

Albert Einstein [Photograph [©] by Fred Stein, New York]

Albert Einstein was born in Ulm, Germany, on 14 March 1879. He received his doctorate from the Swiss Federal Polytechnic Institute at Zurich in 1906. From 1902 to 1909 he was a patent examiner in the Swiss Federal Patent Office. In swift succession he held academic chairs at the University of Zurich, the University of Prague, and the Swiss Federal Polytechnic Institute. In 1914 he accepted membership and a professorial staff position in the Prussian Academy of Sciences at Berlin, one of the most honored and independent academic po-(Continued on page 487)

SCIENCE

Fifty Years of Relativity

Peter G. Bergmann

Albert Einstein published his first paper on the theory of relativity 50 years ago while serving as a member of the staff of the Swiss Federal Patent Office (Amt für geistiges Eigentum) at Berne. To celebrate this anniversary, the professors of theoretical physics of the various Swiss universities organized an international conference at Berne, which took place 11-17 July 1955. This meeting had not been conceived primarily as a formal gathering but rather as a working conference that would bring together most of the active workers in the field of relativity. The untimely death of Professor Einstein served to underline the solemnity of the occasion. The conference plans themselves had been made while Einstein was still living; though he himself had not intended to make the trip from the United States to Switzerland, he had taken part in the preparations

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This article is intended to serve both as a review of the half-century of growth of relativity and its impact on physics as a whole, and as a brief account of the Berne conference.

Special Theory of Relativity

What we call today the special theory of relativity was put forward by Einstein in 1905 in a paper in the Annalen der Physik, which he entitled "The electrodynamics of bodies in motion" (1, 2). At that time, the theory of the electromagnetic field had developed to the point where it came into serious conflict with the foundations of classical mechanics. Attempts at a resolution by experimentation had served to highlight this conflict. The problem was solved by Einstein in a manner that eventually brought about a complete revision of our concepts of space and time. We shall begin with a brief survey of the situation at the turn of the century.

Newton had established that within the framework of his mechanics it was impossible to discover any "absolute" motion that was purely translatory and free of acceleration (the classical principle of relativity). All "inertial" frames of reference were to be considered equivalent. This principle is usually illustrated by the example of a laboratory aboard a moving vehicle. As long as there is no acceleration, the passengers cannot discover evidence of their motion by means of experiments wholly carried out with equipment belonging to their laboratory. Presumably the earth is such a moving vehicle. And though it is quite possible to demonstrate the rotation of the earth about its axis (Foucault's pendulum) and its motion about the sun (solar tides), we have no way of observing the earth's and the solar system's motion (if any) through space. As far as Newtonian mechanics is concerned, it is meaningless to talk of absolute rest and absolute motion, though it is meaningful to talk of absolute acceleration and absolute rotation.

With the advent of electromagnetism, a new situation arose. Maxwell had conceived of the electromagnetic field as a sort of stress produced in a carrier medium, the ether. His equations predicted the existence of transverse elastic waves in this ether (somewhat similar to the seismic waves in the earth) propagating at a uniform speed of about 3×10^{10} centimeters per second in the absence of retarding matter. Maxwell had ventured the guess that visible light was a form

sitions existing in Imperial Germany. While he was still at Berlin, Einstein had accepted a professorship at the newly organized Institute for Advanced Study at Princeton, New Jersey. When the Nazis assumed government power in Germany, Einstein severed his connection with the Prussian Academy and moved to Princeton. He remained with the institute until his death on 18 April 1955.

Einstein is usually identified in the public mind with the theory of relativity; however, he also contributed in an extremely significant manner to quantum theory and to statistical mechanics. He more particularly initiated the theory of the photoelectric effect, the theory of quantum emission and absorption of light, the quantum theory of specific heats, and the theory of fluctuation phenomena.

Fairly detailed information, much of it in semitechnical and nontechnical language, about the significance of his many contributions can be found in the volume *Albert Einstein: Philosopher*— *Scientist*, edited by P. A. Schilpp (Library of Living Philosophers, Evanston, Ill., 1949). This volume contains Einstein's "Autobiographical notes," as well as contributions by leading physicists and philosophers discussing Einstein's works. It also contains the most complete bibliography of Einstein's writings through 1949, a second edition for another two years. Early in 1955 the Institute for Advanced Study published through Princeton University Press a list of publications of its members, which brings the list up through 1954. The most complete list of Einstein's writings, to the time of his death, is the one found in the German translation of the Schilpp volume (Kohlhammer Verlag, Stuttgart, 1956).

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of electromagnetic wave motion. H. Hertz was the first to produce radiofrequency waves and to propagate them across his laboratory. The new field of physics soon proved refractory to all attempts to treat it as part of mechanics. The ether had to be endowed with all kinds of properties unknown in any other elastic medium. It needed to be incompressible (to prevent the possibility of longitudinal waves) and quite rigid; at the same time it should not offer any resistance to the passage of material bodies through it. Attempts to measure its motion relative to the earth (or vice versa) led to contradictory results.

