1902. From 1903 to 1913 he worked and taught in Munich. In 1914 Goldschmidt was selected by Boveri to join him as a member of "the newly founded . . . wonderful Kaiser Wilhelm Institut für Biologie, Berlin-Dahlem." Soon after, for reasons of health, Boveri had to give up his projected move to the Institut. Goldschmidt accepted his appointment, which extended over 22 years; for the last 15 of these he served as a director. In 1936, after the passage of the "Nürnberg Laws," he was invited to join the zoology department of the University of California. "This turned out to be one of the most happy events of my life, crowned by becoming an American citizen in 1942"—thus he wrote in an autobiographical sketch filed, by request, with the National Academy of Sciences in Washington (1948). By then he had, according to American custom, included a middle name in the by-line of his publications—Richard Benedict Goldschmidt.

In Berkeley he taught genetics and cytology for more than a decade and uninterruptedly continued his research for 22 years. Reports on experiments and books on wide-ranging subjects followed one another. Even after his death two papers appeared in print.

Goldschmidt's work covers nearly 60 years of tireless productivity. When, in 1954, he compiled a list of his 17 books and approximately 250 papers, he divided the latter into the following classes: protozoology (1904–07); cytology (1902–50); embryology (1900–35); histology and neurology (1903–10); acrania (1905–33); gynandromorphism (1922–37); intersexuality (1911–51); general sex determination, sex-controlled heredity (1910–53); genetics and evolution (1911–53); genetics: Mendelian analysis and general (1913–54); physiological genetics (1916–52); human heredity (1927–53); biographical, popular science, *varia* (1916–53).

He listed his books under "technical," "textbooks," "popular," and "travel." Among these were such books as *Die quantitativen Grundlagen von Vererbung und Artbildung* (1920); *Mechanismus und Physiologie der Geschlechtsbestimmung* (1920); *Physiologische Theorie der Vererbung* (1927); *Die sexuellen Zwischenstufen* (1931); *Physiological Genetics* (1938); *The Material Basis of Evolution* (1940); *Theoretical Genetics* (1955); *Einführung in die Vererbungswissenschaft* (first edition, 1911; fifth edition, 1928); *Ascaris, eine Einführung in die Wissenschaft vom Leben* (first edition, 1921; third edition, 1953); and *Neu-Japan* (1927). Translations of his books appeared in English, French, Hebrew, Japanese, Polish, Russian, Spanish, and Yugoslavian. His latest volume, the charming *Portraits from Memory, Recollections of a Zoologist* (1956), is in the process of being translated into German. An autobiography went to press this spring.

Goldschmidt's influence on the biology of the 20th century rested on observation and experiment as well as on the theory-building sweep of his imagination. His outstanding experimental accomplishment was the long series of crosses between geographical races of the gypsy-moth *Lymantria*. It led to an analysis of the phenomenon of intersexuality which went far beyond the framework of classical genetics. He had early trained himself to be a revolutionary of science. He reached his height in his endeavors to build a dynamic physiological genetics on the static and material basis which Mendel and Morgan had laid, and which he admired, as such, without reservation. He raised his voice in warning of a too ready acquiescence in apparently established concepts of the gene and some widely held genetic interpretations of evolution. He was willing to face the strong opposition to his unpopular ideas, but he lived to see them move into the forefront of contemporary thought.

He lectured before thousands of eager listeners—students, colleagues, men of other professions, and interested lay people—in Europe, America, Asia, and Australia. Three periods which he spent