27 January 1956, Volume 123, Number 3187

SCIENCE

Edge Waves on the Continental Shelf

Walter Munk, Frank Snodgrass, George Carrier

In 1846, Stokes derived a solution for wave motion over an inclined bank, which differs markedly from the conventional pattern of waves coming onto a sloping beach. In Stokes' solution, the crests are normal to the coast line, and they travel in a direction parallel to the coast. Amplitudes diminish rapidly from the shore seaward and are negligible at a distance of 1 wavelength. Lamb (*1*, p. 447) accordingly calls these waves "edge waves" and states that "it does not appear that the type of motion here referred to is very important." But edge waves appear to be common! Evidence comes from a variety of sources (*2*).

Gust of 6 January 1954

At 3:35 A.M. (0335) on 6 January 1954, an unusual gust was recorded at Scripps pier (Fig. 1). Pressure rose by 2 millibars, and wind speed increased abruptly from 3 to 14 miles per hour. This gust was followed by pressure and wind oscillations of 8-minute period; these have been attributed to internal waves on the atmospheric inversion layer (*3, 4*). Sea level at Scripps responded to the atmospheric pressure as an inverted barometer. This is shown by the trace of the Scripps tsunami recorder (*5*), an instrument that discriminates by means of pneumatic devices against swell and tides and that has a peak response for a period of 45 minutes. Our attention for the moment is on the simultaneously recorded (by radio link) signal from the tsunami recorder at Oceanside, 38 kilometers along the coast to the northwest. Between 3:40 A.M. (0340) and 4:50 A.M. (0450), it showed the familiar features of an impulsively generated dispersive wave train. There was no meteorologic disturbance at Oceanside that could account for the recorded wave train.

It is not difficult to compute roughly the character of edge waves that are caused by an impulsive source. Phase velocity C, wavelength L, and period T of edge waves over a beach of constant inclination β are related by the equations

$$C = (gT \sin \beta)/2\pi, \qquad L = (gT^2 \sin \beta)/2\pi \quad (1)$$

These are identical with the relationships for ordinary deepwater waves when gravity g is replaced by $g \sin \beta$. With this modification, the Cauchy-Poisson treatment of impulsively generated waves in deep water from a line source is applicable. The upper curve in Fig. 2 shows the solution copied from Lamb (*1*, p. 387). Time is in units of $(2y/g \sin \beta)^{1/2}$, and elevation is in units of $Q/\pi y$, where y is the distance from the disturbance and Q is the sectional area of the initially elevated fluid. The second curve is the recorded Oceanside trace corrected roughly for instrument response (on the record, the short late waves are suppressed and slightly delayed relative to the long early waves) and drawn to scale for best agreement with the theoretical curve. Zero time is taken at the first indication of the disturbance at La Jolla. On comparison, we find that

$$(2y/g \sin \beta)^{1/2} = 582 \text{ sec}, \quad Q/\pi y = 0.3 \text{ cm}$$

The mean slope over the continental shelf between La Jolla and Oceanside is 0.02. With this value of $\sin \beta$, we get

$$y = 33 \text{ km}, \quad Q = 3.1 \times 10^6 \text{ cm}^2$$

These values are reasonable; y is nearly the distance between Scripps and Oceanside, and Q corresponds to an initial displacement of 3 centimeters over 10 kilometers. The recorded displacement at La Jolla was about 8 centimeters.

The wave train is barely discernible on a record taken by the Mark IX wave instrument at Camp Pendleton, 3.5 kilometers northwest of Oceanside. With imagination, one can find some traces on tide records in the Los Angeles area. However, these records are hardly useful; the wave instrument is tuned to higher frequencies, and the tide gage is tuned to lower frequencies than the frequency of the disturbance under consideration.

We must point out a number of difficulties in the application of the Cauchy-Poisson theory to our problem. The continental shelf is about 5 kilometers wide, and the assumption of constant slope is tolerable only if $L/2\pi$ is less than 5 kilometers. This is true for the wave that arrived after 4 A.M. (0400), but the first arrival must have extended beyond the shelf, and the theory does not strictly apply.

