6 April 1956, Volume 123, Number 3197

SCIENCE

Standards of Time and Frequency

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Until recently there was only one basic standard of time and frequency, which was used alike for all purposes both ordinary and scientific. The standard was the second, which was defined as 1/86,400 of the mean solar day. The second has now been officially redefined. It now depends not on the day but on the year, and the definition is 1/31,556,925.975 of the tropical year for 1900.0. It is now possible also to speak of an entirely different sort of basic standard of frequency, which has been made possible by the development of devices for measuring the frequency of the natural vibrations that take place in atoms and molecules; for brevity I refer to these new standards as atomic standards.

Thus we have for the first time in the history of science at least two basic standards of time existing side by side and at least three basic standards of frequency. We are not accustomed to dealing with such a profusion of standards, and in order to avoid confusion and misapprehension it is desirable to examine them in the light of our basic concepts of making measurements and to ascertain their advantages and their limitations as well as we can. The danger of confusion and misapprehension is greater because the practical determinations of time and frequency have been in the hands of a small number of astronomers. Astronomers in general, and all scientists in other disciplines, being busy with their own work, have paid little attention to the matter. Thus it now becomes the duty of the few astronomers who know something of the subject to tell what they know, and that is my purpose here.

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Definitions

It will be well to begin by reviewing a few fundamental definitions. Several sorts of definitions may be distinguished, and semanticists have paid considerable attention to the subject, but for our present purpose it is sufficient to speak of two kinds, operational definitions and all others. Operational definitions signify what is actually done while others do not. For example, the operational definition of a meter is the distance between two marks on a bar of platinum that is stored in a specified place in France. Another, earlier, definition of the meter is the 10 millionth part of the arc of the meridian from the north pole to the equator through Paris. Still another is 39.37 inches. When we are trying to reason as precisely as possible, it is always well to use operational definitions, and that is what I shall do for the most part. Therefore it will be no cause for astonishment if some of the definitions to follow are not found in dictionaries.

Any recurring phenomenon, the recurrences of which can be counted, is a *measure of time*. Examples are the passing of trains under the Hudson River, the ticking of a watch, the vibrations of a quartz crystal or of an atom, the meridian passage of a star, and the revolution of the moon around the earth or of the earth around the sun.

The interval between two successive recurrences is a *unit of time*.

A *clock* is any mechanism that counts such recurrences. It often serves as well to subdivide the unit of time into smaller parts. Thus our ordinary clocks subdivide the day into hours and minutes, and some special clocks subdivide the second into 1000 or 10,000 parts.

A *frequency* is the ratio between two different units of time, commonly expressed as the number of one sort of unit occurring during one unit of the other sort. Consider for example an alternating current. One unit of time is given by 1 cycle of the current. If we choose the second as the other unit, then the frequency of the current may be stated to be, for example, 60 cycles per second. While in strictness any two units of time may be made the basis for a statement of frequency, the cases that arise in practice are of only two sorts. (i) A frequency may be used as the definition of one unit of time in terms of another. For example, when we define the (old) second to be 1/86,400 of the mean solar day we might equally well say that the frequency of seconds per mean solar day is 86,400. Frequencies when used as definitions are fixed and invariable. (ii) A frequency may be used to connect an experimental unit of time with a fundamental unit. Frequencies of this sort are either *nominal* or *actual*. Thus the nominal frequency of an alternating current may be 60 cycles per second while its actual frequency may be 59.9998 cycles per second. Notice that actual frequencies must be determined by experiment and hence are necessarily affected by errors of observation.

The *rate* of a clock is the difference between its normal frequency and its actual frequency, conventionally taken in the sense nominal minus actual. For example, a seconds pendulum has a nominal frequency of 86,400 cycles per day. If its actual frequency is 86,401 cycles per day, it has a rate of -1 cycle per day. The pendulum is commonly said to have a gaining rate of 1 second per day, and this statement must be understood to be precisely equivalent to the preceding one, although in its terms it is less precise because the word *second* is used in a double sense: the unit of time given by the clock itself as well as the unit obtained by dividing the mean solar day into 86,400 parts. The rate of the clock could also be expressed by saying that it runs fast by 1 part in 86,400, or that the actual frequency is greater than the nominal by 1 in 8.64×10^4. Thus, frequencies and rates are very closely related, but they are not quite the same thing.

We notice that while a cycle of alternating current and a seconds pendulum are both measures of time they are not both clocks. A seconds pendulum is a clock because its cycles may be immediately counted, but the same is not true of an alternating current. A mechanism may, however, be devised for counting the cycles of an alternating current, and such a mechanism is a clock. The distinction is important because clocks tell the time, or in technical language, establish the epoch—an *epoch* being any specified instant of time—while an alternating current by itself does not. I note, by the way, that *epoch* is also frequently used to denote a more or less vaguely specified interval of time, as in *glacial epoch,* but we are not concerned here with that usage.

