Stokes' Edge Waves in Lake Michigan

William L. Donn and Maurice Ewing

During the past centuries a considerable loss of life and property has been recorded in shore areas from the effects of one or a series of high water waves that appear abruptly and usually without warning. This phenomenon is common to large inland lakes, bays, estuaries, and open coastal zones. When these waves have meteorological origins, they are referred to as "storm surges"; when they are of earthquake origin, they are referred to as "tsunamis."

Large inland water bodies provide ideal places to study these waves, because water and air data may be available from the entire lake boundary as well as from the surrounding region. This is not true for a seacoast.

The disastrous Lake Michigan surge of 26 June 1954, which took a number of lives along the Chicago waterfront, has been explained by us (*1*) on the basis of resonant coupling between a fast-moving atmospheric pressure-jump line (squall line) and the resulting gravity wave generated in the lake. Reflection of this high-coupled wave from the eastern shore of the lake accounts for its unexpected arrival at Chicago about 2 hours after the squall itself passed there.

Ten days later, a new pressure-jump line crossed Lake Michigan from north to south with a speed of about 50 miles per hour. Several water-level instruments maintained by the U.S. Lake Survey, the state of Illinois, and lakeside industrial plants, recorded long-period high waves that can be correlated with the pressure-jump and appear to be explainable on the basis of the theory described in subsequent paragraphs. After a discussion of the theory, we present the new storm-surge data and the application of the theory (*2*).

Stokes' Edge Waves

The theory of long gravity waves (where $c = \sqrt{gh}$) as applied to the earlier storm surge of 26 June does not provide an adequate explanation for the new case. It does not account for the long period of the water waves, and it also seems probable that the effects of resonant coupling between the atmospheric disturbance and such gravity waves would be kept to a minimum, owing to continuous refraction of the waves toward the shore.

Recently, Munk, Snodgrass, and Carrier (*3*) explained certain wave phenomena over gently sloping continental shelves by means of the theory of edge waves, which was first described by Stokes (*4*) in 1846 and has been elaborated by Ursell (*5, 6*) in recent years. These waves, which travel with crests transverse to the shore, are dispersive waves whose phase velocity is a function of the slope of the bottom and of the wave period. They can be thought of as resulting from the constructive interference of refracted gravity waves of different wavelengths. Such interference produces wave crests (edge waves) that maintain themselves transverse to the shore. The refraction effect confines the energy to a distance $\lambda/2\pi$ from the shore (where λ is the wavelength), so that the wave height becomes negligible at this distance.

Following Ursell (*6*), we let β be the inclination of the lake bottom and with origin at the edge, take the x-axis in the water surface normal to the edge, the y-axis downward, and the z-axis parallel to the edge. The velocity potential ϕ for waves propagating with their crests transverse to the shore must be of the form

$$\phi = A \cos (kz - \sigma t) e^{-k(x \cos \beta + y \sin \beta)} \quad (1)$$

where A is the amplitude; t is the time; λ, the wave length, is $2\pi/k$, or k is $2\pi/\lambda$; and T, the period, is $2\pi/\sigma$, or $\sigma = 2\pi/T$; in order to satisfy the equation

$$\nabla^2 \phi = 0 \quad (2)$$

and the boundary conditions

$$\sigma^2 \phi + g \frac{\partial \phi}{\partial y} = 0 \quad (3)$$

at $y = 0$ and

$$\frac{\partial \phi}{\partial y} = \frac{\partial \phi}{\partial x} \tan \beta \quad (4)$$

at $y = x \tan \beta$, provided that

$$\sigma^2 = gk \sin \beta \quad (5)$$

where g is the acceleration owing to gravity.

The phase velocity c (from Eq. 5)

$$c = \frac{gT \sin \beta}{2\pi} \quad (6)$$

and the group velocity U (following Lamb, *7*)

$$U = \frac{d(kc)}{dk} = \frac{C}{2} \quad (7)$$

have the same relationship as they do for gravity waves in deep water. The period T is given by

$$T = \frac{2\pi c}{g \sin \beta} \quad (8)$$

As is well known (see, for instance, Lamb, *7*, p. 413), the principal waves generated in a dispersive system by a moving disturbance are those for which the phase velocity equals the velocity of the disturbance. We may therefore calculate the period of the waves generated by considering c in the period equation as representing the velocity of the disturbance. This was applied in the theory of air-coupled flexural waves by Press and Ewing (*8*) and by Munk, Snodgrass, and Carrier (*3*).