It has been demonstrated by Ursell (*6*) that the Stokes solution represents only the gravest of an infinite number of possible modes of edge waves. The n'th mode has n extrema in elevation between shore and open sea, and its velocity and length are given by

$$C = [gT \sin (2n+1)\beta]/2\pi, \qquad L = [gT^2 \sin (2n+1)\beta]/2\pi \quad (2)$$

This reduces to Eq. 1 for $n = 0$. Note that the computed values of y and Q for the first harmonic are 3 times those for the zero mode and that they are inconsistent with observations. Suppose that the initial disturbance is $f(x,y)$, with x pointing seaward and y measured along the coast. If the disturbance is concentrated very near $y = 0$, then $f(x,y)$ can be written $g(x)\,\delta(y)$, where δ is the delta function. The relative amplitudes of the various modes can be found by expanding $g(x)$ in Laguerre polynomials. The predominance of the fundamental mode points to an exponential-like (at least

The authors are on the staff of the Scripps Institution of Oceanography, La Jolla, Calif. Dr. Carrier is on leave of absence from Harvard University.

monotonic) drop off with distance from shore.

The Cauchy-Poisson theory is derived for such a concentrated line source at $y=0$, but holds reasonably well for a finite width provided that the computed wavelengths are larger than this width. The Oceanside signal ended at 4:50 A.M. (0450) with a period of 10 minutes, which corresponds to a wavelength of about 10 kilometers. This may be taken to mean that the initial disturbance was about this wide.

A search through 3 years of records did not reveal any other example of a comparable gust, nor did it reveal a similar Cauchy-Poisson type of disturbance in sea level.

Hurricanes and Squalls

On 26 August 1954, hurricane Carol formed near the Bahamas and was moving very slowly northwestward. On the morning of 30 August, the hurricane started its rapid trip up the East Coast. According to weather maps (7), Carol was moving at 32 knots when it passed Atlantic City 24 hours later. From the arrival times of the water-level disturbance at various tide gages (8), we obtain 34 knots.

Consider the characteristics of edge waves that might be generated by such a traveling disturbance. The prominent waves will have a velocity C that just equals U, the longshore component of velocity of the traveling disturbance. Precisely the same considerations determine the period and length of waves behind a vessel traveling at velocity U. Accordingly, we replace C by U in Eq. 1 and obtain

$$T=\frac{2\pi U}{g\sin\beta},\qquad L=\frac{2\pi U^2}{g\sin\beta}\qquad(3)$$

for the period and wavelength of the fundamental edge-wave "wake." Off Atlantic City, the 30-fathom line is approximately 100 kilometers offshore, giving a slope of $\beta=5\times10^{-4}$. For $U=32$ to 34 knots, this yields $T=5.8$ to 6.1 hr for the fundamental mode. A. C. Redfield and A. R. Miller have kindly furnished us values of observed minus predicted tide level. They indicate four waves about 2 feet high with a period of 5.5 hours (Fig. 3). Note that these waves do not show the progressive shortening in period that is so characteristic of impulsive generation (Fig. 2).

For the first harmonic ($n=1$), the computed period is one-third of the fundamental. The records do not show any prominent oscillations of 2-hour period.

Further north, near Sandy Hook, the 30-fathom line is more nearly 120 kilometers offshore, and $\beta=4.25\times10^{-4}$. The computed periods are then somewhat longer, 6.9 to 7.2 hours. The measured period at Sandy Hook was also longer, 7 hours. But it is not clear whether the change in topography actually accounts for the change in period, and we have not considered in detail what happens when slope varies gradually along the travel path.

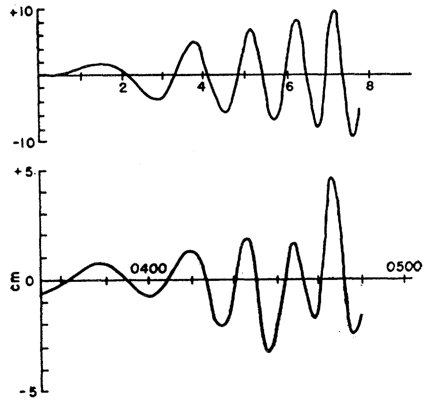

Fig. 2. The upper curve shows elevation against time at a fixed point according to the Cauchy-Poisson theory. The lower curve is the recorded water level at Oceanside corrected for instrument response.

The wavelengths are of the order of 400 kilometers. Edge waves are essentially confined to a strip within a distance $L/2\pi$ from shore; for this reason, we have based our calculations on average slopes over the first 100 kilometers. Beyond the continental shelf, the slope is far larger. To estimate the effect of this increased slope, we have worked out the theory (not given here) for an inclination β_1 to a distance x_1, followed by β_2 beyond x_1. Setting

$$x_1=200\text{ km},\ \beta_1=4.25\times10^{-4},\ \beta_2=5.9\times10^{-2}$$

for the bottom off Sandy Hook leads to a *decrease* in computed period by roughly 2 percent. The approximation of a constant slope is adequate.