If the ratio of two different units of time—that is, the frequency—varies from epoch to epoch, one measure of time is said to be *accelerated* with respect to the other. Such accelerations are the rule rather than the exception. Also, as a general rule, one clock is not only accelerated with respect to another, but the acceleration itself changes from epoch to epoch. Most of the work involved in practical determinations of time and frequency aims at determining the accelerations of clocks and changes in frequencies.

The foregoing definitions may be clarified by noting the analogies with the familiar terms used in connection with lengths. The correspondence is shown in two parallel columns in Table 1.

If we imagine a *scale of time* to be an indefinite sequence of units of time, in one-to-one correspondence with the real numbers, then an epoch is a specified instant in the time scale just as a position is a specified point in a length scale.

If two different units of time are denoted t_1 and t_2, then frequency is given by t_1/t_2, just as velocity is given by L/t_2, say.

Similarly a change of frequency, or an acceleration of t_1 with respect to t_2, may be expressed as t_1/t_2^2, just as ordinary accelerations are expressed as L/t_2^2.

It is worth noting that the reciprocal of a frequency is also a frequency, which is not the case with velocity. Also, the ratio of two frequencies is also a frequency, provided that one unit of time is common to both; for example, the ratio of t_1/t_2 to t_3/t_2 is t_1/t_3, while the ratio of two velocities is a pure number without dimensions, provided that the same unit of time appears in both. Thus the analogy between frequency and velocity is not complete.

It is important to remember that a change of frequency is itself an acceleration of one time scale with respect to another, and it cannot properly be called an acceleration of frequency. An *acceleration of frequency,* if the term is to be used at all, is a *change* in actual acceleration, of the form t_1/t_2^3.

Table 1. Correspondence of terms used in connection with time and length.

Time	Length
Epoch (specified instant, time of day)	Position (specified point)
Frequency, rate of a clock	Velocity
Change of frequency, acceleration	Acceleration
Change in acceleration	Change in acceleration

All that has so far been said about measures of time pertains equally to any measure of time whatever. In practice, it is obvious that some measures are preferable to others. We should not think of basing our fundamental unit of time on the passage of railway trains; in practice, we do the reverse, running our trains more or less to time instead of running our time to trains. Let us ask ourselves what we require of a measure of time in order for it to be suitable as a standard. Obviously we require it to be continuous. Also we require it to be easily accessible; of two measures that are equally good intrinsically, we prefer the one that is more readily accessible. But there is another essential requirement that is not so easily stated. We commonly express it by saying that a standard measure of time should be invariable, but in fact there is no absolute criterion of invariability. Suppose we compare two measures of time with each other and find one to be accelerated with respect to the other; we are able to conclude that one or both of the measures are variable, without knowing which one. If on the other hand no acceleration is observed, we are not able to conclude that both measures are invariable (within the errors of observation); in fact both may vary with respect to a third. Nevertheless, a precise meaning can be given to the invariability of a measure of time. To understand the matter fully it will be helpful for us to review a short chapter of the history of astronomy, which will explain how the second came recently to be redefined in terms of the tropical year instead of in terms of the mean solar day.

Celestial Motions

The equations of motion for any member of the solar system may be immediately derived from Newton's law of gravitation. (I do not speak of the refinements demanded by the general theory of relativity, which are not germane to the present discussion.) They take the form of three differential equations for each body, which express the second derivatives of the three coordinates with respect to the time as functions of the masses and mutual distances of all the bodies. The three equations for any body may be solved by successive approximation if we have sufficient information about the other bodies and if we know the six constants of integration, which may be taken to be the three rectangular coordinates and the three rectangular components of the velocity at any convenient epoch. The solution serves to give the three coordinates of the body as functions of the time reckoned forward and backward from the epoch, and the mathematical expressions are called a *theory* of the motion of the body. Since there are nine planets in the solar system massive enough to affect the motions of one another, it can readily be appreciated that the task of constructing a theory for any one of them is very considerable. Nevertheless, it has been accomplished with quite a high degree of precision. The coordinates of Jupiter, for example, have been calculated to ten significant figures for the years 1653 to 2060, so as to be strictly consistent with the initial conditions (*1*).

Astronomical Units of Mass, Time, and Distance

Let us examine the precise specification of the masses, the time scale, and the distances of our planetary theories. The unit of mass is the mass of the sun, all other masses being expressed in terms of it. The unit of time is commonly said to be the mean solar day, but as we shall see, this specification is not precise enough; the unit actually employed is the mean value of the mean solar day during the 18th and 19th centuries. The unit of distance is the astronomical unit, and it is derived from the units of mass and time by means of the Newtonian law of gravitation. For the present purpose, it will suffice to describe the unit of distance as the distance from the unit of mass at which a body of negligible mass, moving in a circular orbit, would move through an angle of 0.01720209895 radian precisely in a unit of time. The astronomical unit is nearly, but not quite, the same as the earth's mean distance from the sun.

