high levels of the hydroxy compound (1 or 2 mg per tube) show a marked growth-depressing effect. This growth depression by high levels of hydroxylysine is eliminated by addition of larger amounts of L-lysine.

Both *Leuconostoc mesenteroides* P-60 and *Streptococcus faecalis* are commonly used for microbiological assay of lysine in hydrolyzates of foods and tissues and each of these organisms has been accepted, on the basis of earlier work (*6, 7*), as having a highly specific requirement for lysine. However, in view of the results reported here, it seems likely that the presence of hydroxylysine in sample hydrolyzates could interfere with the quantitave microbiological determination of lysine when these organisms are used with basal media that contain no hydroxylysine.

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13 September 1955

What Are Variables and Constants?

The notion *variable* has not in the literature attained the degree of clarity that would justify the almost universal use of that term without explanations. Recent investigations (*1*) have resolved it into an extensive spectrum of meanings, some pertaining to reality as investigated in science, others belonging to the realm of symbols studied in logic—two altogether different worlds. As a corollary, these distinctions yield a clarification of the notion *constant.*

Science and applied mathematics. By *quantity* we mean a pair of which the second member (or *value*) is a number while the first member (or *object*) may be anything. We call a class of quantities *consistent* if it does not contain two quantities with equal objects and unequal values. If $p(\gamma)$ denotes the pressure (in a chosen unit) of a gas sample γ (*2*), then the pair $[\gamma, p(\gamma)]$ is a quantity, and the class of all such pairs for any γ is consistent. This class reflects the physicist's idea of gas pressure, p. Gas volume, v, and temperature, t, can be defined similarly. Consistent classes of quantities, such as p, v, and t, are what scientists and mathematicians in applied fields mean by *variable quantities* and what Newton called *fluents.* The class of all gas samples will be referred to as the *domain* of p; the class of all numbers $p(\gamma)$ as the range of p.

A variable quantity whose range consists of a single number is said to be *constant.* An example is the gravitational acceleration g at a definite point on the earth, defined as the class of all pairs $[\alpha, g(\alpha)]$ for any object α falling in a vacuum, where $g(\alpha)$ denotes the acceleration of α. For any α, this value of g is found to be equal to one and the same number, g. (Throughout this paper, symbols for fluents are italicized and symbols for numbers are printed in roman type.) A less important, because less comprehensive, example is the distance traveled by a specific car C while C is parked. (The distance traveled by C is the class of all pairs $[\mu, m(\mu)]$ for any act μ of reading the mileage gage in C, where $m(\mu)$ is the number read as the result of μ.

In a plane, the coordinates relative to a chosen Cartesian frame are variable quantities whose domain is the class of all points (*3*). The abscissa x is the class of all pairs $[\pi, x(\pi)]$ for any point π. In the equation of the straight line $x - y = 3$ the *3* denotes the constant fluent consisting of the quantities $(\pi, 3)$ of value 3 for each point π.

Of paramount importance are the consistent classes of quantities whose domains consist of numbers. An example is the class of all pairs $[\mathrm{x}, \log \mathrm{x}]$ for any number $\mathrm{x} > 0$, called the logarithmic function or the function *log*. It is capable of connecting consistent classes of quantities—for example, $y = \log x$ along a logarithmic curve, and $w = \log v$ for an isothermic expansion of an ideal gas, if w denotes the work in a proper unit. That is to say, $y(\pi) = \log x(\pi)$ and $w(\gamma) = \log v(\gamma)$ for any point π on the curve and any gas sample γ pertaining to the process. Moreover, *log* connects functions—for example, the exponential with the identity function, and *cos* with *log cos*. Because of their connective power, functions are omnipresent in science as well as in mathematics (*4*). Variable quantities such as p and v (whose domains do not contain numbers or systems of numbers) lack this power and therefore are confined to special branches of science such as gas theory. Denying the significance of this difference would be denying the role of mathematics as a universal tool in quantitative science.