If electromagnetic radiation was to propagate with uniform speed through the ether, a careful determination of the apparent speed of light relative to earthbound laboratory instruments should reveal the relative motion of laboratory and ether. But there were other possibilities as well. A number of ingenious experiments-for example, the experiment by Trouton and Noble (3), and the famous experiment by Michelson and Morley (4)-were conceived and carried out, all based on some "transport terms" that would presumably appear in Maxwell's equations if they were transcribed from the frame of reference provided by the ether to some different frame of reference. Every one of these experiments failed. One might have concluded that the ether was being dragged along locally by such large masses as the earth, but this view was contradicted by the astronomical effect known as "aberration." In careful determinations of the locations of fixed stars in the sky, it is found that the stars carry out an apparent periodic annual migration, with an elliptic path whose major axis has the same value for all stars and whose minor axis depends on the angular distance from the plane of the ecliptic. Aberration can be explained in quantitative detail by the assumption that the ether does not participate in the motion of the earth about the sun (5). Still, it provides no further information about the motion of the ether relative to our whole solar system.

H. A. Lorentz (2, 6, 7) attempted to reconcile this seeming contradiction by the postulate that in moving through the ether bodies contracted uniformly in the direction of motion and that actual time must be replaced by a "local time"that is, the apparent time indicated by clocks that were moving through the ether. Poincaré (8, 9) discovered the "group property" of Lorentz's proposed transformation equations. He established that the transformation equations that led from the "true" lengths and time to the "contracted" lengths and the "local" time of a moving frame of reference, or from these quantities in one moving

frame to another set defined in another moving frame, or finally from some "contracted," "local" set back to the "true" scales of space and time, were all identical. The Lorentz equations had this formal property in common with those introduced by Galileo and Newton: there was no way, by studying the mutual relationships between various frames of reference, to establish a mathematically or physically privileged "state of motion of the ether." All frames of reference whether "at rest" or in uniform translatory motion were equivalent. If Lorentz's transformation equations described correctly the behavior of actual scales and clocks, then any experiment concerned with purely electromagnetic phenomena was bound to confirm this (Poincaré's) principle of relativity.

The physicist Lorentz had explained the negative outcome of all "ether wind" experiments without giving up the notion of the ether itself as the medium of transmission of all electromagnetic disturbances. The mathematician Poincaré had formulated a new principle of relativity for the realm of electromagnetic phenomena, without attempting a detailed physical analysis or interpretation. It remained for Einstein to provide an integrated mathematical-physical analysis (1, 2). Without bothering with a detailed review of the unsuccessful though ingenious ether-wind experiments (none of them is even mentioned individually in his paper), he started with the remark that to the first order the outcome of quite elementary experiments depends only on relative motion; for example, the electromotive force generated in a conductor by a nearby magnet depends only on the relative motion of wire and magnet; it is the same whether the wire is moved in the field of the magnet at rest or whether we move the magnet and leave the wire stationary. All other firstorder effects fall into the same pattern. Einstein then postulated that this symmetry was valid not only to the first order (in the relative velocities) but exactly; he also retained the universal validity of the law of uniform propagation of electromagnetic waves. He discovered that the apparent contradiction could be resolved by a more penetrating analysis of the meaning of space and time measurements; he began by exposing the relative nature of the concept of simultaneity as applied to distant events.

To establish the simultaneity of happenings in faraway places, we must, in principle, possess a system of clocks distributed throughout space and synchronized with each other. To accomplish such synchronization, we require signals that will permit the speedy transmission of knowledge over large distances. There being no means of transmission faster than light, we generally use light for this

purpose. Two clocks will then be considered synchronous if light takes (apparently) the same time to travel either way. Let us now consider two sets of clocks, each set distributed over a large region of space, but one set "stationary," the other set traveling all in the same direction at a constant rate of speed. If, then, we merely assume that the latter set of clocks all run at the same speed (not necessarily the same speed as the stationary set), we may synchronize the traveling clocks with respect to one another, using the same light signals as we did for synchronizing the stationary clocks. But if the traveling clocks are synchronized with respect to one another, they will not be synchronous with the stationary clocks. The farther back a traveling clock is located (as viewed in the direction of forward motion) the farther ahead it must be set (as observed by a stationary observer) in order to be synchronous with the other traveling clocks. If two events are timed relative to each other, the result of this measurement will obviously depend on which of the two sets of clocks we employ as our standard.

Once Einstein had discovered that simultaneity was a relative concept, depending on the state of motion of the observer, he found it easy to show that comparison of *lengths* of moving scales as well as of rates of moving clocks depended on judgments of simultaneity. In other words, two observers measuring the length of a moving rod will in general disagree, and there will be no way to tell which one is "right." Einstein then proceeded to rediscover Lorentz's transformation equations, but with a new physical interpretation. Instead of leading from "true" lengths and times to "apparent" or "local" lengths and times, the equations were now found to lead from one set of valid coordinates to another set of equally valid coordinates (describing both space and time). In this interpretation, the Lorentz equations contradicted flatly the old (Galilean) transformation equations, which had been based on the (tacit) assumption of a universal, "absolute" time. Either of these transformation laws was purely "kinematic" -it purported to make statements about the relationship between measurements by two observers moving relatively to each other, without reference to the dynamics of particular physical systems. The scales and clocks used by either observer were to be "good" instruments: a scale was a solid body that retained its shape under appropriate safeguards (constant temperature, absence of mechanical stresses, and so forth), and a clock was any system that possessed a reproducible period. The contradiction between old and new transformation laws would have to be settled eventually. This

task was not completed until 1916, when Einstein presented a new theory of gravitation.