To a physicist, the system of units of mass and length used by the astronomer must seem like a queer one. The reason for the choice of units is the peculiarly restricted nature of astronomical observations. With the exception of measurements of velocity in the line of sight, all that astronomers know about the motions, distances, and masses of celestial objects has been deduced from a single kind of direct measurement: the measurement of the angle between two lines of sight from the observer. The result of an observation is a statement giving the value of an angle at an epoch. The angle is in some cases the angular distance between two celestial objects and in some cases the angular distance between one object and the plumb line. From such measurements alone, with the assistance of theory, the ratios of masses and the ratios of distances are deduced, but they cannot be expressed in grams or centimeters by any astronomical method whatever. It is true that we can state, to five or fewer significant figures, the masses of celestial objects in grams and their distances in centimeters, but to calculate them we must rely on physicists and geodesists to tell us the mass and size of the earth; such information is not needed for any astronomical purpose and astronomers never refer to it, except in answering questions put to them by nonastronomers.

Ever since the invention of mechanical clocks, astronomical observers have used the mean solar day as their unit of time. There are several reasons for this choice. In the first place, astronomers require a natural unit of time and not an arbitrary one. An arbitrary unit of time cannot be taken from place to place as easily as an arbitrary unit of length. Furthermore, all arbitrary units of time—that is to say, man-made ones such as those given by mechanical or electric clocks—are variable and evanescent; it has not been possible to build two mechanical or electric clocks that keep the same time or one clock that runs indefinitely without stopping. Hence a natural unit of time is a necessity in order that astronomical observations made at different places and at different epochs may be compared with one another. Of all natural units of time available to the astronomer, the period of rotation of the earth is the most accessible and can be observed with the highest precision. It is only necessary to observe the meridian passage of a star on two successive nights in order to have the period of rotation of the earth immediately. It is true that stars cannot be observed in foul weather, but man-made clocks are good enough to carry on with from one clear night to the next, and also to subdivide the period of the earth's rotation into 86,400 parts. It is interesting to notice, by the way, that the development of man-made clocks has had its greatest successes in a country that is noted for its inclement weather, England.

The mean solar day is not quite the same as the period of the earth's rotation, and not quite so accessible, the sun being less readily observed than the stars. But astronomers have been glad to make the very slight sacrifice required in order that the unit of time shall be suitable for the regulation of their daily lives. To pass from the period of the earth's rotation to the mean solar day in practice, it is only necessary to multiply by the number 1.0027378118868. This number, which is an actual frequency, is one of the most accurately determined constants in physical science, if not the most accurate; only the thirteenth decimal is in doubt. It is important to note that the mean solar day is defined in such a way as to make the number an absolute constant—that is, the mean solar day faithfully follows the rotation of the earth, and anything that disturbs one disturbs the other.

Observation and Theory

Throughout the course of history astronomers have sought to calculate theories of the motions of planets and satellites that would agree with actual observations. With each major contribution to fundamental theory has come renewed hope of complete success, but continual increases in the precision of observations have doomed all attempts to failure, at least until our own generation, and of course what will happen in the future is not known. After the formulation of the law of universal gravitation, it seemed that all practical problems of celestial mechanics had been reduced to arithmetic. Then the advance of Mercury's perihelion was discovered, which was not to be explained for another half century. Also, it is one thing to reduce a problem to arithmetic and quite another thing to solve it. The calculation of a planetary theory or a lunar theory is one of the most formidable tasks known to scientists, at least in cases where eight or more significant figures are required in the coordinates. It is not only a task of multiplying millions of numbers—which was formidable enough in itself until a very few years ago—but of proving the results correct, and even more difficult, of planning the work in such a way that it could be accomplished at all. It has taken a decade of hard work to produce our best planetary theories, and our best two lunar theories have each been the work of 25 years. The motion of the moon has been particularly troublesome; probably as much effort has been devoted to the moon as to all other celestial objects combined.

About 80 years ago it began to be suspected that the difficulty in getting theory and observations to agree might be ascribed, at least in part, to defects in the measure of time instead of to inadequacies of theory. In order to help us understand the consequences of a defect in the measure of time, let us imagine several things that are not the case. Suppose first that we have theories of the motions of all celestial bodies that agree with observations; suppose further that the rate of rotation of the earth is decreased imperceptibly, so that it is not detected by man-made clocks; and suppose that observations of the sun, moon, and planets are suspended until mean solar time has fallen an hour in arrears. What will be noticed when observations are resumed? Evidently the sun, moon, and planets will all be observed in advance of their calculated positions by the angles that they move through in 1 hour. The moon, for example, in an hour moves through an angle that varies from about 0.48 to 0.68 degree during a month, and the variations are very precisely known; it would be readily apparent that the moon was ahead of its course, and we might be inclined to suspect an error in the lunar theory until we observed the sun. The sun would be ahead of its calculated position by an amount varying through the year from 136 to 166 seconds of arc; the discrepancy and its variation would easily be detected. Furthermore, an eclipse of the moon would be observed precisely 1 hour in advance of the predicted time. The planet Mercury would be observed sometimes east and sometimes west of its predicted position, depending on whether its apparent motion was direct or retrograde, and the amount and variation of the discrepancy would indicate that at any epoch Mercury was in the position calculated for an epoch 1 hour later. Similar observations and conclusions would be made for the other planets. The inescapable conclusion would be that either our clocks were 1 hour slow or else that the moon and planets had accelerated in their orbits until they were an hour in advance, and had then returned to their normal velocities. In other words, either the measure of time was at fault, or else the theories of the motions of the moon and planets contained errors of a very curious sort.