The motion of a particle of water can be determined from the components of velocity parallel to the coordinate axes, $v_x = \partial\phi/\partial x$, and so on, and those perpendicular (v_p) and parallel (v_s) to the bottom. Thus

$$v_p = v_x \sin \beta - v_y \cos \beta = 0 \quad (9)$$

$$v_s = v_x \cos \beta + v_y \sin \beta = - kA \cos (kz - \sigma t) e^{-kD} \quad (10)$$

and

$$v_z = - kA \sin (kz - \sigma t) e^{-kD} \quad (11)$$

where $D = x \cos B + y \sin B$, the distance from the edge to a line through x, y perpendicular to the bottom. Thus, any water particle moves in a circular orbit parallel to the bottom with a radius that decreases exponentially with distance from the shore.

Storm-Surge Data

Pressure data. Isochrones for the 6 July pressure-jump line are shown in Fig. 1. These are interpolated positions based on the exact times and magnitudes of the recorded jumps given in Table 1.

The data indicate that the pressure-jump line moved southward at about 50 miles per hour and crossed the southern

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Table 1. Pressure-jump data for 6 July 1954.

Station	Time (CST)	Magnitude (in.)
FAR (Fargo)	6:50 A.M.	0.06
STC (St. Cloud)	8:39 A.M.	0.09
MSP (Minneapolis)	9:54 A.M.	0.07
GRB (Green Bay)	11:00 A.M.	0.04
TVC (Traverse City)	12:30 P.M.	0.02
MSN (Madison)	1:02 P.M.	0.08
MKE (Milwaukee)	1:24 P.M.	0.06
MKG (Muskegon)	2:00 P.M.	0.04
CHI (Chicago)	3:30 P.M.	0.10
BTL (Battle Creek)	3:30 P.M.	0.04
SBN (South Bend)	3:55 P.M.	0.06

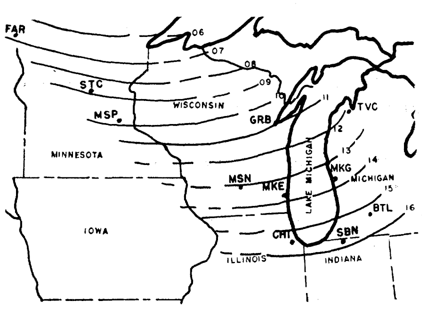

Fig. 1. Isochrones for Lake Michigan pressure-pump line of 6 July 1954.

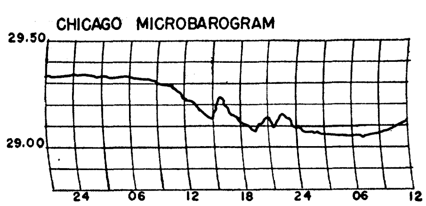

Fig. 2. University of Chicago microbarogram showing pressure-jumps of 6 July 1954. (In this figure and also in Figs. 3, 5, 6, 7, and 9, time is designated in the 24-hour system.)

end of the lake just prior to 4 P.M. Furthermore, most of the microbarograms examined show that the pressure-jump was oscillatory, with the primary jump being followed by at least two oscillations of lesser magnitude. A copy of the University of Chicago microbarogram showing these oscillations is reproduced in Fig. 2, which shows that they continued until 12 midnight on 6 July. A copy of the microbarogram from Muskegon recorded on an open time scale is shown in Fig. 3. This location, on the east side of Lake Michigan and much farther north than Chicago, shows a fairly quiet record from 4 P.M. to the end of the day. The significance of this record is discussed following presentation of the wave data.

Wind data. A study of the available anemograms shows that a considerable increase in wind speed occurred with the passage of the pressure-jump. The Wilson Avenue (Chicago) anemogram, for example, shows an increase of from 20 to 70 miles per hour between 3 and 3:30 P.M. A wind shift from south to north accompanied the speed increase. Smaller oscillations in speed and direction occurred with the subsequent pressure oscillations that are described in the preceding paragraph.

Wave data. The stations from which water-level records were studied in detail are shown in Fig. 4, which also shows depth contours (in feet) for Lake Michigan. The contours are based on U.S. Lake Survey chart No. 7, "Lake Michigan." To obtain the wave periods, which are given in subsequent paragraphs, the periods of several wave trains along the record were read and averaged.

