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High Atmosphere Densities

To explain satellite observations, models of the thermosphere must allow for diffusion and other factors.

Marcel Nicolet

Although the densities of the high atmosphere which are extrapolated from rocket data usually lead to low values at altitudes above 300 km, recent values obtained from observations of satellites by Harris and Jastrow (*1*) and by Paetzold (*2*) are best explained by assuming that the densities are relatively high—greater than 10^{-15} g/cm^3 at 500 km.

For example, Kallmann, White, and Newell (*3*) derived a value of 4.4×10^{-15} g/cm^3 at 300 km, and more recently, Miller (*4*) obtained only 2.7×10^{-16} g/cm^3 at 500 km. Because of the current impossibility of finding an atmospheric model based on the usual concept which can explain the satellite observations and even certain ionospheric properties, it is thought that the upper thermosphere has a very high temperature; only thus can the escape time of helium (of the order of 2 million years) be explained as reported by Nicolet (*5*).

This article shows how the concept of the thermosphere must be considered according to the view of Chapman (*6*) on the extension of the solar corona toward the earth's atmosphere and according to the atmospheric structure determined by Nicolet (*7*), who introduced diffusion distributions for all constituents at altitudes above 110 km.

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The Thermosphere

The region above the deep minimum of temperature, near 85 km, corresponds to the upper atmosphere, in which the temperature increases with height. If the temperature distribution depends only on the rate of absorption of the energy emitted by the sun in the ultraviolet, it is not possible to have any increase in temperature above 300 km. For this reason, the atmospheric model adopted by Johnson (*8*) is limited to a temperature maximum of about 1100°K at 300 km. Furthermore, according to Lowan's (*9*) calculation, this model cannot be applied to night conditions since there is no solar energy available. Besides, Bates (*10*) has shown that it is difficult to maintain a large temperature gradient without very large amounts of energy being absorbed by the atmosphere at 200 km.

All the atmospheric models based on rocket results up to 200 km rest upon an assumption which requires a constant ratio—namely,

$$\frac{n(\mathrm{O}) + 2n(\mathrm{O_2})}{n(\mathrm{N}) + 2n(\mathrm{N_2})} = \tfrac{1}{4},$$

where n is the concentration—that is to say, they rest on conditions prevailing during perfect mixing such as is found in the lower atmosphere. Such a hypothesis leads to a deficiency of atomic oxygen in the thermosphere if the concentration of molecular oxygen observed up to 180 km by Byram, Chubb, and Friedman (*11*) is taken into account in the computations. However, a restriction such as that imposed by a constant oxygen-nitrogen ratio does not apply to aeronomic conditions in which transport is involved. First, the vertical distribution of atomic and molecular oxygen in the thermosphere above 100 km must be considered to be due to diffusion. [Nicolet and Mange (*12*) have shown that the oxygen distribution departs from photochemical equilibrium conditions]. Second, the production of almost 10^{11} oxygen atoms per square centimeter per second above 100 km modifies the constancy of the oxygen-nitrogen ratio since, as shown by Nicolet (*13*), atomic oxygen must recombine below 100 km after a downward transport from the highest altitudes. Finally, the absolute values of the atomic oxygen concentration should be fixed by the secular equilibrium between the total production of oxygen atoms and their recombination. Comparing the possible concentrations of atomic oxygen between the 100-km and 110-km levels, at which the downward transport and equilibrium conditions prevail, respectively, it is seen that

$$\tfrac{1}{2}n(\mathrm{N_2}) \leq n(\mathrm{O}) \leq n(\mathrm{N_2});$$

that is to say, the oxygen-nitrogen ratio is greater than the value given by a constant mixing ratio for which $2\,n(\mathrm{O}) \leq n(\mathrm{N_2})$.