Something like the hypothetical example just described has actually occurred, only it has not been a case of the clocks and the earth's rotation losing precisely an hour. In order to reconcile observations with theory, it is necessary to suppose that the clocks are sometimes fast and sometimes slow, the earth rotating at faster than its average rate for some years, and then changing rate rather quickly, so that it rotates more slowly, or even faster. Two thousand years ago, according to observations of solar eclipses made then, a clock keeping mean solar time would have been 2.6 hours slow. At about 1750 the clocks were on time; at 1850 they were 2 seconds slow; at 1900 they were 3.9 seconds fast; and at 1940 they were 24.5 seconds slow (*2*). The relatively large error at the beginning of the Christian era will not cause astonishment if it is remembered that we have adjusted our measure of mean solar time to make it fit the average duration of the mean solar day during the 18th and 19th centuries; at very remote epochs the errors may be expected to be much larger than in our own time.

Astronomers have had no hesitation in ascribing the discrepancy to errors in the measure of time rather than to errors in the theories of motion. It is fair to ask why. Several answers are possible, according to the point of view taken. To those who believe in Occam's razor (economy of hypotheses) the answer is obvious; either we must suppose the earth's speed of rotation varies unpredictably or we must suppose that the moon and planets vary in their orbital motions, also unpredictably, but in concert, and that the astronomical unit of length varies accordingly, accompanied by variations in the velocity of light. To those who believe in general relativity, it is enough to point out this last consequence, for the constancy of the velocity of light is a fundamental postulate of general relativity. To those who accept a hypothesis only if it is accompanied by a suitable mechanism, it was not possible to say anything until a very few years ago; even now the mechanism that produces changes in the earth's rate of rotation is scarcely understood. It is thought, however, that turbulence in the liquid core of the earth, accompanied by electromagnetic coupling between the core and the mantle, is sufficient to account for the changes. Finally, to practical horologists who recall the days before radio time signals when ships carried three chronometers, and when confidence was placed in the two that agreed with each other in preference to the one that disagreed, it is only necessary to point out that the rotation of the earth and the revolutions of the moon and planets are in fact clocks. Four of them —the moon, Mercury, Venus, and the earth's revolution—agree, while the earth's rotation disagrees with all the others.

The outer planets are clocks just as the three inner ones are, and in fact all celestial bodies are clocks, but most of them have such slow angular motions in consequence of their great distances that the motions during a second or two cannot be measured with sufficient precision for checking the earth's rotation. Thus we see that there is still another requirement that a practical measure of time must fulfill; in addition to being continuous, accessible, and "invariable," the recurring phenomenon that is being counted must recur moderately often. The meaning of *moderately often* depends on the precision of astronomical observations. In the present state of astronomy, a year is the longest natural unit of time that is of practical value. If the precision of astronomical observations were, however, to increase tenfold, then the revolution of Jupiter, which is accomplished in about 12 years, would provide a measure of time sensibly as good as the revolution of the earth is at present.

Ephemeris Time

We are now in a position to understand why the second has recently been redefined as a definite fraction of the tropical year rather than as a definite fraction of the mean solar day (*3*). And we are in a position to understand in what sense a measure of time may be said to be invariable; an invariable measure of time is simply the measure that brings the theories of the motions of celestial bodies into agreement with observations. Stated even more concisely, it is the independent variable of the accepted equations of motion. There is, however, no practical necessity for speaking of an invariable measure of time at all, and we shall see later why it may be inadvisable to do so. The word *invariable* implies something absolute in the minds of most persons, and it may be better to avoid it when we are speaking as precisely as possible. All that is really necessary for practical and scientific purposes is to choose a measure of time that appears suitable and to define it precisely. For convenience, a special name has been given to the measure of time that is the independent variable of the equations of motion; it is called *ephemeris time* in contradistinction to mean solar time. An *ephemeris* is a table of the positions of a celestial body at various epochs, calculated according to the accepted theories of motion; ephemeris time, then, is merely the measure of time defined by an ephemeris.

In redefining the second, it was specified that the tropical year for 1900.0 should be used, the decimal indicating the beginning of the year 1900. The reason for the qualification is that the tropical year (which corresponds to the seasons) is decreasing at the rate of 0.530 ephemeris second per century, or 1 in 5.95×10^9 per year. The variation being known, it is easy to relate any particular year to the tropical year for 1900.0 with a precision of 1 in 10^{13}.