The sea-level disturbance at Atlantic City and Sandy Hook consisted of four waves and lasted until about 24 hours after the passage of the hurricane. A rough calculation of the expected duration of an edge-wave wake is as follows. Hurricane Carol started its rapid trip up the east coast on the morning of 30 August from about latitude 31°N. The hurricane arrived at Atlantic City about 24 hours later. The front of the water disturbance arrives with the hurricane; the rear travels with group velocity, which is one-half the phase velocity for edge waves. Hence the rear will take twice 24 hours, and the duration is 24 hours, as observed. For the general case, let D be

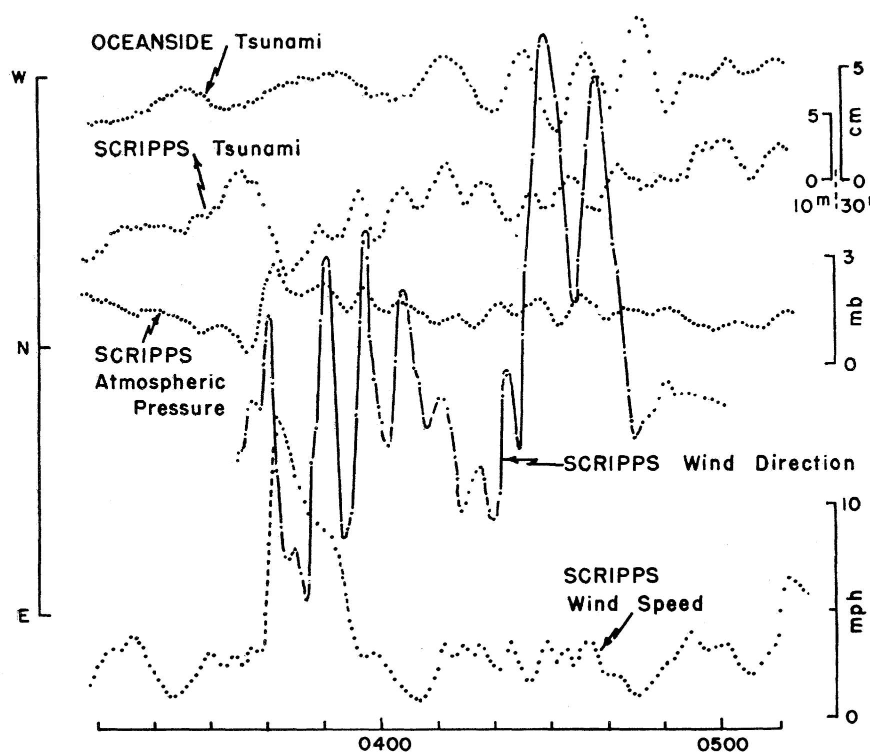

Fig. 1. Meteorologic and sea-level records of 6 January 1954. The Scripps and Oceanside tsunami recorders are peaked for sea-level oscillations of 45-minute period. The scale for oscillations of 30-minute and 10-minute periods is indicated. A gust recorded at Scripps at 3:35 A.M. (0335) is followed by a dispersive sea-level oscillation at Oceanside, 38 kilometers to the northwest.

travel time and Y travel distance at a velocity of U. The duration τ of the disturbance is given by

$$\tau = D \text{ or } \tau = Y/U \tag{4}$$

These expressions are equivalent for constant U, as assumed. For Carol we set D=24 hr, Y=540 naut. mi., U=33 knots; the two expressions give τ = 24 hr and τ = 16 hr. The numerical values are summarized in Table 1, together with similar calculations for three other hurricanes (*9*). The values for Edna are similar to those for Carol. The hurricanes of 1938 and 1944 had similar paths and were farther out at sea than the 1954 hurricanes. We have taken their starting time when the storms' centers came over the shelf just south of Cape Hatteras. This yields durations that are considerably shorter than the observed values. To obtain agreement, one must take the starting time 12 hours earlier, when the hurricane centers were about 250 miles from shore.