It may be pointed out that an arbitrary hypothesis has been made to explain rocket data—that is, a low mean molecular mass as low as $M = 22.3$ at 180 km. Such a low mass corresponds to a constant mixing ratio in which dissociation of nitrogen is introduced. However, atomic nitrogen is not present to such an extent that the mean molecular mass is less than the minimum mass $M = 24$. Horowitz and Lagow (*14*), in analyzing the rocket data of densities, adopted, for the region between 100 and 180 km, the constant oxygen-nitrogen ratio used by Johnson (*8*). However, evidence for the dissociation of N_2 does not exist, and atomic nitrogen is only a minor constituent. When atomic nitrogen is produced in the thermosphere, it reacts immedi-

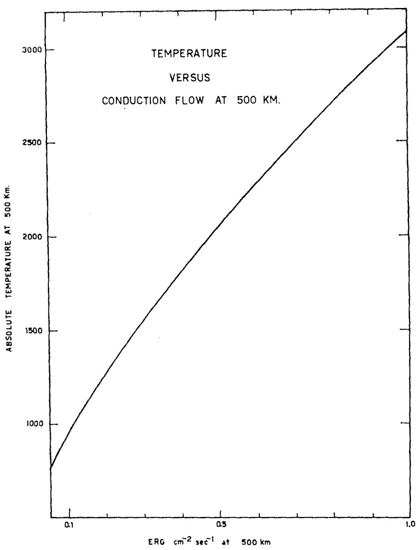

Fig. 1. Flow of energies between 0.1 and 1 erg cm^{-2} sec^{-1} in the atmosphere at an altitude of 500 km, where the principal constituent is atomic oxygen, leads to temperatures between 1000° and 3000°K.

ately, above 110 km, with molecular oxygen according to the process

$$N + O_2 \rightarrow NO + O$$

and disappears by the reaction

$$NO + N \rightarrow N_2 + O.$$

The life-time of a nitrogen atom is less than one day at altitudes below 200 km. Therefore there is insufficient dissociation of nitrogen, and any working model of the atmosphere can incorporate atomic nitrogen only as a minor constituent, while the concentration of nitric oxide is a definite fraction of the concentration of molecular oxygen in the whole atmosphere. A dissociation beginning at 120 km and being complete at 220 km, such as that adopted by the Rocket Panel (*15*), is not acceptable for studying the behavior of the thermosphere. In fact, vertical distributions corresponding to linear increases of the dissociation of oxygen or nitrogen cannot be stable when the diffusion times, as determined by Mange (*16*), are used. Nicolet (*7*), using the criteria of Mange, has shown that times corresponding to a change from a mixing distribution to diffusion can be found. Diffusion of all constituents from the mixing state is complete in about one day when it begins at an altitude of 120 km, but requires at least one week when it begins at an altitude of 100 km. In other words, whatever the mean molecular mass may be, it is not possible to maintain mixing above an altitude of 100 km when the process requires less than one day, and, therefore, diffusion of all constituents must take place between altitudes of 100 and 110 km. In the light of these results, the observational data of Townsend (*17*) on the argon-nitrogen ratio are interpreted by a diffusive separation which may begin at altitudes as low as 110 km.

Consequently, it may be concluded that the atmospheric mean molecular mass varies with altitude in the thermosphere and is associated with the varying ratio of the concentrations of atomic oxygen and molecular nitrogen.

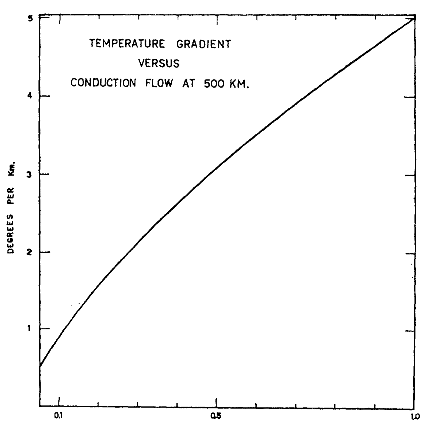

Fig. 2. The gradient of the temperature at 500 km depends on the conduction flow in an atmosphere of atomic oxygen. Gradients of between 1° and 5° per kilometer correspond to energies of between 0.1 and 1 erg cm^{-2} sec^{-1}.

Thermal Conductivity and the Thermosphere

If there is no thermal flux arriving at the top of the atmosphere, the production of heat by absorption of solar radiation does not permit, as was shown by Bates (*10*), very high temperatures above the electron peak of the ionospheric F layer to be inferred, for no ultraviolet radiation absorption process occurs there. The energy supply must be in accordance with Chapman's (*6*) deduction of heat flow by conduction due to the extension of the solar corona. Chapman found that it is possible for 2.4×10^{19} erg/sec to be available to the whole terrestrial atmosphere at a distance of five earth's radii. Thus, if any significant fraction of this energy is trapped, a high temperature above 300 km is possible. Furthermore, Chapman (*18*) has shown that atomic hydrogen must be involved in heat transport from the external corona to the earth's atmosphere, and since the thermal conductivity, λ_I, of the ionized gas is, according to Chapman (*6*),

$$\lambda_I = 5.2 \times 10^{-7}\ T^{5/2} \text{ erg cm sec °K}$$

and that λ_0 of neutral atomic oxygen is

$$\lambda_O = 3.6 \times 10^2\ T^{1/2} \text{ erg cm sec °K}$$

it is certain that at 500 km (the atmospheric region that we are considering here), the conductivity corresponds to a heat transport in the atomic oxygen gas.