Ephemeris time is determined in practice by observations of the moon; the moon moves more rapidly than the planets, and hence the time can be determined from the moon with greater precision. A single observation of the moon does not, however, fix the ephemeris time with the required accuracy. Until very recently it has been necessary to observe the moon for a year in order to accumulate enough observational material to be of value. Some time is also required for working up the observational material; hence our determinations of ephemeris time are about 2 years in arrears. The recent development by Markowitz of a new photographic technique for observing the moon (*4*) has resulted in a considerable increase in precision; it now appears that in a very few years we shall be able to determine ephemeris time from month to month as accurately as we have done from year to year. Even at best, however, ephemeris time cannot be determined as accurately as mean solar time. The earth rotates about 27 times as fast as the moon revolves, which is a considerable advantage; an error of 0.1 second of arc in observing an equatorial star corresponds to an error of 0.007 second of mean solar time, while the same error in observing the moon corresponds to an error of 0.18 second of ephemeris time. Ephemeris time is less accessible than mean solar time. Thus, in redefining the second so as to derive it from the year instead of the day, we have substituted a less accessible invariable unit for a more accessible variable one.

Precision of Determinations of Time

The practical determination of mean solar time consists in noting the instant, according to some mechanical or electric clock, when a star crosses the local meridian. The actual mean solar time of the star's meridian passage is known in advance by means of an extended series of special observations and calculations that will not be discussed here. The discrepancy between the actual time of meridian passage and the clock time gives the error of the clock. The clocks used for the purpose need not be set to the correct time, but instead, a record may be kept of their errors, which are continuously changing. This record permits any other clock, after comparison with the clock whose error is known, to be set to the correct time, preparatory to controlling the emission of radio time signals. But the time signals are not absolutely correct because of the errors in the astronomical observations and because of the errors in extrapolating the errors of the clocks.

The most precise instruments for determining mean solar time are the photographic zenith tube (*5*) and the Danjon astrolabe (*6*). With the photographic zenith tubes of the U.S. Naval Observatory, it is the practice to observe about 15 stars on every clear night, and the probable error of the mean result of a night's work is about 3 milliseconds. By *probable error* is meant the quantity that exceeds half of the actual errors, and is ex-

Table 2. Rough estimates of the probable relative errors with which different intervals of mean solar time are determined.

Mean solar interval	Probable error
1 day or less	1 in 10^8
30 days	1 in 4×10^8
365 days	1 in 10^{10}

ceeded by half of them. The best quartz-crystal clocks run with a precision greater than this, and they are used to smooth out the random errors in the astronomical observations from night to night, so that at a single observatory possessing the best instruments and clocks, the mean solar time can be determined with a probable error of, say, 2 milliseconds. The International Time Bureau at Paris, which is sponsored by the International Astronomical Union, intercompares the data supplied by the various national time services and thus is able to make, a year or so in arrears, still further improvements in the knowledge of mean solar time, the error at this stage being probably less than 1 millisecond. But for the present discussion, I shall assume a probable error of 2 milliseconds.

What has been said thus far relates to the establishment of the epoch; physicists and engineers in general are more interested in the establishment of frequencies or time intervals. For this purpose, the quartz clocks are more accurate than the astronomical observations over intervals of at least a few weeks, while for longer intervals the astronomical observations are more accurate than the clocks. So far as astronomical observations alone are concerned, any interval of mean solar time may be determined with an absolute error that has a probable value of 3 milliseconds, which is found by multiplying the probable error of a determination of the epoch by the square root of 2. The relative error of a determined time interval follows quite a different law. If observations were restricted to the two ends of the interval, the relative error would vary inversely as the duration of the interval, but if the interval is long enough so that many observations can be obtained within it, the accuracy is further improved, and is nearly proportional to the 3/2 power of the duration. Table 2 shows some rough estimates of the probable relative errors with which different intervals of mean solar time are determined by combining the indications of the best quartz clocks with the best astronomical observations.

The high accuracy indicated for an interval of 365 days is, however, of no practical value because the length of the day varies in an unpredictable manner every few years by as much as 1 in 10^8 or even more. For example, if the length of the day in 1936 is taken as the standard, then in 1923 it was longer by 1 in 10^8 and decreasing at the rate of 2 in 10^9 per year, while in 1940 it was also longer by 1 in 10^8, but increasing at the rate of 2 in 10^9 per year. The practical question is on the precision with which the newly defined second, or equivalently the length of the year, can be determined by observation. As has already been mentioned, the precision is less than it was for the old second, chiefly because of the relatively slow motion of the moon. In the past it has been necessary to collect observations of the moon for an entire year in order to determine ephemeris time with a probable error of about 100 milliseconds. This precision has been attained only in recent years; it corresponds to a probable error of 140 milliseconds in measuring the length of a single year, or 1 in 2×10^8.