It is known that the rise in water level is of the same order as that given by the inverse barometer rule: the water level rises 1 centimeter for each millibar of pressure drop. The measured rise is often larger, and the difference is attributed to winds. Here we limit ourselves to the question whether a traveling pressure spot of reasonable dimension could generate edge waves with amplitudes of the order of the inverted barometer. Consider a pressure spot

$$p = p_0 \frac{a(x+a)}{(x+a)^2 + (y-Ut)^2}$$

traveling from $y = -\infty$, $t = -\infty$ with velocity U parallel to the coast. The resemblance to actual hurricanes is poor, but the Fourier transform is simple. The isobars are nonconcentric circles. The pressure deficiency is p_0 at $x=0$, $y=Ut$, and $\frac{1}{2}p_0$ on a circle of radius a centered at $x=0$, $y=Ut$. It can be shown that the elevation of the sea surface is given to good approximation by

$$2ka(p_0/\rho g)e^{-k(x+a)} \cos k(y-Ut)$$

where $k = g \sin \beta / U^2$ is the wave number $(2\pi/L)$ of the fundamental mode of edge waves traveling with a velocity $C = U$. Only the fundamental mode is generated to the present approximation. This is, of course, a consequence of the assumed pressure pattern, but it is doubtful whether any monotone (nonwiggly) pressure pattern resembling surface isobars of hurricanes would be an effective generator of harmonics. The crest height at the coast line differs from that given by the inverted barometer rule by a factor of $2\ ka\ e^{-ka}$. This has a maximum value of $2/e$ for $ka = 1$. Thus, with wavelength having been determined by the speed of the hurricane, its effectiveness depends on the hurricane's dimension. The waves are largest when the radius of the half-pressure isobar equals

$$k^{-1} = L/2\pi = U^2/(g \sin \beta)$$

For the hurricanes under consideration here, this equals 30 to 40 miles. The observed half-pressure radius is more like 100 miles, too large for optimum generation. The afore-mentioned values give ka = 2.5 to 3.0, and $2\ ka\ e^{-ka} = 0.41$ to 0.30. Thus, the inverted barometer rule yields the correct order of magnitude.

The over-all agreement for the four East Coast hurricanes is good. The observed and computed periods agree, and the longest period is associated with the fastest hurricane. Duration and amplitude cause no particular difficulties. The outstanding feature is that the periods are large, and this is attributed to the very gentle bottom slope.

For contrast, we consider very briefly the disturbance caused by a traveling squall over Osaka Bay, Japan, on 29 August 1953 (*10*). Coming from the southwest at a speed of very roughly 30 knots, it gave rise to a pressure drop of 3 millibars at Wakayama. In this region, the coast tends north-south, and the longshore projection of the velocity was of the order of 40 knots. The offshore topography is irregular. We replace it by an idealized profile consisting of a constant slope to the 30-fathom line 4 miles offshore, and constant depth seaward. The theory for this geometry (not included here) gives a period of 31 minutes for a phase velocity of 40 knots. If one assumes the slope to extend indefinitely, Eq. 3 applies, and the period is 29 minutes. The observed periods along this stretch of coast line were 20 minutes at Shimatsu, 30 minutes at Kainan, and 25 minutes at Wakayama.

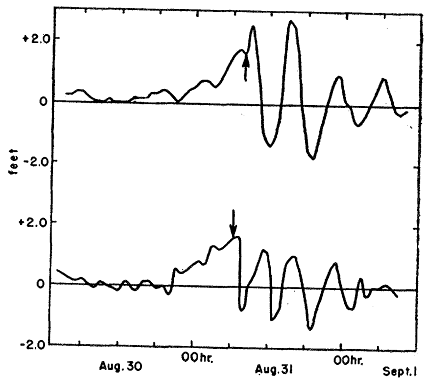

Fig. 3. Observed minus predicted tide-gage level at Atlantic City (lower curve) and Sandy Hook (upper curve) during hurricane Carol. Arrows indicate time of passage of storm center.

On comparison with Table 1, we find the velocities of the disturbance to be commensurate, but the bottom slope of Osaka Bay is 15 times larger than that off Atlantic City. In accordance with edge-wave theory, the excited periods should be one-fifteenth as large, and they are. In most regions of the world, the slope is even larger than it is off Wakayama, and the observed open-sea seiche periods are shorter. Typical values are $\beta = 0.015$ and T = 10 min; this requires a longshore component of velocity of 27 knots for the traveling disturbance.

Redfield and Miller (*8*) discussed the oscillatory wake under the heading "resurgences." They stated "these phenomena are of particular importance because they tend to catch one unaware, coming as they do after the storm appears to be subsiding. . . . Should a storm pass along the coast at low tide the accompanying rise in water level might be inconsequential, but the resurgence occur-

Table 1. Periods and durations of sea-level disturbances caused by four hurricanes.