In the meantime, Einstein discovered the famous relationship between energy and mass, publishing a brief paper on the subject and suggesting that experimental evidence might be found in radioactive substances (10). Minkowski discovered that mathematically the Lorentz equations represented rotations of the coordinate system in a four-dimensional continuum (space and time combined) with an indefinite metric (11). Accordingly, he constructed a vector and tensor calculus for such a space and succeeded in showing that in terms of this new formalism the laws of the electromagnetic field take a particularly simple and beautiful form. In the four-dimensional continuum, the electric and the vector potentials together form a single vector field, whose curl, a tensor, possesses altogether six components, ordinarily designated as the components of the magnetic induction and the electric field strength. The four-dimensional divergence of this latter tensor (in empty space) equals a new vector field whose components are proportional to electric charge and current density. Finally, from the six components of the electromagnetic field, we can construct the four-dimensional analog of Maxwell's stress tensor of the electromagnetic field, a set of ten quantities, six of which are the components of the original stress tensor (including radiation pressure), three of which represent the flux of energy (Poynting's vector), and the last of which is the energy density of the field.

Let us return to Newton's mechanics with its absolute space and time scales. Whereas Maxwell's theory is concerned with a *field* that extends throughout space and is governed in its dynamics by partial differential equations, classical mechanics is concerned with separated mass points, the forces they exert on each other, and their motion under the influence of these mutual forces. The laws of the field are local laws-that is, the field changes at a given space point in the course of time because of the fields and their gradients in the immediate vicinity. In contrast, the appropriate dynamic laws of mechanics describe action at a distance: across empty space one mass point experiences the influence of other mass points. Experience had taught that this force was an attraction or repulsion between the interacting mass points, depending in magnitude on their intrinsic properties (mass, electric charge) and their mutual distance only. Mechanics does not recognize forces that depend on the velocity. For the formulation of any dynamic law in mechanics, an absolute concept of simultaneity appeared to be fundamental; to tell the distance between

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two mass points that are moving relative to each other, one must first be able to tell unambiguously where both of them are at the same time. The new theory of relativity thus appeared in direct conflict with classical mechanics.

If we consider the actual range of classical mechanics, we find three wide areas of application. The first of these is the theory of motion of celestial bodies, the second electrostatics and magnetostatics, and the third the short-range action of bodies on each other, as in gears, levers and similar machines important in everyday engineering. Of these areas, electrostatics and magnetostatics are limiting cases of electrodynamics, the one field in which the new theory of relativity had proved itself so successful. In general, electric charges affect each other dynamically only indirectly. A charge will give rise to an electromagnetic field in its vicinity; this field will propagate throughout space in accordance with Maxwell's laws; wherever this spreading field encounters another charge, it will exert a force on it, which depends only on local conditions. In the limiting case of negligible velocities, Maxwell's laws simplify so that a direct relationship may be established between the force acting on the second particle and its distance from the first particle (the source of the field), and this is Coulomb's law. Only in this limiting case can we omit the field from the mathematical formulation of the laws of motion without serious error.

It is at least conceivable that the apparently purely mechanical law of gravitation (Newton's inverse-square law) represents a similar limiting case of a more generally valid field law. This conjecture eventually led Einstein to the general theory of relativity.

As for the third area of applicability of classical mechanics, Einstein assumed that the laws of conservation of energy, linear momentum, and angular momentum, which are usually sufficient to describe the laws governing short-range (impulsive) interaction, would remain; the question was how the detailed expressions for the energy and the momenta should have to be modified so that their conservation would be valid for any observer regardless of his state of motion. These modifications were developed by Einstein in 1906. He found that the mass of a body, if defined as the ratio between its linear momentum and its velocity, would have to depend on its state of motion and would, therefore, be different for different observers. Again he found that the increase in mass due to motion was proportional to the (relativistic) kinetic energy (12).

Thus the special theory of relativity was capable of absorbing two of the three areas of classical mechanics. Its theoretical development was thereby es-

sentially completed. In the decades to follow, experimental physicists worked with ever higher energies. The deviations between the old and relativistic mechanics, which were barely observable in 1905, assumed ever larger proportions as physicists succeeded in producing particle velocities approaching the speed of light. Eventually, it became commonplace to measure nuclear energy losses as mass defects and to observe the conversion of material particles into energy and vice versa. The latest discovery in this respect, the antiproton and its recombination with a proton, was announced but a few weeks ago.

The relativistic variability of mass was originally a design limitation for Lawrence's first cyclotron. This limitation was overcome through the invention of the phase-modulated cyclotron (synchrocyclotron) and the true synchrotron as well as through the development of linear accelerators and the betatron, devices that are capable of operating in the extreme relativistic energy range. There has been further careful work on the old kinetic effects explained 50 years ago by Lorentz, Poincaré, and Einstein (13-15). Interesting as these experiments are, they can no longer be considered crucial for the verification of special relativity. In our time, every new accelerator that works according to design is existing proof of the validity of Einstein's theory of relativity. As for purely kinetic effects, determinations of meson life-times at relativistic energies demonstrate the slowing-down of moving clocks more impressively than the delicate canal ray experiments by Ives (15). Although we cannot rule out the further development of any physical theory, there is little question that we shall never witness the return of physics from relativity to the Newtonian-Galilean concepts of space and time.

