Book Reviews

The Moon. H. Percy Wilkins and Patrick Moore. Macmillan, New York, 1955. 388 pp. Illus. $12.

This is the first major treatise on descriptive selenography to appear in English for a quarter-century. The field has badly needed a modern summary, for a host of isolated observational facts await ordering. Perhaps this disorganization was a major cause of the comparative disinterest, until very recent years, of many astronomers in lunar problems.

Both authors are well-known English amateur observers of the moon, and Wilkins is director of the lunar section of the British Astronomical Association. They are specialists in charting minute lunar surface detail from visual examination. From this fact come both the strength and weakness of their book—the excellent qualitative descriptions of surface features, and the neglect of means of attack other than direct inspection. For example, photometric and polariscopic observations are not considered. However, such topics presumably will be thoroughly covered by G. P. Kuiper's forthcoming section on the moon in *The Solar System,* which is now appearing under his editorship.

The bulk of *The Moon* consists of topographic descriptions of more than 700 of the principal craters and other formations. Here are enumerated interior details such as peaks, mounds, clefts, craterlets, and pits. Recent observations are comprehensively covered, but only a selection of the older findings is given. As the authors point out, their book supplements but does not supersede earlier treatises; the reader will still want to consult the classics by Neison, Elger, and Goodacre. Wilkins' great 300-inch map of the moon is reproduced on a reduced scale in 29 sections; there are many detailed charts of individual craters, as well as lunar photographs of exquisite quality.

In the main portion of the book, quantitative data are scarce, and there are far too few references. Surely the sources of the basic observations should have been listed for questions such as the problematical transformation of the crater Linne. New names have been introduced for a number of craters without the sanction of the International Astronomical Union, but their unofficial status is not made clear to the reader.

A 30-page introduction surveys the history of lunar observations and sketches the classification of surface formations. It also contains a rather confused collection of many attempted explanations of the origin of lunar features. Here the opportunity was lost to clarify a subject badly muddied by controversy. But we are treated to curiosa such as coral-atoll hypotheses of crater formation, and the recent studies by Kuiper and Urey are not mentioned.

E. A. Whitaker of Greenwich Observatory has contributed a valuable appendix on the techniques of lunar photography. A collection of biographical sketches of leading selenographers has been added. The choice of names strongly reflects the personal tastes of the authors; some rather minor figures appear, while Gaudibert, Weinek, Hayn, and Banachiewicz are passed over.

While one must turn elsewhere for a full treatment of the theoretical and astrophysical aspects of lunar research, unquestionably *The Moon* will long remain a standard reference work on descriptive selenography.

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Handbuch der Laplace-Transformation. vol. II, pt. 1, *Anwendungen der Laplace-Transformation.* Gustav Doetsch. Birkhäuser, Basel–Stuttgart, 1955. 436 pp. 48 illus. DM. 52.

This book, by the distinguished professor of mathematics at the University of Freiburg, is the second of a trilogy which is to comprise an exhaustive account of the theory and application of the Laplace transform. The first volume, *Theorie der Laplace-Transformation* (1950), encompasses an inclusive treatment of fundamental mathematical theory of the Laplace transform. The present second volume and the forthcoming third volume constitute, in principle, an integrated account of a further body of theory, both complementing and supplementing that of volume I, which is particularly pertinent to solution of physical and technologic problems and exemplification of its use by examples drawn from these domains. In such fact the word *handbuch* must be translated as "treatise" rather than as "handbook" in the conventional American usage of this term.

Volume III, now in press, includes partial differential equations, difference-equations, integral equations, and functions of exponential type. The present volume, partly in preparation for the material of this third volume and partly as an entity in itself, constitutes a thorough treatment of asymptotic representations, convergent representations, and solution of ordinary differential equations as effected by Laplace transform techniques.

The context is divided into one short and three major sections. The range of content, scope of treatment, division of material, and relative emphasis on different subjects is well summarized by giving a *free* translation of the 16 chapter headings, corresponding page numbers, and concise note of the principal purpose underlying the exposition of each section. Thus, the first section, "Introduction," comprises a single chapter, "Correspondence of fundamental operations [in the original domain] to functions [in the transform domain] through the Laplace-transform and its inverse" (pp. 15–26). This is given over to a well-organized statement, without proof, of certain basic properties, manipulations, and theorems, derived formally in volume I and brought together here for convenience of use in volumes II and III.

Part I, "Asymptotic representations," includes nine chapters: "General considerations concerning asymptoticity" (pp. 29–44); "Abelian asymptoticity of the one-sided Laplace-transform: behavior of $f(s)$ at infinity" (pp. 45–96); "Abelian asymptoticity of the one-sided Laplace transform: behavior of $f(s)$ at a finite point" (pp. 97–100); "Abelian asymptoticity of the two-sided Laplace transform and of the Mellin transform" (pp. 101–108); "Abelian asymptoticity of the V-transform comprised by the complex inversion integral for functions with isolated singularities" (pp. 109–140); "Abelian asymptoticity of the V-transform comprised by the complex inversion integral for functions with algebraic and logarithmic singularities" (pp. 141–173); "Abelian asymptoticity of the V-transform for $t \to 0$" (pp. 174–180). "Tauberian asymptoticity of the Laplace transform (pp. 181–192); "Asymptotic expression in various fashions of the original function and the associated Laplace transform" (pp. 193–200).

This unit comprises a thorough exploration and integration of the essential theory, including much original work, pertinent to three major methods of effecting asymptotic representations—as rooted in the one-sided and two-sided Laplace transforms and in the complex inversion integral—and various exemplifications of this theory, especially as it can be used to effect asymptotic solutions of boundary-value problems that would be difficult to solve by conventional classical procedures. Other applications range over mathematical topics as diverse as the Gaussian error integral and the prime number theorem and over physical topics as different as current distribution in electric cables and the wave functions of the continuous spectrum of the hydrogen atom.