Two very recent advances at the U.S. Naval Observatory have resulted in a considerable increase in precision. One is a precise survey of the marginal zone of the moon by Watts, which makes it practicable to allow satisfactorily for the irregularities of the surface; it must be understood that observations of the moon are referred to the bright edge of the visible disk, and any mountains at the points where measurements are made contribute to the uncertainty in ephemeris time. The other advance is the new observational technique already referred to, which relates the moon to several stars in the immediate neighborhood, while eliminating the errors of measurement caused by the moon's motion among the stars. It is now possible with a single telescope to determine the length of the year with a probable error of 1 in 4×10^8. Only one telescope is now working, but it is planned to put twenty in operation during the International Geophysical Year, 1957–58, so it may be expected that the length of that particular year will be determined with a probable error of 1 in 2×10^9. It is reasonable to suppose that at least four telescopes will continue to work indefinitely. Assuming that number, then the probable relative errors of determinations of ephemeris time intervals are those shown in Table 3.

The probable error for 1/12 year or less is based on the assumption that quartz-crystal clocks are used to subdivide the year; the astronomical observations are not so precise for short intervals. It must be understood that determinations having this precision are possible only in arrears, the delay being perhaps a year. Although the delay does not detract from the permanent value, it is a drawback in the short term.

Table 3. Probable relative errors of determinations of ephemeris time intervals.

Ephemeris time interval	Probable error
1/12 year or less	1 in 10^8
1 year	1 in 10^9
5 years	1 in 10^{10}

Atomic Standards

During the past few years, techniques have been devised that make it possible to obtain access to the natural vibrations of atoms and molecules. There is no reason to doubt that such vibrations have a very high degree of reliability, equal to the revolution of the planets around the sun and the moon around the earth. The practical difficulty lies in counting the number of them occurring in a second, which is of the order of 10^{10}. Several techniques have recently been devised for the purpose, and the natural resonant frequency of the cesium atom has been measured with a stated precision of 1 in 10^9 by Essen and Parry at the National Physical Laboratory in England (*7*). Essen and Parry state that the potential precision is considerably higher, but special electronic techniques will have to be developed before the higher precision can be utilized.

Thus there has become available a new standard of frequency, which can be used for the calibration of another frequency with the same precision being attained in a few minutes as can be obtained from astronomical observations in a year. This important advantage indicates that for some purposes atomic standards of frequency will soon be used in preference to astronomical ones. The atomic standard does, in fact, supply us with a natural unit of time independent of the second, and of quite a different character, being independent of the motions of celestial bodies, at least in an operational sense.

A very important question for basic science is whether atomic frequencies, stated in terms of the astronomical (ephemeris) second, are constant or variable. Physics provides us with no certain answer to the question, which will have to be settled by experiment. E. A. Milne and his collaborators constructed an elaborate physical theory called kinematic relativity (*8*), in which two natural time scales appear, one being continuously accelerated with respect to the other, so that the ratio of the two units of time is continuously increasing; the change at the present epoch is supposed to be a little less than 1 in 10^9 per year. Dirac, Milne, Jordan, and others have suggested that one of these units may be identified with atomic frequencies and the other with astronomical ones. If that is in fact the case, then it should be possible to measure the acceleration in about 5 years, and our earlier notions about invariable units of time will have to be drastically revised. We shall have no reason to favor one unit over the other by calling it invariable, and the word will no doubt drop out of use. Obviously, if such an acceleration is observed, the effect on basic physical theories and on cosmogony will be great and far reaching.

Atomic Unit of Time

Before atomic standards of frequency can be used in preference to astronomical ones, it is necessary to determine the frequency of the atomic standard itself in terms of the second. As noted before, this has already been done in terms of the old mean solar second, but it is desirable to do it also in terms of the new ephemeris second, which will require some time. Once it is done, then in all probability the astronomical second will soon be forgotten by most persons who work with the atomic standard from day to day, and there will be danger of confusion, particularly if the atomic and astronomical units should not have a variable ratio, or if the variation should be so small that a long time is required for its detection. Similar confusion does already exist to some extent with units of length.

The meter is defined as the distance between two marks on a certain bar of platinum. But some physicists have found it more convenient, instead of actually using the meter as a unit of length, to use instead the wavelength of a specified spectral line of cadmium or mercury; in many cases, wavelengths can be compared with one another with greater precision than that with which a wavelength can be compared with the meter. The number of standard wavelengths in a meter has been measured to a certain precision, and that is sufficient for the time being. But the number of wavelengths in a meter is an experimentally determined quantity, subject to revision and improvement. What is to be done each time a revision is made? Shall we attempt to revise every measurement in which wavelengths have been employed? Evidently such a procedure would be entirely impracticable. In order to prevent confusion from occurring, it is only necessary to adopt a new standard of length, say the angstrom, which is no longer to be regarded as precisely 10^{-10} meter, but is defined to have a fixed and invariable relation to the standard wavelength, the number of angstroms in a meter being experimentally determined and subject to revision. Thus, when measurements of lengths are stated in angstroms, it is to be understood that they are referred to atomic wavelengths, and when they are stated in meters, to the standard meter.