Relativity played a vital role in the development of modern quantum theory. Within a very few years after the emergence of wave mechanics (16), Dirac showed that the relativistic theory of the electron differed fundamentally from the nonrelativistic theory (17). The relativistic electron must be a particle of spin $\frac{1}{2}$ if the probability density for a single particle is to be nowhere negative. He also recognized that such a relativistic electron possesses states of large negative energy, states into which a single free electron will drop-in contradiction to our experience-unless electrons obey Fermi statistics (that is, each possible state of an electron accommodates no more than one actual electron) and unless all negative energy states are ordinarily occupied. An occasional unoccupied state of negative energy appears to the observer as if it were a particle of positive charge and positive energy, a socalled positron. When an electron drops into this free "hole," both the electron and the positron disappear from the scene of observable particles, and we speak of the "annihilation of an electron-positron pair." The reverse process is known as pair creation. Thus Dirac's relativistic theory predicts the observed qualitative properties of electrons in a completely satisfactory manner. Proton and antiproton are another instance of Dirac particles.

Within the last 10 years, a number of difficulties in the quantum theory of electromagnetic radiation as well as of electrons have been greatly ameliorated by means of newly devised, consistently relativistic procedures known as "renormalization procedures" (18-22). Though the theory is not yet completely satisfactory, it is fair to say that it agrees well with the facts and that it is superior to any nonrelativistic theory.

General Theory of Relativity

When Einstein tackled the theory of gravitation, he recognized as early as 1907 that the extension of the new spacetime concept to that area would not be routine (12). A steady concentrated effort directed toward the riddle of gravitation began about 1911, culminating in the first comprehensive presentation of the general theory of relativity in 1916 (23).

For small velocities, Newton's law of gravitational interaction and Coulomb's law of electric interaction are similar in that they are both inverse-square laws. This fact is undoubtedly not accidental. It encouraged Einstein to search for a relativistic field law that would resemble Maxwell's laws of the electromagnetic field. The source of the gravitational field is the distribution of gravitating masses. But a mass distribution in relativity is described by a tensor with ten components, of which one represents the density of mass, three its flux, and six the stresses present. Accordingly, the gravitational field must also possess ten potentials, a conjecture that has been borne out by the completed theory. In the meantime, the task of constructing field equations for a ten-component potential field, with possibly 40 components representing field strengths, appeared overwhelming, not because it could not be done but because there are so many different logical possibilities. Instead of carrying on a formal investigation of this multiplicity, Einstein turned his attention to the physical peculiarities of gravitation. True, the static law resembled that of the electron field. But there was one significant difference. The acceleration of an electrically charged particle in a given electric field

is proportional to the ratio of its electric charge to its mass (e/m); different particles will accelerate differently in the same electric field. For gravitational effects the corresponding ratio, between "gravitational mass" (the source of the gravitational field) and "inertial mass" (the resistance of the body to acceleration) is 1 for all particles; hence in a gravitational field all bodies accelerate at the same rate. On the surface of the earth, for instance, this universal rate of acceleration is approximately 980.6 centimers per second, per second. Newton was well aware of this fact, but it was confirmed to some eight significant figures in the present century.

It followed that in a local experiment a gravitational field is indistinguishable from inertial effects, such as centrifugal and Coriolis forces. If a large box without windows were falling freely in a gravitational field, passengers inside the box could not distinguish their actual situation from unaccelerated motion in a space free of gravitational fields. Einstein has called this indistinguishability the "principle of equivalence." If taken seriously, this principle casts doubt on the validity of the concept of inertial frames of reference, which plays such an essential role both in Newtonian-Galilean physics and in the special theory of relativity. After some hesitation, Einstein accepted the principle of equivalence and discarded the concept of inertial frames, at least in the presence of gravitational fields. Whereas the restricted principle of relativity requires that the laws of nature should take the same form in all inertial frames of reference (and these, in turn, are connected with each other through Lorentz transformations), we must now require that any frame of reference will serve as well as any other. This new requirement, much more stringent than the former, is variously called the "general principle of relativity" or, in its mathematical execution, the "principle of general covariance." The term frame of reference, which originally denoted a Cartesian coordinate system along with a set of synchronized clocks, now comes to denote any (curvilinear) four-dimensional coordinate system.

To find laws of nature that are identical in any such coordinate system is a task that requires both mathematical and physical ingenuity. Einstein looked for a set of laws that would describe the gravitational field and its dynamics in such a manner that for weak fields the laws would take a simple special-relativistic form, and that if the gravitating bodies had velocities small compared with c, Newton's laws of gravitation would result. He succeeded in this program by introducing geometric concepts originally due to Gauss and to Riemann. These mathematicians characterized the curvature of a space as an intrinsic propertythat is one that could be recognized without viewing the space "from the outside." If we define a "straight line" (more properly speaking a geodesic) as the shortest curve connecting two points, figures constructed from such geodesics in a curved space will not possess all the properties that they have in a flat (Euclidean) space; for instance, the sum of the three angles of a triangle will not equal 180 degrees, but will be smaller or greater, depending on the type of curvature of the space. In such a curved space there are no real straight lines, and its properties are therefore described more conveniently if we make no attempt to approximate Cartesian coordinates but rather use any curvilinear coordinate system that comes to hand.