Hurricane	Velocity U (knots)	Path Y (naut. mi)	Travel time D (hr)	Wave period T (hr): Computed, $\sin\beta = 5.0 \times 10^{-4}$	Computed, $\sin\beta = 4.2 \times 10^{-4}$	Observed, Atlantic City	Observed, Sandy Hook	Duration (hr): Computed	Observed, Atlantic City	Observed, Sandy Hook
30 Aug.–1 Sept. 1954 (Carol)	32–34	540	24	5.8–6.1	6.9–7.2	5.5	7.0	16–24	20	26
11–12 Sept. 1954 (Edna)	32	530	24	5.8	6.9	6.0	7.0	17–24	23	?
14–15 Sept. 1944	33	360	12	6.0	7.1	5.6	7.2	11–12	23	30
21–22 Sept. 1938	40	360	9	7.3	8.6		8.0	9		16

ring six hours later would arrive at high tide, and might thus prove more destructive than the original rise in water level."

The prediction of the resurgences is then a matter of practical importance, and our calculations for four hurricanes must be extended to other cases. The results shown in Table 1 give some hope that the arrival time of resurgent crests can be predicted on the basis of Eq. 3 and that the number of such crests can be estimated from Eq. 4. The amplitude will be more difficult to predict; it is of the order given by the inverted barometer rule. We expect resurgences only if the hurricane travels over the shelf for a time exceeding $T\,[=2\pi\ U/(g\ \sin\beta)$, Eq. 3]. Hurricane Hazel came over the shelf at 40° incidence and quickly traveled inland. There were no resurgences.

In closing this discussion, we should point out the connection between our treatment and previous investigations. Redfield and Miller (*8*) contoured the depth h^* for which $(gh^*)^{1/2}$ equals the speed of the disturbance. They correctly emphasized that effective generation of the water-level disturbance takes place only where the shelf depth is less than h^*. Our interpretation of h^* (for constant slope) is the depth at a distance $L/2\pi$ from shore, the *mean* distance of the wave elevation. Ewing, Press, and Donn (*11*) drew similar contours to explain the disturbance caused by a squall line over Lake Michigan. They attributed the disastrous wave to the unusually high velocity of the squall. This has the effect of coupling the atmospheric disturbance to the relatively flat areas near the lake bottom rather than to the steeper edges. Edge-wave theory is not applicable to this situation, but it might be profitable to look for a normal mode solution that is.

Edge-Wave "Noise"

The causes discussed so far have dealt with clear-cut and exceptional meteorological disturbances. But the Scripps and Oceanside tsunami recorders show some activity at all times. The root-mean-square height ranges from less than 1 centimeter on quiet days to more than 5 centimeters on noisy days. There is a correlation between the general level of activity at Scripps and Oceanside, but there is no coherence between individual waves at the two stations. The records are irregular and quite similar in appearance to ordinary wave records, but the periods are about 100 times longer. M. J. Tucker of the National Institute of Oceanography, Great Britain, has kindly made a frequency analysis of some of the records. The spectra show a band of activity between periods of 10 and 30 minutes.

If this activity is the result of edge waves, then it should diminish with distance from shore in the predicted manner. To test this, we have made simultaneous recordings from a shore-based and a ship-based recorder. The portable shore unit was placed on the bottom in about 30 feet of water. The fluctuating pressure acts on a system of capillaries and air volumes that discriminate against swell and tides and give peak response for 10-minute periods. A strain-gage transducer converts the filtered pressure fluctuations into fluctuating voltages, which are transmitted by cable to a bank of shore-based resistance-capacitance filters of very long time constant. Their output is recorded in two broad frequency bands, one centered at 13-minute period, the other at 22-minute period.

This kind of instrument is not readily adapted to great depth. It is hard to provide for an adequate volume of compressed air, and even if one does, one is apt to have a thermometer on one's hand instead of a pressure recorder. We have accordingly abandoned any compliance and prefiltering at depth for the ship-based instrument. A Vibrotron transducer on the sea bottom consists essentially of a stretched wire oscillating at 15,000 to 30,000 cycles per second. The "raw" pressure fluctuations modulate this frequency, and the oscillations are counted and printed on shipboard. The instrument has been used up to depths of 2500 feet. For 1-minute counts, we can detect changes in pressure by 1 millimeter of water.

The Vibrotron output includes the effects of swell and tides as well as the effects of the intermediary frequencies that are under consideration here. For comparison of records with those of the shore-based recorder it is necessary to subject the Vibrotron output to equivalent frequency filters. This has been done numerically by forming a convolution of the record with the Fourier transform of the impedance of the shore-based recorder. Details are elaborate and will be submitted for publication elsewhere.