It is very desirable to treat the units of time similarly. We should reserve the word *second* to mean the astronomical second, and adopt a new unit, which I shall here call the *essen,* and which would have a fixed and invariable relation to the frequency of the cesium atom, being very nearly a small aliquot part of a second, the exact number of essens in a second being subject to experimental determination and revision. Then any frequency stated in cycles per essen would be understood to be referred to the atomic standard, while frequencies stated in cycles per second would be understood to be referred to the astronomical second. The distinction between the atomic unit of time and the astronomical unit of time is all the more necessary because both will remain in use for an indefinitely long time.

Consequences of Adopting an Atomic Unit of Time

Two important consequences would result from the adoption of an atomic unit of time that is to be used concurrently with the astronomical second. The first arises from the fact that the atomic unit of time is not independent of the atomic unit of length in the sense in which the meter is independent of the second. Wavelengths and frequencies are connected by a definite physical relationship: the product of the two is the velocity of light. Thus, if the velocity of light were known with sufficient precision, a wavelength could be deduced from a frequency, and vice versa. In fact, the velocity of light (in terms of the astronomical second) is uncertain by a part in 10^4 (recent determinations that have stated probable errors of 1 in 3×10^5 should not be taken at their face value until the discrepancy with earlier determinations is explained), and so this procedure cannot be used. But it would appear that the frequency of the cesium atom (in terms of the astronomical second) might be combined with the wavelength (in terms of the meter) so as to obtain an improved value for the velocity of light.

In discussing this matter, the greatest care is necessary in order to avoid circular reasoning. Let us assume that an atomic unit of time has been adopted, together with the atomic unit of length, and let us ask ourselves, In what sense, if any, is the product of a wavelength and a frequency, expressed in these units, to be regarded as the velocity of light. The answer is that such a product of numbers is not the value of the velocity of light, and in fact has no physical significance whatever; it is merely a number that was determined in advance when the atomic units of time and length were adopted. Thus, to say that the velocity of light is so many meters per second is to express an experimental result, but to say that the velocity of light is so many angstroms per essen (using the words in the sense I have suggested) is a tautology.

It is well known that the constancy of the velocity of light is a fundamental postulate of general relativity. What, then, becomes of general relativity if atomic time should turn out to be accelerated with respect to astronomical time? No doubt we shall hear it said that general relativity has been refuted, but that will not necessarily be the case. There are three valid ways of expressing the velocity of light: in meters per second, in meters per essen, and in angstroms per second; all three are the expressions of experimental results. The questions will be: (i) In which of the three modes of expression is the velocity of light variable? and (ii) Which of the three is referred to in the postulate of general relativity?

The second consequence of the adoption of an atomic unit of time arises from the fact that the atom, while being a natural standard of frequency, and in this respect much superior to arbitrary standards such as quartz crystals, which have to be continually compared with a natural standard, cannot be made to control a natural clock. In other words, there is no property of atomic vibrations that can be used to establish an epoch, as is done by the passage of a star over the meridian of Greenwich. It is true that atomic clocks can be built and that they will be very useful as secondary standards of time, but it will not be possible to adopt them as fundamental standards. They will not run indefinitely, but will stop occasionally because of failures in electric power of one sort or another. When they are started again, it will not be possible to ascertain the amount of time lost except by comparing them with other clocks that have continued running in the meantime. The only way of comparing clocks at some distance from one another is by means of radio time signals, which have variable velocities of transmission. Thus, after the adoption of an atomic standard of frequency, two units of time will be in use, one for frequencies and the other for measuring time itself.

Measure of Performance Q

There is a quantity Q that I venture to mention here only because it has occasionally been used (and more often misused) as a measure of the excellence of clocks. Among the more important legitimate applications of Q is the measure of the performance of resonant electronic circuits. In this application, Q is a measure of the sharpness of tuning—the sharper the tuning the higher the Q. Here

Q is equal to the ratio of the resonant frequency to the bandwidth between the frequencies on opposite sides of resonance where the response of the circuit differs by 3 decibels from that at resonance. Quartz-crystal clocks employ an electronic circuit that is tuned to the frequency of the crystal. Evidently, the higher the Q of such a clock, the more accurately the clock will count the vibrations of the crystal. With the techniques employed for atomic standards, the variation of response with frequency is similar to that of a resonant circuit, and so it has been found convenient to define the ratio of the frequency of maximum response to the 3-decibel bandwidth as an equivalent Q, which effectively measures one characteristic of such a standard.

Now there is another quantity δ, which is the symbol for logarithmic decrement, that is of importance in connection with damped oscillations such as spark discharges. It measures the amount of damping and is equal to the natural logarithm of the ratio of the amplitudes of two consecutive oscillations. It turns out that for a transient discharge through a resonant circuit, $Q = \pi/\delta$, π being the familiar constant 3.14159. This relation makes it possible to measure the damping by Q as well as by δ, a large Q corresponding to a small δ.