Figure 5, which is a copy of the Waukegan wave record, shows some activity above background between 2 and 3 P.M., with the principal wave disturbance beginning at 3 P.M. and continuing with decreasing amplitude until 10 P.M. on 7 July. The wave period recorded is about 110 to 120 minutes. The waves of higher frequency, which are superimposed on these long waves, have a period of close to 20 minutes.

The Wilson Avenue (Chicago) record (Fig. 6) shows a preliminary peak at 3 P.M. and a major wave that began at

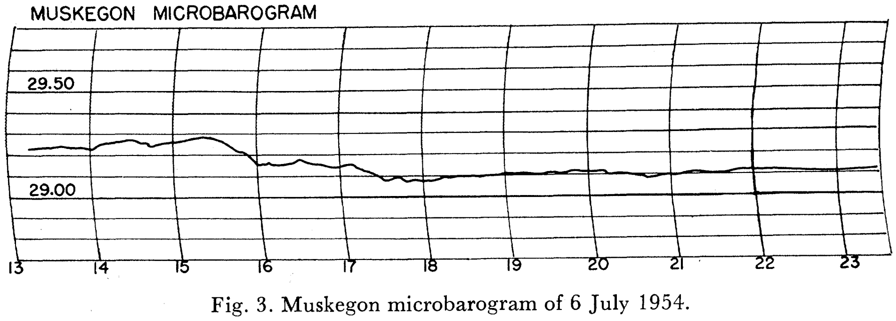

Fig. 3. Muskegon microbarogram of 6 July 1954.

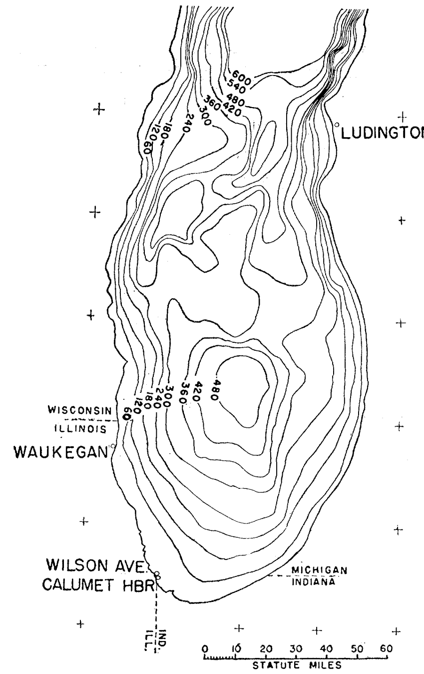

Fig. 4. Bottom topography of Lake Michigan.

3:30 P.M. and reached a crest just after 4 P.M. Again, this initial surge is followed by a prolonged train of higher than normal waves until the close of 7 July. The period is about 97 to 100 minutes, which is close to that for the background waves preceding the surge. Waves of shorter period (18 to 20 minutes) are also present as on the Waukegan records.

Preceding and following the surge at all the southern stations in the vicinity of Chicago and Waukegan, the background waves show a nearly steady state of low-amplitude, long-period oscillations. The period varies from about 2 to nearly 3 hours. Shorter oscillations of 15 to 20 minutes are superimposed on these, just as they are on the higher waves that comprised the surge.

The Calumet Harbor (Chicago) record (Fig. 7) also shows some preliminary activity at 3 P.M., prior to the major surge that began about 3:30 P.M. and reached a maximum at 4 P.M. Above-normal oscillations of the water, which continued through most of 7 July with decreasing amplitude, have a period of about 115 minutes, with waves of shorter period (20 minutes) superimposed. Figure 8 shows the water level at Chicago recorded on a scale convenient to illustrate both the surge and the background oscillations preceding and following it, although the scale does not permit detailed study.

The total duration of the wave disturbance is estimated to have been 29 hours, although the background oscillations continued somewhat higher than normal for a longer time. The reason for the latter is discussed later. The Ludington record (Fig. 9) shows a group of small waves of about 50-minute period beginning at 1:30 P.M. and continuing until 4 P.M. A group of much larger waves of 84-minute period began about 8 P.M., with a maximum crest at 10:30 P.M., and these waves persisted with attenuating heights into 8 July.