The observed values in Table 2 have been obtained by the crude method of selecting portions of the record for which some frequency predominates for a few cycles, and then comparing amplitudes and phases. The last columns give theoretical values. The computed ratios in the column headed "edge waves" are $\exp(-2\pi\,\Delta x/L)$, where Δx is the distance between the recorders projected along a line normal to shore and L is the wavelength of the fundamental mode, $n=0$, for slopes of 0.02 and 0.03, respectively. The shallow-water values are computed on the assumption that wave heights are proportional to $h^{-1/4}$ and that the phase velocity equals $\sqrt{gh}$.

For nearby stations, the coherence was sufficiently good to permit positive identification of detailed features on both records, but for shorter periods at distant stations, one ought to compare spectra. The complex submarine topography off La Jolla, Calif. (Fig. 4) is as much a

Table 2. Comparison between selected portions of wave records of ship-based recorder at stated distances and depths and wave records of shore-based recorder at 1200-foot distance from the beach and 22-foot depth beneath mean lower low water; *r* is the ratio of offshore to near-shore amplitude, and Δt is the offshore minus near-shore arrival time of a coherent feature. For comparison, the values based on edge-wave theory and ordinary shallow-water propagation are also tabulated.

Station	Distance (ft)	Depth (ft)	Observed			Edge wave		Shallow water	
			Period (min)	r	Δt	r $\beta=0.02$	r $\beta=0.03$	r	Δt
A	1,450	35	2	1.0	*	0.39	0.49	0.89	8^s
			12.5	0.81	*	0.97	0.98		
B	4,700	135	2	0.65	30^s	0.00	0.00	0.64	1^m15^s
			3.5	0.65	-40^s	0.01	0.04		
			4.0	0.70	-12^s	0.02	0.08		
			16	0.85(?)	*	0.79	0.86		
C	6,500	155	3.5	0.56	2^m	0.00	0.00	0.61	1^m50^s
			4	0.62	-30^s	0.00	0.02		
			5	0.50	*	0.03	0.09		
			10.5	0.80	*	0.45	0.56		
			22.5	0.84	1^m	0.83	0.88		
D	10,200	295	2	0.37	2^m(?)	0.00	0.00	0.52	2^m15^s
			13.7	0.50	*	0.44	0.58		
E	22,900	275	2	0.40	†	0.00	0.00	0.53	3^m40^s
			12	0.18	*	0.07	0.18		
			15	0.40	*	0.19	0.33		
			22	0.43	*	0.47	0.60		
			25	0.58	*	0.55	0.67		

* The phase lag was too small to be measurable. † No coherence.

liability to us as it is an asset to the submarine geologists. The recorded distances in Table 2 are referred to the beach at Scripps; had we measured them from the nearest land, the values would have been appreciably smaller. We are planning a series of measurements in regions with simpler topography, as well as an analysis involving co- and quadrature spectra of the two records. But even so, certain results stand out clearly from the present crude analysis.

1) There is a marked distinction between waves of period less than 5 minutes and those of period more than 10 minutes.

2) The 2- to 5-minute waves have been called surf beat (*12–14*). Their spectrum is related to the spectrum of the *envelope* of the incoming swell, and their energy just outside the surf zone is 1 percent of the wave energy. According to edge-wave theory, these waves should be all but eliminated at stations B, C, D, and E; according to the conventional shallow water theory, the amplitude should be reduced only by a factor of two at the farthest stations. The observed values are certainly not in accord with edge-wave theory. There is rough agreement with regard to amplitudes (but not with regard to phase) with shallow water theory.

3) The 10- to 30-minute waves are believed to be a quite different phenomenon. Certainly their general level of activity is found to be unrelated to that of the surf beat. Table 2 shows that the observed amplitude ratios are in general agreement with those given by edge-wave theory. Particularly at station E, a shift toward the longer periods was noticeable. The shift with distance from shore is analogous to a similar shift with *depth* in the case of ordinary surface waves.

4) Results were similar at stations B and C, which were located at opposite sides of the Scripps submarine canyon (Fig. 4).