At the time curved spaces were first investigated, there was no concrete reason to believe that such spaces would ever play a role in the physical sciences. But now that the role of inertial frames was being questioned, curved spaces appeared as a possible geometric model for the situation in the physical space-time continuum in the presence of gravitational fields. Absence of a gravitational field would be equivalent to a flat space, its presence equivalent to space curvature. The laws of the gravitational field would presumably appear in the theory as laws dealing with the curvature of space-time. Because such laws would have to have a form independent of the choice of coordinate system, there were very few possibilities; for a physicist it was not very difficult to choose the one that would also go over into Newton's theory for small velocities and small fields.

The completed theory is known as the general theory of relativity. Although it is primarily a theory of gravitation, it permits the simultaneous consideration of any other fields whose special-relativistic formulation is known. The modifications required for these purposes are minor and relatively routine. The new theory leads to observable deviations from Newtonian results only in three instances. The first is a very slow precession of the orbit of Mercury in its own plane. This effect was known before Einstein had completed his theory, but it had remained unexplained until then. The second is the deflection of light rays that pass close to the limb of the sun. The third is a reddening of light originating in a small dense star. The latter effects were not looked for until Einstein had predicted them. These three effects are so minute that they require elaborate instrumentation for their observation and extremely careful work and analysis for their quantitative determination. The deflection of light rays can be observed

only during total eclipses of the sun and has led to costly and highly publicized expeditions to the sites of eclipses. At present the prevailing opinion is that the effects have been verified, both qualitatively and quantitatively. Because of their importance—so far they are the only possible experimental verifications of general relativity—work will undoubtedly be continued until the decision is clearcut.

The two stages of relativity have brought about a profound reevaluation of our ideas concerning the nature of space and time. Early in the 19th century Kant had proclaimed space and time as the unavoidable framework of human thought processes, prior to any specific observations and cognitions of the external universe. Space and time were conceived as absolutes. Temporal and spatial order of events was to be an inherent property, not a function of the observer. Special relativity first of all interlaced space and time so intimately that the only absolute relationship between two events is a single quantity, the space-time "interval" between them. The interval is the same for all observers, whereas distances in space alone and distances in time alone are not. However, special relativity retains the notion of uniform translatory motion and, by implication, the absolute character of rotatory motion and of translatory acceleration. It also retains the conceptual separateness of space-time and its absolute geometry on the one hand, and the dynamics of physical processes on the other. In general relativity, all that is left of the space-time continuum is the concept of the space-time point (the "event"). The geometric structure of the space-time continuum is no longer uniform, no longer the same everywhere but it depends on local physical conditions, the density of matter, and the strength of the gravitational field. The seriousness of this "geometrization" of physics has probably not yet been fully comprehended. Such statements as the one that in the absence of external forces bodies will move at constant speed in a straight line have no longer any simple meaning. By its very presence a body modifies the geometry of an otherwise flat space, so that there cannot be a straight line. And even motion along a geodesic, an assumption made in the early version of relativity, is a meaningful concept only in the case of test bodies sufficiently small that their own presence does not affect the local geometry. Einstein, Infeld, and Hoffmann showed in 1938 that the field equations of general relativity by themselves lead to equations of motion not in spite of but precisely because of the effect each body has on the local geometry (24-30). Unless a body moves in a particular fashion, the field equations in the surrounding space cannot be satisfied.

Aside from a new approach to the equations of motion, general relativity forces us to reconsider the meaning of all conventional laws of physics. Hitherto the ideal of a good theory had been to predict the value of any physical quantity at any time in the future (as identified in terms of some conventional clock time) at any place in the universe (as identified by a suitable coordinate system) from data supplied at some earlier time. A general-relativistic theory cannot possibly make such predictions because the identification of a space-time point in terms of its coordinates is not unique. It has been shown recently that the equations of general relativity determine the future uniquely (if enough is known about the past) save for the mathematical ambiguity of the coordinate system; but just what quantities are appropriate for the description of the dynamics of a general-relativistic theory is not yet known. This problem is bound to arise in any field theory that possesses general covariance; it is not a result of the particular form of Einstein's original theory of relativity.

General relativity has given a strong impetus to the fields of cosmogony and cosmology (31). These fields concern themselves with the origin and with the structure of the whole universe. Previously it had been thought that the universe was infinite, the alternative being a definite "end of the world," in space or in time. But once it became clear that gravitational fields caused space to be locally curved or buckled, it was no more than reasonable to inquire whether space and time might not also possess a curvature in the large. If so, it was feasible to think of models of the universe that were finite without having boundaries, in analogy to the surface of an ordinary sphere, which is also finite but has no edge. In the course of the last 30 years, quite a number of different models of the universe have been suggested and investigated. The principal observational effect we know of is the red shift of distant objects (galaxies); their spectra indicate that these objects recede from us at speeds that are roughly proportional to their distance from us. Present observations extend to distances of the order of roughly 1000 million to 2000 million light-years. At these distances, the observed speed of recession is about onefifth of the speed of light. Whether these spectral shifts are indicative of a real expansion of the universe is not quite clear, though the preponderance of opinion is that this very intuitive interpretation is correct. If this speed of expansion had been sustained in the past, backward extrapolation would lead to the result that some 5000 million to 10,000 million years ago the universe was very much denser than it is today. Another school of thought (F. Hoyle, H. Bondi) suggests that the universe is in a steady state and that the expansion is compensated by a process of continuous creation of matter to the extent that the density of matter in the universe, averaged over a cosmic scale, remains constant. These questions are all under very active investigation, both observational and theoretical, and cannot be considered settled.