Having applied Q to damped electric oscillations as a measure of the damping, it was only a step to apply it by analogy to measure the damping of any damped oscillation whatever, such as the vibrations of a tuning fork or of a pendulum. It has been asserted, as proof of the superiority of atomic standards over pendulum clocks and quartz clocks, that the latter have a Q of only 10^6, while the Q of the former is much larger, various figures between 10^7 and 10^{18} being cited. Of these figures, 10^{18} is the Q approached in molecular transitions, while 10^7 is a value that may reasonably be expected to be attained in practical applications. These entirely misleading comparisons produced considerable confusion because the fact is that Q has about as much to do with the performance of a clock as the size of the battery has to do with the performance of an automobile; a certain size for either is a necessary, but not a sufficient, condition.

There are two principal reasons why Q is not useful as a measure of the performance of clocks. The first is that in clocks the natural damping of the pendulum or quartz crystal is counteracted by applying power in such a way as to maintain the oscillations at a nearly constant amplitude. Thus Q has a meaningful value only during the interval between successive applications of power, which may be made as often as we please. The way in which the power is applied is the most important single factor in determining the performance of a pendulum clock. The second reason is that the amplitude of the oscillations has nothing to do with the precision of a clock, provided that the changes in amplitude do not affect the frequency, or that they do affect it in a determinate manner. The clock that is provided by the rotation of the earth, for example, is supposed to be retarded by tidal friction, the corresponding value of Q being about 10^{13}, and the frequency changing by 1 in 5.3×10^9 per year. This retardation, provided that it is known and allowed for, does not detract from the excellence of the clock in the slightest degree. It is rather the irregular unpredictable changes of frequency that have led to the redefinition of the second in terms of the tropical year.

All that can properly be said of Q in connection with standards of time and frequency, and of the size of batteries in automobiles, is that they should not be so small that they constitute an effective limitation on performance.

Carbon Clock

For completenesss, it may be desirable to mention an entirely different sort of measure of time from any discussed thus far: the measure that is furnished by the radioactive decay of isotopes of various elements. In recent years, an isotope of carbon has been extensively used for the purpose, the proportion of the isotope present at any time being a measure of the time elapsed since the carbon was deposited. The principal use of this measure of time has been for dating fossils and geologic deposits. It is worth noting that the carbon isotope is in fact a clock, according to the definition stated earlier. The recurring phenomenon that is counted is the decay of an atom of the isotope. It is true that the decayed atoms can be counted only statistically and not individually, but this is not a drawback in principle, although it does severely limit the precision; if a technique could be developed for counting the individual transformations as they occur, the clock might become quite precise. The carbon is also a natural clock; the epoch it establishes is the epoch at which the carbon was deposited. It is not, however, suitable as a fundamental standard because it is not unique; there are as many different epochs as there are deposits.

Requirements for a Standard of Time

We may now reformulate the requirements for a satisfactory standard of time. We have seen that in fact there is no requirement for the invariability that was mentioned earlier because there is no way of determining whether a measure of time is invariable or not. For invariability we may substitute the requirement that, of two satisfactory standards, the acceleration of one on the other must be constant (or zero). Thus, a satisfactory standard of time must be continuous, must be accessible, must have a constant acceleration on other satisfactory measures, must be based on a unit that is neither too long nor too short, and must establish a unique epoch. Of two satisfactory standards of time having a mutual acceleration, the one to be regarded as fundamental is the one that leads to no contradiction between observations and physical theories. If it should turn out, for example, that the measure of atomic time is consistent with quantum mechanics while the measure of astronomical time is consistent with general relativity, one being accelerated on the other, then it will be the worse for at least one of the theories; it will be necessary to remove the contradiction in the theories before it can be decided which of the two measures, if either, is the fundamental one.

The requirements for a satisfactory standard of frequency and for a fundamental standard are the same as those for a standard of time, except that a standard of frequency need not establish a unique epoch. The requirement for continuity may also be somewhat relaxed; a standard of frequency need be continuous only over any time interval that it is required to measure.

References

1. W. J. Eckert, D. Brouwer, G. M. Clemence, *Astron. Papers Amer. Ephemeris vol. 12* (Government Printing Office, Washington, D.C., 1951).
2. D. Brouwer, *Astron. J.* 57, 125 (1952).
3. *Trans. Intern. Astron. Union*, in press.
4. W. Markowitz, *Astron. J.* 59, 69 (1954).
5. G. M. Clemence, *Am. Scientist* 40, 260 (1952).
6. A. Danjon, *Compt. Rend. Acad. Sci. Paris* 227, 320 (1948).
7. L. Essen and J. V. L. Parry, *Nature* 176, 280 (1955).
8. E. A. Milne, *Kinematic Relativity; a Sequel to Relativity, Gravitation, and World Structure* (Clarendon Press, Oxford, 1948).

Nothing has tended more to retard the advancement of science than the disposition in vulgar minds to vilify what they cannot comprehend.—SAMUEL JOHNSON.