How is the edge-wave noise generated? One is tempted to look again for some coupling between atmosphere and ocean. This time we do not look for a single impulse or a well-defined traveling disturbance, but for the random superposition of many such disturbances. A possible source is the microbarographic oscillations. The microbarograph at Scripps traces a noisy record containing frequencies of the same order as those on the sea-level record. About ten times a year, these oscillations show rather well-defined patterns with amplitudes of as much as 1 millibar. One such case is shown by the pressure and wind direction traces on Fig. 1. There is quite good evidence (*3, 4*) that these oscillations are the surface (actually bottom) manifestation of

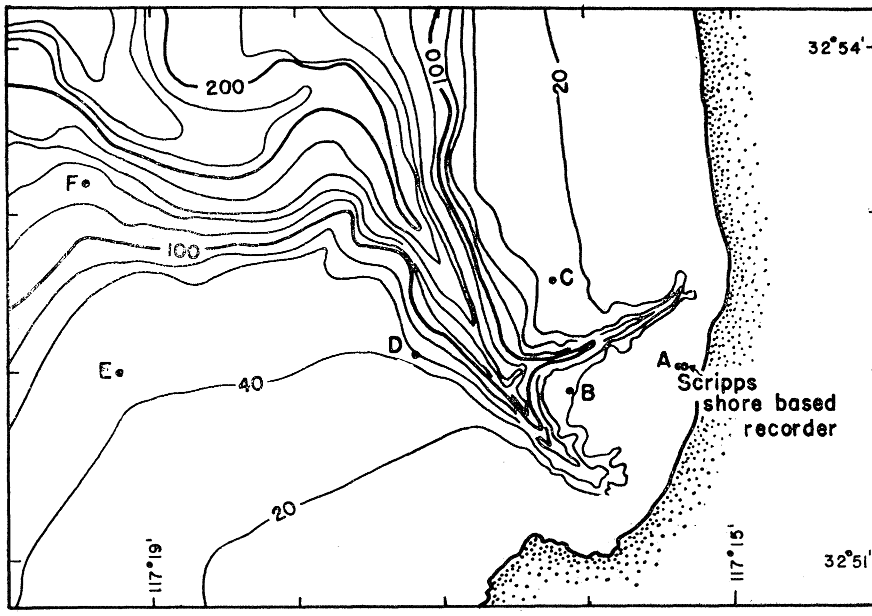

Fig. 4. Depth contours in fathoms off La Jolla, California. Positions of ship-based tsunami stations, *A* to *F*. The permanent shore-based recorder is located near station *A*, as shown.

internal waves traveling on the atmospheric inversion layer with the "shallow water" velocity

$$C_a = (g h_a \Delta \ln \theta)^{1/2} \qquad (5)$$

where h_a is the height of the inversion layer, and $\Delta \ln \theta$ is the logarithmic change in potential temperature across this layer. Ultimately, these waves may be caused by a variety of meteorologic events such as cyclogeneses offshore or break-throughs of marine air (*4*).

We now suppose, without evidence, that the usual microbarographic noise in this area is also the result of internal waves on the inversion layer. Let α be their direction relative to a normal with the coast line. There is coupling with edge waves, provided that $C_a \csc \alpha = C$, or in view of Eqs. 2 and 5, if

$$T = \frac{2\pi(g h_a \Delta \ln \theta)^{1/2}}{g \sin \alpha \sin (2n+1)\beta} \qquad (6)$$

Typical values are $h_a = 400$ m, $\Delta \ln \theta = 0.075$, $\sin \beta = 0.02$. For the fundamental mode, these give $\alpha = 90°$ (glancing), $T = 8.7$ min; $\alpha = 60°$, $T = 10.0$ min; $\alpha = 30°$, $T = 17.4$ min; $\alpha = 15°$, $T = 33.5$ min.

The observed values of T are 10 to 30 minutes. The short-period limit of the spectrum is determined by the speed of the atmospheric waves. Longer periods are the results of the "scissors effect" between atmospheric wave crests and the coast line for other than glancing incidence. The computed periods can be made arbitrarily large for near-normal incidence; but for equal probability of wave direction, the spectral densities of very long periods are then very small. Without further measurements, the peculiar mechanism proposed here—that of coupling the atmospheric inversion to the continental shelf—must be regarded as speculation.

Tsunamis

So far, we have dealt with edge waves generated by atmospheric disturbances over the shelf. The reader may have wondered whether tsunamis recorded on coastal tide gages are, at least in part, edge waves excited by long waves from the open sea.