Part II, "Convergent developments," embraces a short "Introduction" (pp. 201–202) and two chapters: "Gamma-function series" (pp. 203–235) and "Special series" (pp. 236–254). The first-named series play an important role in the solution of difference-equations, numerical analysis (especially in connection with modern, high-speed automatic digital computers), and the asymptotic representation of a major class of Laplace-transforms. The special series treated encompass developments of the Laguerre polynomials and confluent hypergemetric functions in terms of Bessel functions; series developments in terms of Laguerre polynomials, Hermitean polynomials, and confluent hypergemetric functions; and other kindred topics that are of prime interest in numerous branches of classical and modern mathematical physics.

Part III, "Ordinary differential equations," is made up of four chapters: "Ordinary differential equations with constant coefficients in a one-sided infinite interval under prescribed initial conditions" (pp. 255–344); "Ordinary differential equations with constant coefficients in a two-sided infinite interval under prescribed initial and boundary conditions" (pp. 345–363); "Ordinary differential equations with variable coefficients in the domain of the original variable of the Laplace transform" (pp. 363–385); "Ordinary differential equations with variable coefficients in the domain of the transform variable of the Laplace transform" (pp. 386–404).

These chapters comprise a good exemplification of the Laplace-transform for the solution of both ordinary linear differential equations with constant and (certain types of) variable coefficients under prescribed initial or boundary conditions and of systems of such equations. A very desirable conciseness and elegance of treatment of systems of equations is achieved by use of matrix notation, in recognition and support of the rapidly increasing use of such in modern engineering analysis. Illustration of application of this theory is drawn largely from two areas of electrical engineering: basic feedback systems theory (which affords an area for evidencing the usefulness of asymptotic and convergent developments in investigating the stability of both ordinary and dead-time control systems) and ladder-connections of four-terminal networks (which afford excellent opportunity for evidencing the utility of the conjunction of matrix notation with Laplace transform analysis). These applications are complemented by a brief, but excellent, account of the theory of the impulse function in its modern rigorous formulation on the basis of Laurent Schwartz's distribution theory. Since the impulse function plays a central role in modern theoretical physics (under the guise of Dirac's δ-function) and in various branches of engineering (electric network analysis and synthesis, information theory, mechanical vibration theory, and so forth) this section of the text should prove of especial interest to those physicists and engineers who are dissatisfied with the usual sketchy and vague treatments of the δ-function given in most textbooks.

The major context is rounded out by a preface, a short appendix on "Lagrange-Burmänn's theorem," a lengthy "Literatureal and historical commentary" comprising pertinent critical remarks relative to various points of the content, a subject index, and a list of emendations to volume I. A detailed bibliography is not given in this volume; references are keyed to the lengthy alphabetically arranged list of articles and books in volume I and to an additional list which will close out volume III.

The physical aspects of the volume are of superior order: a high-quality paper; sturdy board covers and attractive green cloth binding; superlative typography; well-displayed equations; finely executed line drawings; and a convenient size of page. The textual content is covered in a lucid, easy style that enables rapid grasp of theoretical development and of illustrative example. Finally, the accuracy of theory and completeness of treatment evidence the broadness of knowledge, depth of mathematical originality, and command of exemplification to be expected of the foremost European authority on Laplace transform theory and its application.

Naturally, this book will prove of greatest interest to mathematicians—especially those interested in analysis or applied mathematics, or both. By virtue, however, of the power and usefulness of the Laplace transform for the solution of problems or the development of general theory in all branches of modern-day physics and engineering (as manifested by the illustrative applications drawn from such diverse domains as wave mechanics, vibration theory, information theory, spectral theory, electric network and transmission-line theory, automatic control theory, oscillator and amplifier theory), this volume merits a careful study by all physicists and engineers who wish to keep abreast of the advances in mathematical analysis that underlie the analytic foundations of their own areas of professional work.

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Topological Dynamics. American Mathematical Society Colloquium Publications, vol. XXXVI. Walter H. Gottschalf and Gustav A. Hedlund. American Mathematical Society, Providence, R.I., 1955. 151 pp. $5.10.

A flow consists of a space X together with a one-parameter group T of transformations of X into itself. Historically, the study of flows arose from the consideration of systems of ordinary differential equations. Poincaré was the first to study such dynamical problems using topological methods; later G. D. Birkhoff undertook a systematic study of the dynamics of flows.

In the present book there is given the first systematic treatment of the work of the last 20 years in topological dynamics. The primary concern in topological dynamics is with "recursive" properties of flows, and of transformation groups in general. In the case of a dynamical system, the study of a particular recursive property arises when one specifies how often a point comes back, as time passes, into any specified neighborhood of the point. Thus there are the periodic points, the almost periodic points, the recurrent points, and so forth.

The book is divided into two parts, the first of which gives the general theory. A feature of this portion is its striking generality. The general framework is that of a (topological) transformation group T operating on a space X. The authors are able to build a theory that generalizes the classical theory and in which T is not only necessarily a one-parameter group, but not even abelian. After preliminary work, in Chapter III the authors define and study the fundamental concept of recursion. There follow specializations of recursion to yield theories of almost periodicity, regular almost periodicity, and recurrence. Next the concept of incompressibility is treated, and its relation to recursive properties is studied. The theory concludes with chapters on transitivity, asymptoticity, and function spaces.

The second part of the book gives the examples. The role of examples in a treatment of topological dynamics is