General Relativity and Quantum Theory

The development of quantum field theory in the early 1930's has brought to the forefront a certain measure of contradiction between the general theory of relativity and quantum theory. Relativity was conceived originally as a classical field theory, and the subject matter of its description was to be a real physical universe, characterized by physicogeometric fields in a four-dimensional continuum; by contrast, the quantum theory that emerged from the work of the latter 1920's deals with probabilities of events. Quantum theory asserts that it is fundamentally impossible to measure simultaneously all the quantities that classically would characterize a physical system, and further that predictions concerning some future time (based necessarily on partial information concerning the present) will in general deal only with the likelihood of various results of observations. Quantum theory does not assert that some of the physical quantities of classical physics should be discarded: on the contrary, it retains all of them and asserts that any one may be measured with perfect accuracy. What is impossible is the simultaneous observation of a coordinate and its associated momentum (which in turn is closely related to the rate of change of that coordinate in time). Quantum mechanics purports to describe the state of a physical system completely by means of a "wave function," knowledge of which will permit the most nearly complete prediction of the future. The wave function is not a classical field, in that every single observation modifies it for the complete physical system.

As long as the geometry of the spacetime continuum was fixed and distinct from the physical fields, no contradiction arose between (special) relativity and quantum theory; in fact, no serious quantum theory of fields or particles would today be conceived nonrelativistically. But any attempt to provide a quantum theory of the whole of nature including gravitation must either exempt the gravitational field specifically from the approach valid for all other physical phenomena, provide a probabilistic interpretation of the geometric properties of the space-time continuum, or produce a nonprobabilistic modification of quantum theory. The first of these three theoretical approaches must be excluded because it leads to internal contradictions in the foundations of the theory. The third approach is the one championed by Einstein. It involves a complete reconsideration of the current method of representing elementary particles as well as a new interpretation of what is to constitute a "complete description" of the state of a physical system. Although a number of theories have been put forward (32-34), none of them has been worked out to the point where it can be tested critically. On the whole, one must consider nonprobabilistic quantum theory right now more of a program than a definite and complete theory.

The second possible program consists of the extension of standard quantization procedures to the geometry of space-time (28, 35). If this program should succeed, then the distance between two neighboring points in space-time would not be a definite number; the best the theory could do is to predict the likelihood of obtaining various values if this distance is actually measured. Actually, even this statement is an oversimplification. Normally we can identify a point in spacetime only because of events taking place there, fields having certain values, and so forth, or, alternatively, we can attempt an identification in terms of the geometric relationships of a point to its surroundings. If both the physical fields and the geometric relationships are uncertain in a general-relativistic quantum theory, then the space-time point loses much of its conceptual substance, and we may no longer be justified in retaining it as a basic element in our description of nature.

Aside from the clarification of a number of technical points surrounding quantization, we must then face this question: How can we identify (that is, describe unambiguously) a total physical situation in general relativity independently of the (accidental) choice of a particular coordinate system and independently of any a priori assumed identifiability of space-time points? Once we have answered this question, we have presumably found those variables that express the substance of a physical situation. Quantization should be applied to these quantities, rather than to the usual field variables, whose values depend both on the physical situation and on our accidental method of description. I have worked on this problem for several years, as have a number of other workers, and the end is not yet in sight.

Unified Field Theories

The general theory of relativity has provided us with a completely satisfactory theory of gravitation and, incidentally, with the logically most satisfactory example of a field theory to date. Conceptually, it suffers from the defect that if it is extended to include electromagnetic and nuclear dynamics, then all these fields appear as mathematically distinct entities. A number of workers, foremost among them Einstein himself but also Kaluza, H. Weyl, Schrödinger, P. Jordan, and many others, have attempted to enrich the geometric structure of space to leave room for at least the electromagnetic field (but preferably the other known fields as well) within a conceptually unified structure. This enrichment has been undertaken in a variety of directions.

The earliest was probably to increase the number of dimensions of the spacetime continuum from four to five, and even to six, and then to explain why macroscopically these additional one or two dimensions are not observed (36-38). Weyl and others modified the geometric structure of Riemann by denying the length of a vector absolute significance (39). Eddington built a geometry without any metric at all, leaving as a basic geometric procedure not the measurement of a distance but the parallel displacement of vectors (40). Most recently, Einstein (and coworkers) (41) and Schrödinger (42) have introduced a "metric tensor" (that is, the set of coefficients by which squares and bilinear products of the coordinate differentials must be multiplied in order to yield the square of the infinitesimal distance between two neighboring points) that no longer leads to a symmetric form but to an asymmetric form. Whereas in four dimensions a symmetric quadratic form has ten independent coefficients, an asymmetric form has 16. It was conjectured that these six additional variables have some relationship to the six components of the electromagnetic field. Einstein spent the last 5 years of his life investigating this theory (the "asymmetric" theory) without arriving at clear-cut answers. At the present time, all unified field theories must be considered speculative. But for a scientist who believes passionately in the intrinsic unity of the physical universe, this speculative inquiry has an irresistible attraction.

Semicentennial Jubilee at Berne

Preparations for the conference in Berne began early in 1954. In the first printed prospectus, the principal topics of the conference were enumerated as follows: (i) methods and solutions of the equations of general relativity; (ii) projective and similar unified field theories; (iii) asymmetric unified field theories; (iv) canonical formalism, general relativity, and field quantization; (v) mathematical structure of the Lorentz group; (vi) cosmology; (vii) deflection of light; (viii) physics and relativity.