Table 2 gives a comparison of the arrival times of the pressure-jump and the arrival times of the major water-wave group at each wave station, together with duration and period data.

Table 2. Summary of pressure and wave data.

Station	Pressure-jump arrival time (P.M.)	Time of first surge wave (P.M.)	Period (min)		Duration of wave disturbance (hr)
			Long-period waves	Short-period waves	
Waukegan	about 3	3	110–120	20	29
Wilson Ave.	3:30	3:30	97–100	18–20	29
Calumet Harbor	3:30	3:30	115	20	*
Ludington	about 1	8–10	84		*

* Record incomplete.

Application to Surge of 6 July 1954

To apply edge-wave theory to the surge of 6 July 1954, it is noted that in the vicinity of Chicago and Waukegan the depth increases uniformly with distance from shore. The distance at which wave height becomes negligible ($\lambda/2\pi$) is estimated to be 16 miles. The bottom slope was determined along six profiles out to the 300-foot contour, from just north of Chicago to 50 miles north of Waukegan. These values of sin β, from north to south, of 0.0029, 0.0034, 0.0035, 0.0024, 0.0021, 0.0016, when inserted in the period equation together with 72.5 feet per second as the value of the phase velocity c, give a mean edge-wave period of 103 minutes for the Chicago-Waukegan area. The mean period of the observed waves, from Table 2, is 109 minutes, giving very good agreement between theoretical and observed values. Although the waves of shorter period (20 minutes) appear explainable numerically, at least on the basis of several different approaches, such as a second mode effect or generation over the steeper northern lake shore, we refrain from discussing them until more data have been compiled from other cases.

By using the velocity equation, we obtain a theoretical edge-wave velocity of 61 miles per hour for the southern half of the lake and 50 miles per hour for the northern half. Because the observed pressure-jump velocity is 50 miles per hour, and because the pressure-jump line is, in reality, a band of some width, it is expected that a considerable resonant transfer of energy would occur from the disturbance in the air to the resulting disturbance in the water. This accounts for the relatively great heights of the water waves that began with the arrival of the pressure jump in the air.

It seems evident from Fig. 4 that the bottom topography northwestward from Ludington would not be appropriate for coupling—hence, the small size of the water waves recorded just after the passage of the pressure-jump, about 1 P.M. The much larger waves after 8 P.M. at Ludington can be explained as arriving from the south. The pressure-jump and associated high waves resulting from resonant coupling reached the southern end of the lake about 4 P.M. These waves would then either be reflected or simply continue to run northward around the eastern edge of the lake. However, they would travel the 165 miles from the southern end of the lake to Ludington, with the group velocity (one-half of the phase velocity c) and would arrive there from 5 to 6 hours after reaching the southern end, or about 9:30 P.M. This is just about the time of the beginning of the major water-wave arrival. Thus, we have a case of high water waves generated through resonant coupling and reaching the station long after the passage of the generating disturbance as the result of reflection. This is similar to the explanation of the disastrous surge at Chicago about 10 days earlier (*1*).

The total duration of the observed water disturbance in the Chicago-Waukegan area cannot be stated precisely. Although the surge started abruptly, it declined gradually, appearing to merge into the background oscillations of higher than normal amplitude. A duration of approximately 29 hours seems to be a reasonable estimate. A minimum theoretical duration can be determined. Thus, the interval of oscillations of atmospheric pressure, according to the Chicago microbarogram (Fig. 2) and others, not shown,

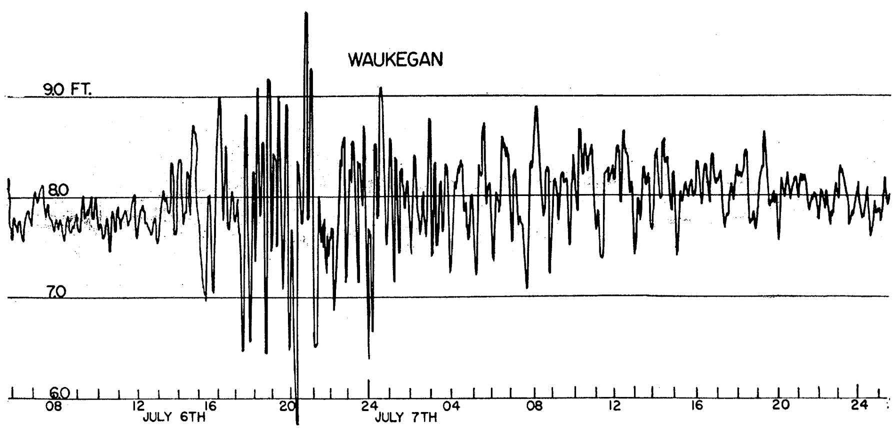

Fig. 5. Waukegan water-level record showing disturbance of 6–7 July 1954.