Logic and pure mathematics. The formula

$$3^2 - 1 = (3 + 1) \cdot (3 - 1)$$

is a statement about specific numbers designated by 3 and 1. In the more general statement

$$\mathrm{x}^2 - 1 = (\mathrm{x} + 1) \cdot (\mathrm{x} - 1) \text{ for any number } \mathrm{x},$$

the remark "for any number x" stipulates that, in the formula, x may be replaced by the designation of any number, for example, by 3 or $\sqrt{5}$, each replacement yielding a valid statement about specific numbers. A symbol that, in a certain context and according to a definite stipulation, may be replaced by the designation of any element of a certain class is what, following Weierstrass, logicians and pure mathematicians call a *variable.* The said class is called the *scope* of the variable. The scope of x in the formula $\log \mathrm{x}^2 = 2 \log \mathrm{x}$ for any $\mathrm{x} > 0$ is the class of all positive numbers. In $(\pm\sqrt{3})^4 = 9$, the symbol $\pm\sqrt{3}$ is a variable whose scope consists of the two numbers $\sqrt{3}$ and $-\sqrt{3}$.

A variable whose scope consists of a single number designates that number and, in pure mathematics, is referred to as *a constant.* Examples include numerals (1, 3, . . .); e, designating the base of natural logarithms; and brief designations of numbers with unwieldy symbols, such as $2^{\sqrt{2}} + \mathrm{e}^{\mathrm{e}^{\mathrm{e}}}$—abbreviations introduced for the purpose of just one discussion involving repeated references to that number. As such *ad hoc* constants, one customarily uses the letters a, b, c, . . . , which, just as x and y, serve as variables in other contexts, for example, in the statement involving two variables

$$\mathrm{x}^2 - \mathrm{a}^2 = (\mathrm{x} + \mathrm{a}) \cdot (\mathrm{x} - \mathrm{a}) \quad \text{for any x and any a.}$$

Because of its vicarious character, a variable may always be replaced, without any change of the meaning, by any otherwise unused letter, for instance, a by y or x by b.

Variables whose scopes are classes of numbers are called *numerical variables.* The definitions of p and g make use of variables γ and α whose scopes consist of gas samples and falling objects, respectively. Statements that are valid for many fluents are conveniently expressed in terms of *fluent variables*; for example,

$$\text{If } z = \log u, \text{ then } \mathrm{d}z/\mathrm{d}u = 1/u \quad \text{for any two fluents } u \text{ and } z \qquad (1)$$

In Eq. 1, u and z may be replaced by x and y: if $y = \log x$ (which holds along the logarithmic curve), then $\mathrm{d}y/\mathrm{d}x = 1/x$; or by v and w: if $w = \log v$ (which holds for an isothermic expansion), then $\mathrm{d}w/\mathrm{d}v = 1/v$. But u and z in Eq. 1 must not be replaced by numbers such as e and 1: although $1 = \log$ e is valid, $\mathrm{d}1/\mathrm{de} = 1/\mathrm{e}$ is nonsensical.

Confusion in the literature. No clear distinction has heretofore been made between numerical and fluent variables. Moreover, in the literature, numerical variables and variable quantities, notwithstanding the profound differences between them, are indiscriminately referred to as "variables"; and the two concepts have actually been confused. For instance, numerous attempts (5) have been made to define gas pressure as a symbol that may be replaced by any value of pressure, thus as a numerical variable, p, whose scope is the range of what herein has been called the fluent *p*. But a fluent is not determined by its range. Could Boyle have connected pressure and volume on the basis of mere information about the ranges of *p* and *v*? It was by transcending these ranges and referring to the domains, namely, by comparing $v(\gamma)$ and $p(\gamma)$ for the same sample γ, that he discovered $v(\gamma) = 1/p(\gamma)$ (in proper units) for any γ of a certain temperature or, without reference to a sample variable, $v = 1/p$. Neither formulation involves numerical variables. Boyle's Law connects specific fluents.

Analogously, constant fluents have been identified with their numerical values even though what primarily interests the physicist in gravitational acceleration clearly is the fact that g is its value for any α rather than the number g as such.

Numerical variables and variable quantities belong to worlds that are not only different but nonisomorphic. The former are interchangeable, the latter are not:

$$x^2 - 4y^2 = (x + 2y) \cdot (x - 2y) \text{ and } y^2 - 4x^2 = (y + 2x) \cdot (y - 2x)$$

for any x and y are tantamount. But $x - y = 3$ and $y - x = 3$ are different straight lines and $w = log\ v$ is incompatible with, and not tantamount to, $v = log\ w$.

Only the consistent maintenance of all these distinctions makes it possible to formulate mathematical analysis as well as its applications to science as a system of procedures following articulate rules (6).