These topics were represented by individual hour-long talks delivered by invited speakers. In addition, some 20 contributed papers enriched the program; there was a good deal of time permitted for discussion both inside and outside the lecture room. Attendance at the conference had been restricted to active workers in some field of relativity; the number of those present at the working sessions was thus held below a hundred. The final public session and formal celebration was open to the press and the general public. The largest lecture hall of the University of Berne, seating several hundred, was filled to overflowing.

W. Baade of Mount Wilson and Palomar Observatories reviewed the experimental evidence on the expansion of the universe. His lecture gave the theoretical physicists some better appreciation of the enormous difficulties involved in making valid quantitative observations on the most distant nebulae even with the new powerful mirror of the Palomar Observatory. The new report that there is some evidence of a leveling-off of the expansion rate at extreme distances, which was reported in the newspapers a few weeks ago, had not yet been established at the time of the Berne conference. Quite clearly, in this kind of work, the chain of reasoning that leads from the original data to the final result is long and tenuous, and it requires both imagination and extreme caution to arrive at valid conclusions. The raw data consist principally of curves that relate apparent magnitudes of objects to their spectra and also to their numbers per unit area of the sky. The apparent magnitude is presumably an indication of distance, at least statistically, if we are willing to assume that most galaxies are about the same size and possess similar star populations. Depending on the recording device (photographic or electronic), apparent magnitude may, however, also be affected by color and, therefore, by the red shift, and it depends possibly on the presence of absorbing materials in the vast intergalactic spaces. H. P. Robertson in a separate paper reviewed the principal cosmological theories and their relationship to the information obtained by the astronomer.

A. Lichnérowicz delivered a major paper concerned with the properties of both the general theory of relativity and the asymmetric unified field theory. He showed that the ambiguity of the solutions of either of these field equations is precisely that required by their general covariance, and that otherwise the future is uniquely determined by the present, a result that I have already mentioned in the section on general relativity and quantum theory. Lichnérowicz and members of his school also have obtained "global" results concerning static solutions of the equations of general relativity. It had been known that there are no solutions of the gravitational field equations representing the field of a central mass distribution without an infinity at or near that center. Lichnérowicz has greatly extended and strengthened these results.

Concerning experimental verifications of the general theory of relativity, R. J. Trumpler of Mount Wilson gave a summary of eclipse expeditions to date and of observational material on the gravitational red shift. The purpose of the eclipse expeditions is to observe and measure the deflection of light rays from fixed stars that pass close to the limb of the sun. These rays can be observed, of course, only when the sun itself is blotted out in a total eclipse. The observed apparent displacements of the fixed stars photographed are at best about 1 second of arc (the theoretical deflection precisely at the rim of the sun's disk would be 1.75 seconds); the evaluation of the data must begin with a very precise measurement of the star images on the exposed plate, to be followed by an evaluation of all conceivable sources of error (thermal expansion of the plate, distortion during development, atmospheric refraction, lack of resolution of the instrument, and so forth) and a statistical adjustment of the data from individual stars. Trumpler concluded that by now the evidence was all in favor of the predicted effect, and with an accuracy approaching ±5 percent. He reached similar though less definite conclusions concerning the red shift of spectral lines originating in regions of high gravitational potential. His conclusions were vigorously attacked by E. Finlay-Freundlich, himself a veteran of eclipse expeditions in the years 1919 and 1922. The preponderance of opinion among the observing astronomers appears to be with Trumpler's conclusions, but obviously these scientific questions will not be settled by majority vote but by ever-improving skill (plus luck with the weather) in future eclipse expeditions.

A lively discussion of the problem of motion in general-relativistic field theory followed a series of contributed papers presented by L. Infeld of Warsaw and V. A. Fock of Leningrad. In the section of this article on the general theory of relativity, it was explained that in that theory the motion of particles is governed by the laws of the gravitational field surrounding them. Although this basic fact appears reasonably clear, actually there are a number of thorny questions left. Through a coordinate transformation we

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can alter the description of an orbit in terms of coordinate locations in an almost arbitrary manner. In their first paper, Einstein and his coworkers achieved definiteness of the particle trajectories by specializing the choice of coordinate system, requiring that certain divergencelike expressions of the gravitational potentials vanish everywhere. They later. found that this restriction was unnecessary and that it could be replaced by a much milder one, that in lowest approximation the coordinate system should be Cartesian, and in the higher approximations it should deviate from Cartesian type no more than necessitated by the curvature. However, it was not quite clear what that meant. Infeld and Scheidegger had tried to show that there was no need for gravitational waves in any problem involving the motion of mass points, but this result was not accepted by others. More recently, Fock and Papapetrou have resurrected the original coordinate conditions. The whole issue of motion is further complicated by the fact that the internal structure of a particle will also affect its motion, aside from the effect of coordinate choice. Intuitively, we may speak of the gravitational dipole or quadrupole moment of a mass distribution. If these higher moments do not vanish, the particle will be affected not only by the gravitational field but also by the field's gradient and higher derivatives. To give these concepts precise mathematical expression is again complicated by the fact that it is not yet known to what extent they possess any intrinsic covariant meaning. In other words, the problem of choice of coordinates is mixed up with the problem of describing invariantly the internal structure of a particle. The discussion of these problems at the conference was stimulating but inconclusive.