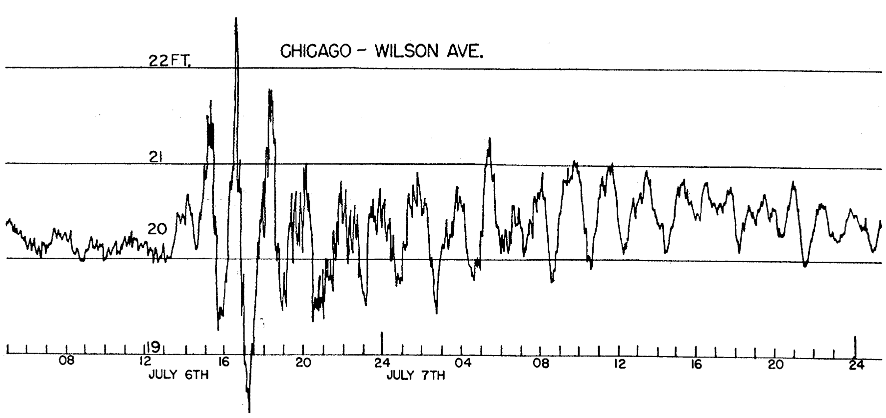

Fig. 6. Water-level disturbance at Chicago (Wilson Avenue) on 6–7 July 1954.

was about 9 hours. Also, it is estimated that the pressure-jump entered the lake about 260 miles north of Chicago. Although the front of the disturbance traveled with the phase velocity, or about the velocity of the pressure-jump, the rear portion of the water disturbance would travel with the group velocity, at about 25 miles per hour and would arrive 5.2 hours after the front of the disturbance. Furthermore, edge waves undoubtedly were generated along the sloping lower half of the eastern margin of the lake. The rear of this disturbance would travel westward around the southern margin of the lake at the group velocity and would increase by about 4 hours the duration of the direct disturbance that traveled down the western edge of the lake. The total theoretical duration is thus estimated to be about 18 hours.

The theoretical edge-wave duration is thus significantly less than the duration of the observed disturbance. But recall that the background water-level oscillations have a period of from 2 to 3 hours, which is close to that of the edge waves. Applying the well-known seiche formula,

$$T = \frac{2l}{\sqrt{gh}}$$

where T is the period, l is the width of the lake (taken here along a profile east of Waukegan), and $\sqrt{h}$ is the mean of the square roots of the depths along the same profile, the theoretical seiche period for the southern portion of the lake is 160 minutes. This agrees well with the observed period of the oscillations having amplitudes above background. It seems reasonable to expect resonance to occur from the addition of energy by edge waves whose period is so close to that of the lake seiche. This would account for the prolonged duration of slowly damped water oscillations having amplitudes above background. An earlier and more general discussion of Lake

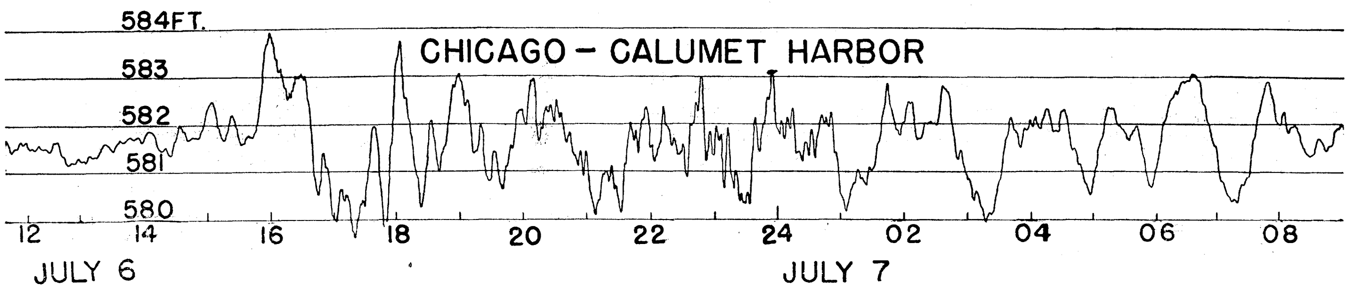

Fig. 7. Water-level disturbance at Chicago's Calumet Harbor on 6–7 July 1954.