Discussion and Results

We may therefore consider that there is an energy flow descending as heat in the upper part of the thermosphere. For example, the temperature distribution in the neighborhood of 500 km is maintained in a steady state in the neutral atomic oxygen gas by downward conduction which removes the heat received from the region surrounding the earth at the same rate as that by which it is introduced. At these altitudes, the heat production by absorption of ultraviolet radiation is certainly negligible, and the heat $R(O)$ radiated by atomic oxygen

$$R(O) = 1.65 \times 10^{-18} n(O) \text{ erg/cm}^3 \text{ sec}$$

can also be neglected.

Consequently, the heat flux F at a distance r from the earth's center being

$$F = 4\pi r^2 A T^{1/2} \frac{dT}{dr}$$

in which $A = 3.6 \times 10^2$ is the coefficient for atomic oxygen, the law of the vertical

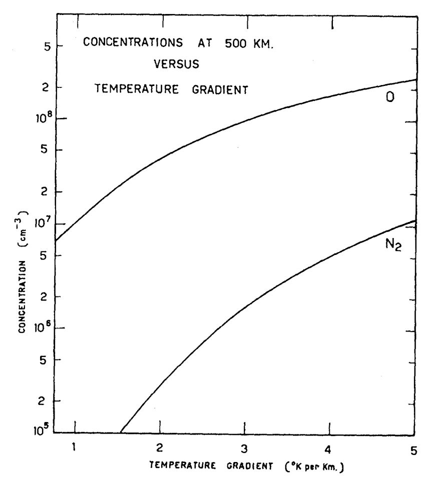

Fig. 3. The number of nitrogen molecules and oxygen atoms depends on the temperature gradient in the thermosphere. The variation is shown for temperature gradients of between 1° and 5° at 500 km.

distribution of temperature is given as follows (*9*):

$$\frac{T_z^{3/2} - T_0^{3/2}}{T_h^{3/2} - T_0^{3/2}} = \frac{z}{h}\,\frac{r_h}{r}$$

where T_h and T_0 are, respectively, the temperature at reference levels $z = h$ and $z = 0$, T_z is the temperature at any level above $z = 0$, and r_h and r are the respective distances from the center of the earth to the specific level h and the general level z.

The two preceding relations lead to the following relation for the rate of conduction of energy E_h (ergs per square centimeter per second) at level h (*10*).

$$E_h = \frac{r_0}{r_h}\,\frac{2}{3}\,A\,\frac{T_h^{3/2} - T_0^{3/2}}{h}$$

If one identifies $h \equiv 500$ km and adopts a rate of conduction of energy in the possible range from 0.1 to 1.0 erg cm^{-2} sec^{-1}, temperature values from 1000°K to 3000°K are derived (see Fig. 1). On this basis, the temperature gradients at 500 km vary between 1° and 5°K per kilometer, as shown in Fig. 2.

By considering specific values for the concentrations of molecular nitrogen and atomic oxygen at level $z = 0$—namely, an altitude of 140 km where the temperature is 560°K—it is possible to obtain the concentrations at 500 km. With $n(\mathrm{O}) = 5.60 \times 10^{10}$ cm^{-3} and $n(\mathrm{N_2}) = 4.15 \times 10^{10}$ cm^{-3}, the concentrations of oxygen atoms and nitrogen molecules at 500 km are given in Fig. 3. The results are (i) a low concentration of N_2; and (ii) a relatively small variation of atomic oxygen concentration for temperature gradients between 3° and 5°K per kilometer.

If the concentrations so obtained are related to the mass density ρ by the relation $\rho = \Sigma nm$, where m is the atomic or molecular mass, the densities at 500 km are obtained for the temperature range 1000 to 3000°K. It can be seen from Fig. 4 that densities greater than 10^{-15} g/cm^3 require temperatures higher than 1500°K. Therefore, temperatures of not less than 2250°K, inferred by Nicolet (*5*) for the escape of helium, lead to densities not less than 4×10^{-15} g/cm^3 at 500 km.