O. Klein and I reviewed the work on the quantum theory of general relativity. As mentioned in the section of this article on this subject, the problem of quantization leads back to the nonquantum problem of an invariant description of physical situations. In this connection, T. Géhéniau of the Free University of Brussels contributed a paper in which he showed that one could characterize points of the four-dimensional continuum by means of the values of four scalars that can be constructed from the curvature tensor (which in turn consists of second derivatives of the gravitational potentials). Once this identification has been accomplished, scalars of even higher differential order would provide a description of a distinct gravitational field. It is clear that a description of a physical situation in terms of scalar fields necessitates the introduction of very high differential invariants; Géhéniau's work may be an indication that an invariant description can be given more adequately

in terms of integro-differential invariants or even more general functionals.

In the area of unified field theories, P. Jordan of Hamburg gave the principal talk on five-dimensional field theories, while Bruria Kaufman reported on the work she had done with Einstein during the last years of his life on the asymmetric theory. A. Tonnelat of the Sorbonne reported on some mathematical results she had obtained on this theory independently of Einstein and Kaufman. Briefly, Jordan modified the original Kaluza theory so as to obtain a theory with 15 field variables. Ten are the gravitational potentials, four are electromagnetic potentials, and the fifteenth is a scalar that is not present in the original Kaluza theory. This scalar appears to play a role similar to the constant of gravitation (which determines, for instance, the gravitational effect of the energy density of the electromagnetic field). Jordan has conjectured that if this scalar should change slowly during cosmological periods, the ratio of e/m for elementary particles should also have changed slowly in the course of the several thousand million years that represent the "age of the universe." This idea, originally proposed by Dirac, would relieve the theorist of the embarrassing necessity of "explaining" or deriving the value of a dimensionless constant of the order of magnitude of 10²⁰ from pure theory. Jordan has followed up his speculation and considered its cosmological and other consequences. The papers by Kaufman and by Tonnelat are too technical to be reported here.

E. P. Wigner of Princeton University talked on the relativistic invariance of quantum-mechanical equations, restricting himself to Lorentz covariance. It is well known that the requirement of Lorentz covariance restricts the possible form of Schrödinger wave equations; several workers have examined all possible types of particles and laws obeyed by them. The ensuing classification may represent a preliminary classification of elementary particles, though it is likely that elementary particles are characterized by other properties than their relativistic transformation law as well, such as their transformation law under isotopic spin transformations.

Max Born spoke on the subject of physics and relativity, but equally so on the life and work of Albert Einstein. He discoursed at some length on the history of relativity (as sketched in the early part of this article), and then on the philosophical attitude of Einstein toward the probabilistic nature of current quantum theory. The last published discussion of Einstein on the epistemological foundations of quantum mechanics is contained in a volume of papers dedicated to Max Born on his retirement from the University of Edinburgh in 1953 (32, 43, 44).

Born's talk was followed by the summary of the conference by its president, W. Pauli. He reviewed the status of the diverse areas that had been the subject of the conference and indicated briefly his own views concerning future developments.

The foundations of the special and the general theory of relativity may be considered as generally accepted, the experimental confirmations conclusive for the special theory, and more and more convincing for the general theory. The special theory forms by now an integral part of physics as a whole and is used in everyday work in atomic and nuclear physics. The general theory of relativity for many years appeared to have its applications principally in cosmology and cosmogony, themselves fields as yet in a highly unsettled state. But recently general relativity is also being considered in connection with questions affecting microphysics. Its relationship to quantum theory is still highly problematical. But the quantum theory of the atomic nucleus and of elementary particles is not in such a satisfactory state that it can afford to disregard possible assistance from whatever source. General relativity offers us a new approach to the ultimate properties of space and time, and these may bear on the physics of the very small as much as we know they do on the physics of the very large. Many of our present efforts are still in a very early stage. The very fact that interest in general relativity has recently increased throughout the world is indicative of the fact that its implications have not yet been fully worked out and exploited for our understanding of the physical universe as an organic whole.

The conference had been the common meeting ground of workers from the four corners of the earth. The countries represented included the United States, the United Kingdom, Belgium, Denmark, Finland, France, both halves of Germany, India, Ireland, Israel, Italy, Japan, the Netherlands, Poland, the Soviet Union, Sweden, and Switzerland. The languages of the conference were English, French, and German. The technical preparation of the conference had been mostly in the hands of André Mercier, its secretary, and the other physicists at the University of Berne. Aside from the excellent technical preparation and the heartwarming hospitality of the hosts, the success of the conference as a clearing house for an active field of physics was due in no small part to the enthusiasm that the participants brought to the subject matter. In the concluding words of Max von Laue, who directed his words of thanks in the name of the foreign participants to the Swiss sponsors and hosts, Einstein would certainly have enjoyed the scientific discussion of his principal field of work, but he would have considered equally important the fact that scientists from all countries could get together and in a spirit of common endeavor help each other with their problems.

The foregoing report is necessarily incomplete. The full proceedings of the Berne Conference will appear this spring as a special issue of Helvetica Physica Acta, approximately 300 pages in length, including both the prepared talks and the discussions. It is to be hoped that in the future similar meetings of workers in relativity can be held every few years.

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It stands to the everlasting credit of science that by acting on the human mind it has overcome man's insecurity before himself and before nature.-ALBERT EINSTEIN.