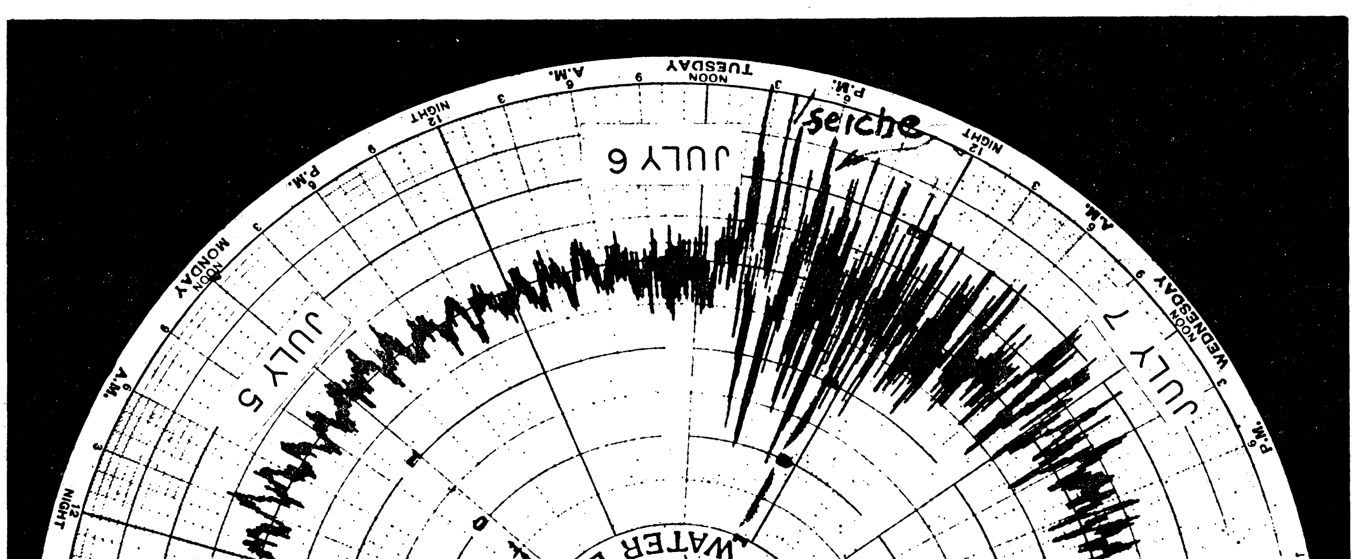

Fig. 8. Chicago (Montrose Harbor) water-level record for 4–7 July 1954.

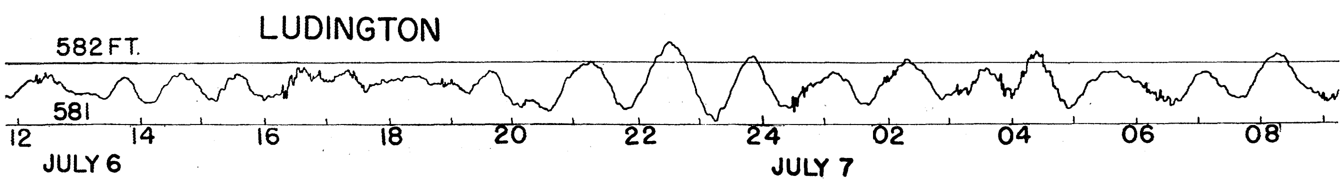

Fig. 9. Ludington water-level record for 6–7 July 1954.

Michigan surges and seiches has been given by Harris (9).

Conclusions

The theory of edge waves appears to explain most of the recorded effects of the storm surge of 6–7 July 1954. It seems reasonable to expect that relatively strong and fast-moving atmospheric disturbances, with large components parallel to the lake, have excited such edge waves in the past and will do so in the future. In cooperation with the U.S. Lake Survey, a program of instrumentation is under way to check this theory further and to provide empirical data for a more complete study of the Great Lakes surges. Edge-wave theory also appears to be applicable to the total reflection of deep-water waves near the beach as described by Isaacs *et al.* (10).