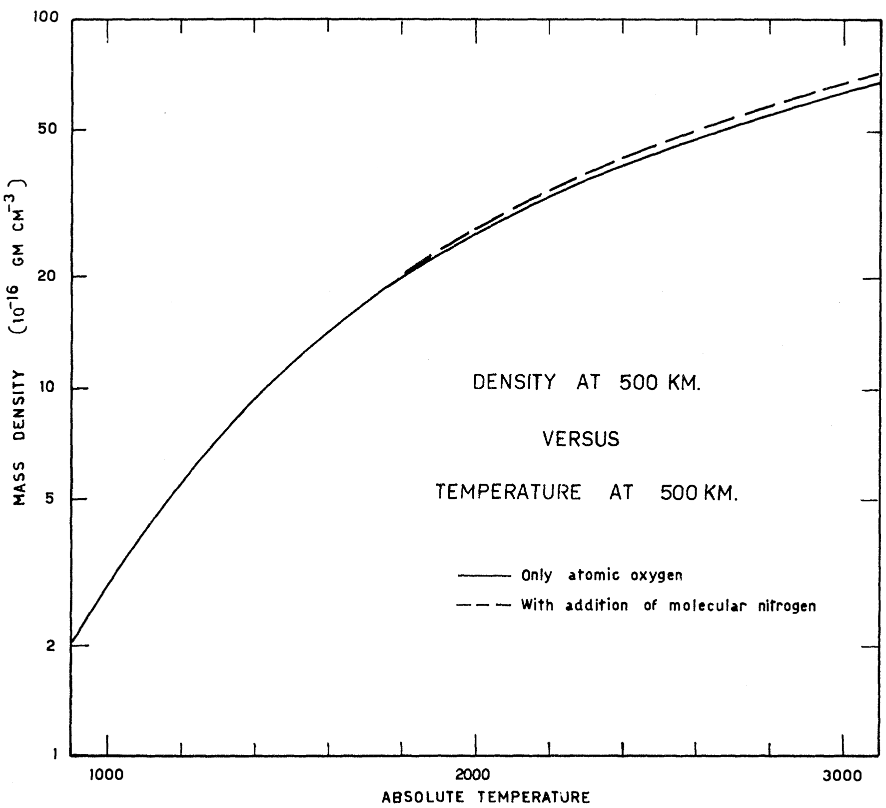

Fig. 4. The density of the atmosphere at 500 km varies by a factor of about 25 when temperatures vary between 1000° and 3000°K.

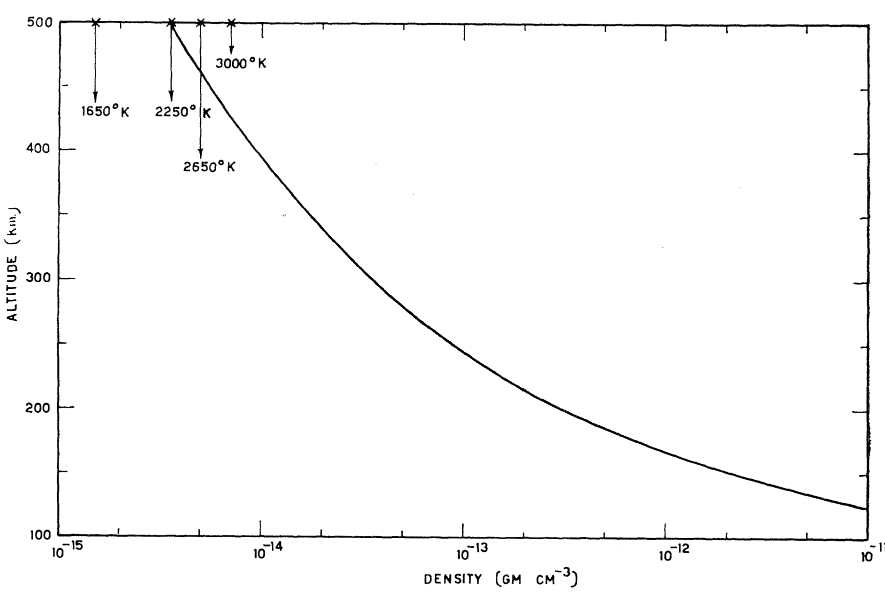

Fig. 5. The vertical distribution of the density can be determined when diffusion of constituents and conduction of heat are applied to the thermosphere. Density decreases relatively slowly at altitudes above 200 km. The crosses at 500 km show the effect of temperatures higher or lower than 2250°K on density values at 500 km.