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References and Notes

1. Compare K. Menger, "The ideas of variable and function," *Proc. Natl. Acad. Sci. U.S.* 39, 956 (1953) and "On variables in mathematics and in natural science," *Brit. J. Phil. Sci.* 5, 134 (1954). A paper "Random variables and the general theory of variables" is in print *Proc. 3rd Berkeley Symposium Math. Statistics* (1955). A systematic exposition of the new theory of variables is contained in K. Menger, *Calculus. A Modern Approach* (Ginn, Boston, 1955).
2. The term *gas sample* means gas in a specific container at a definite instant.
3. Depending on whether a physical, a postulational, or a pure plane is under consideration, a "point" is a physical object (for example, an ink dot in a paper plane), or an undefined object satisfying certain assumptions, or an ordered pair of numbers.
4. Even if, following the suggestion of some mathematicians, one called all consistent classes of quantities "functions," one obviously would need a special term for "functionally connecting functions" such as *log*.
5. Compare, for example, R. Courant, *Differential and Integral Calculus* (Interscience, New York, 1954), vol. 1, p. 14.
6. Compare K. Menger, *Calculus. A Modern Approach* (Ginn, Boston, 1955).

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Prenatal Diagnosis of Sex Using Cells from the Amniotic Fluid

In most mammals, including human beings, males normally have the sex chromosome constitution XY, and females the sex chromosome constitution XX (*1*). It has been shown in a variety of tissues in human beings and some other species (2) that there is a sex difference in the percentage of cells with chromocenters, especially those at the nuclear membrane; this presumably is due to this difference in sex chromosome constitution in males and females. A determination of the percentage of cells with chromocenters can therefore give, in sexually normal individuals, a diagnosis of sex.

The present study was undertaken in order to show whether, in human beings, such a diagnosis can be made before birth, not only for aborted fetuses from which pieces of tissue can be removed for examination, but also for viable fetuses by an examination of cells from the amniotic fluid. In order to establish whether amniotic fluid contains cells suitable for diagnosis, fluid was taken before delivery by puncture of the membranes from women in the ninth month of pregnancy. The fluid was centrifuged, and the cells were smeared on slides, fixed in alcohol-ether, and stained with Feulgen and fast green (Fig. 1).

Our analysis has shown that cells suitable for the diagnosis are present, and an examination of 35 cases in the ninth month, which include those reported previously (*3*), has given 35 correct diagnoses of the sex of the fetus.

It therefore seems that this method is particularly reliable, especially since there appears to be no theoretical objection to it. The only apparent exception that occurs to us at present is the rare case of an intersex in which the sexual phenotype does not correspond to the sex chromosome constitution. When one is collecting the amniotic fluid it is, of course, essential to avoid contamination with cells from the mother.

Amniotic fluid can be obtained from viable human fetuses from 12 weeks to term (*4*). We have found, from an examination of the fluid obtained from viable human fetuses in the sixth and seventh months, that a prenatal diagnosis of sex can be made at these stages. We have also found suitable cells in the fluid of an aborted 8-week-old human embryo.

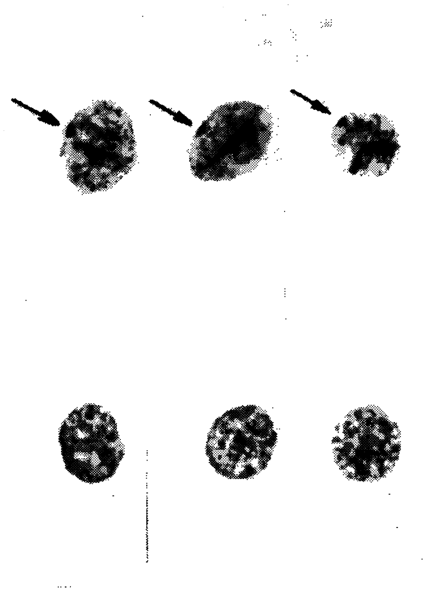

Fig. 1. Photomicrographs of cells from the amniotic fluid (× 1500). (Top row) Nuclei with a chromocenter at the nuclear membrane; from female human fetuses in the ninth month. (Bottom row) Nuclei without a chromocenter; from male human fetuses in the ninth month.

It may be possible to apply this method for the prenatal diagnosis of sex to domestic animals.

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