This study shows again that potentially dangerous waves may be expected at a shore station long after the passage of the generating disturbance in the air. It should be noted that the disastrous surge of 26 June 1954, which was described earlier (1) and was explained as originating from resonant coupling to gravity waves, does not permit explanation by the present mechanism, owing to the configuration of the lake bottom at the southern end of the lake and to the transverse path of the disturbance across the lake.

References and Notes

1. M. Ewing, F. Press, W. Donn, *Science* 120, 684 (1954).

2. This article is Lamont Geological Observatory contribution No. 188. This study was made possible by the help and close cooperation of the U.S. Weather Bureau and the U.S. Lake Survey. We are especially grateful to Morris Tepper and Gordon Dunn of the U.S. Weather Bureau and to W. T. Laidly and Frank Blust of the U.S. Lake Survey. All financial support was obtained through a grant from the Engineering Foundation of New York City.

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Proteins and Calcium in Serums of Estrogen-Treated Roosters

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In 1935 Laskowski (1) discovered phosphoprotein, which he termed *serum vitellin*, in the plasma of laying hens. Riddle (2) in 1942 correlated the appearance of this protein with the elevated calcium that is seen in laying birds. Since that time, the protein-calcium system has received further elucidation. Several reports have appeared relating estrogen to elevated serum calcium (3). Other reports have related estrogen specifically to the appearance of phosphoprotein and other protein components in serum (4). Recently an interrelationship has been seen among all three entities, estrogen, phosphoprotein, and elevated calcium (5). In 1956, Clegg and coworkers demonstrated that injection of diethylstilbesterol into cockerels resulted in the introduction to the serum of phosphoprotein that complexed relatively large amounts of ionized calcium-45 that had been added to an *in vitro* system.

In connection with investigations on the comparative physiology of endosteal bone formation by Urist and McLean (6), studies utilizing the ultracentrifuge have been made of calcium-binding proteins in serums of roosters injected with massive doses of estrogen (7). With this material and the partition cell developed for the preparatory ultracentrifuge by Schjeide and Dickinson (8), further information has been obtained on calcium-binding components that are produced by avian species in response to estrogen (9).

Materials, Methods, and Results

A microsuspension of 125 milligrams of USP estrone (Ayerst) was injected intramuscularly into roosters. Beginning within 24 hours and rising nearly to a maximum within 5 days, there was a large increase in serum chylomicrons (10) and beta lipoproteins (10), nearly complete disappearance of alpha lipoproteins (10), and appearance of at least two new components (X_1 and X_2, as resolved by ultracentrifugation in sodium chloride (88 mg/ml, *p*H 6.0). In Fig. 1 are depicted schlieren patterns typical of control and estrone-injected roosters after removal (by floatation) of the chylomicrons and the beta lipoproteins. The concentration of albumin is practically the same in injected and control birds, despite a tenfold increase in phosphorus.

The X_1 and X_2 components display sedimentation rates of S 7.5 and S 15.0, respectively, when extrapolated to infinite dilution and corrected to 20°C (Fig. 2).

Analyses for calcium (11) in chylomicrons, beta lipoproteins, and alpha lipoproteins reveal that these bind less than 5 percent of the total calcium in the serum. Similar analyses on other components indicate that albumin and the denser globulins also bind comparatively small amounts of the serum calcium in estrone-injected birds. Within approximately 3 days after treatment, the calcium content of the serum in compartments 1, 2 and 3, 4 of the partition cell, after 9 hours of centrifugation in 10-percent NaCl at 70,000 times gravity, was as shown in Fig 3.

The elevated serum calcium seems to be mainly associated with the X_1 component. Correlations between the concentration of this species and protein-bound calcium are illustrated by the schlieren patterns shown in Fig. 4.

The X_1 species appears to be a phosphoprotein as shown by comparative studies of alkaline hydrolysis (after removal of lipids) on centrifugal fractions of serums from two different birds. One bird (A and B) has a very high level, and one bird (C) selected for comparison, has a relatively low level of protein phosphorus (Fig. 5).

The X_2 species is a very dense lipoprotein. Correlation of such data as total nitrogen (micro-Kjeldahl), schlieren patterns (ultracentrifuge), total lipid

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