By assuming a specific value for the heat flow, namely 0.6 erg cm^{-2} sec^{-1}, leading to a temperature of the order of 2250°K and a temperature gradient of 3.5°K per kilometer at 500 km, the variation of density with height may be used to provide an average vertical distribution such as that shown in Fig. 5. The crosses indicate the degree of the effect of temperatures higher or lower than 2250°K on the density at 500 km.

Conclusions

The conclusions are these: (i) The atmospheric models of the thermosphere that neglect diffusion cannot explain ionospheric and satellite observations. (ii) The dissociation of molecular nitrogen cannot be introduced to decrease the mean molecular mass. (iii) The oxygen-nitrogen ratio cannot be considered as constant. (iv) An atmospheric model must be founded on heat transport such as suggested by Chapman. (v) It is possible to estimate temperatures and temperature gradients in the neighborhood of 500 km and, consequently, conservative values of densities, if various values of the heat conduction are adopted.

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New Approach To Teaching Intermediate Mathematics

It is based on a resolution of the spectra of meanings of the letter x and the term *variable*.

Karl Menger

Nothing is more distasteful to an active mathematician or scientist than discussions of symbolism and notation, and that dislike is perfectly understandable. After having overcome in his youth whatever difficulties the formal expression of ideas presents, the mathematician finds that certain ways of writing have become his second nature and regards any suggestion of a change, even if he recognizes its merits, as nothing but a trivial nuisance.

There are, however, situations in which a thorough discussion of such matters on the highest level is inevitable. They occur when, at turning points in the history of culture, it becomes imperative to make certain techniques and ideas of mathematics available to wider strata of the population. In the large groups to be initiated, many persons lack the ability to overcome the difficulties that the specialist overcame in his youth. Moreover, an immense collective benefit results if even persons with that ability are spared unnecessary complications.

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Such a turning point affected arithmetic when, during the Renaissance, mercantilism and experimental science were born. In banks and laboratories, the letters introduced by the Greeks and Romans as numerals proved to be utterly inefficient, even though they had served arithmeticians for over 2000 years. Unfortunately, medieval mathematicians misinterpreted the specialists' manipulative facility as intrinsic simplicity of the ancient numerals and regarded the Hindu-Arabic ideas as a pure nuisance. "Even in the 15th century," wrote G. Sarton, "there were still any number of learned doctors and professors who claimed that the Roman letters were much simpler than the Hindu numerals." Such prejudices confined the knowledge of arithmetic to a small elite and retarded its democratization as well as its progress. Eventually, however, as everyone knows, practical exigencies prevailed —incidentally, to the ultimate benefit of pure mathematics too.

The middle of the 20th century appears to be another such turning point. This is the time when scientific and technological progress has reached proportions necessitating the dissemination, on a large scale, of intermediate mathematics. A considerable part of the population should learn certain techniques of algebra, analytic geometry, and calculus, as well as some basic ideas of those theories. The attempts toward this aim, which follow traditional lines, are generally regarded as not sufficiently successful. In my opinion, the principal stumbling block is the fact that most of those great mathematical ideas and techniques are being presented in their 17th century form.

Uses of x

A principal feature of those antiquated formulations is the indiscriminate use of the letter x (as well as of the letter y) in diverse meanings and according to discrepant rules. What enhances the confusion are references to those diverse types of x and y by one and the same term, namely, *variables*.

Algebra. In algebra, beginners learn that in the formula $x+1=1+x$ they may replace x with numerals, thereby obtaining formulas such as $4+1=1+4$. But they find that this practice must not be applied to the statement that the function $x+1$ is nonconstant, since replacement therein of the "variable" with 4 would lead to the false statement that the function $4+1$ is nonconstant. Beginners further learn that squaring the equation $y=x^4$ yields $y^2=x^8$. But they find that the square of what often is referred to as the function $y=x^4$ is the function $y=x^8$. This contradiction is so blatant that many mathematicians altogether refrain from referring to the said functions as $y=x^4$ and $y=x^8$, and rather call them, briefly, the functions x^4 and x^8. As a result, however, x frequently has various