We have considered in some detail the problem of waves from the open sea (depth H_0) impinging on a coastal ledge of some specified width D and depth H_1. This geometry makes the algebra simple and the comparison with observed conditions difficult. A detailed treatment is out of place here, but we shall sketch our results. Suppose a regular wave train arrives from the open sea with a velocity $C_0 = (gH_0)^{1/2}$ from a direction α relative to the coast normal. After a time interval of several wave periods, a disturbance is built up over the shelf. The disturbance is driven along the coast line with a speed $C \csc \alpha$ that is far greater than the speed of free edge waves of comparative length.

The amplitude at the coast line depends on α and the relative dimensions of the shelf and waves. For our simple geometry, Brewster's angle

$$\arctan (C_0/C_1) = \arctan (H_0/H_1)^{1/2}$$

enters in a critical manner. For incidence more nearly glancing than Brewster's angle, the coastal amplitude is always smaller than the deep-sea amplitude. For incidence more nearly normal, it is always larger. The maximum amplification is $(H_0 H_1)^{1/2}$. This occurs for normal incidence if the wavelengths are such as to give an antinode at the coast and a node at the outer edge of the shelf, in close analogy with the open organ pipe formula.

We must therefore expect that only the first few waves can give some indication of the deep-sea disturbance. Thereafter, selective amplifications of those particular frequencies that are in resonance with local topography should dominate the record. This is more or less what one observes. For example, the measured time interval between the first and second crest usually shows a systematic increase with travel time (*15*) in accordance with what is to be expected for slightly dispersive waves (*16*). But the frequencies in the interior of the record differ erratically from station to station. A spectrum analysis by M. J. Tucker of the La Jolla and Oceanside tsunami records for the Japanese tsunami of 4 March 1952 (*17*) revealed sharp peaks at periods of 30 and 21 minutes. Application of the open organ pipe formula yields resonance amplification of the continental borderland ($H_0 = 3000$ m, $H_1 = 1000$ m, $D = 260$ km) for periods of 1.2 hr, 2.5 hr, and so forth; the continental shelf ($H_0 = 1000$ m, $H_1 = 100$ m, $D = 1.5$ km) is resonant for periods of 3 minutes or less. The former periods are larger, and the latter are smaller than the periods believed to be effectively generated by tsunamis, and this may explain why the average amplitudes in the California region are only a fraction of those at Hawaii. There is no clue here for the observed spectral peaks of 30 and 21 minutes. Our interpretation is that the continental waves are due to off-resonance coupling and that the spectral peaks were already inherent in the offshore disturbance.

The important point is this: the customary procedure is to estimate deep-sea amplitudes by assuming a coastal amplification of $(H_0/H_1)^{1/4}$ according to Green's law, or some equivalent rule based on ray optics. Since we find that the amplification may be much larger or much smaller, the usual estimates of deep-sea amplitudes may be off by orders of magnitude. Useful estimates can perhaps be made from stations on isolated small islands because the wave height observed there would be nearly that of the open sea.

One more point needs to be made. Irregularities in the coast line such as bays and headlands will scatter the waves on the shelf and convert some of their energy into *free* edge waves. In this manner, the deep-sea disturbance is trapped into low-velocity propagation along the edges (*13*). A curvature in the coast line that is concave shoreward (a bay) should be effective in trapping; convex curvature would lead to a loss of trapped energy by radiation back to the open sea. Major tsunamis are followed by something like 5 days of enhanced activity, whereas the sea-level disturbance in an infinite ocean of constant depth should be a matter of hours, at most. Some of the prolongation is undoubtedly associated with reflection from continental borders, as shown by Cochrane and Arthur (*18*); some may be the result of scatter from oceanic islands and sea mounts. We suggest that some of the afterglow, at least on continental margins, results from a conversion by coastal irregularities of fast open-sea waves into slow edge waves. There is a close analogy here with the experimental demonstration by Tatel (*19*) that some of the complexity of seismic records can be traced to a conversion by surface irregularities of fast body waves into slow surface (Rayleigh) waves. But there are difficulties with this suggestion. Travel times between La Jolla and Oceanside for tsunami waves do not check out well; and how is one to account for the afterglow at Hawaii?

References and Notes

1. H. Lamb, *Hydrodynamics* (Cambridge Univ. Press, Cambridge, 1932).

2. Contribution from the Scripps Institution of Oceanography, new series No. 826. This work was supported by the Office of Naval Research and the Air Force Cambridge Research Center, contract AF (19) 604 (1163). We wish to express our appreciation to A. C. Redfield and A. R. Miller of the Woods Hole Oceanographic Institution, who furnished us with the sea-level data for the four hurricanes; to M. J. Tucker of the National Institute of Oceanography, Great Britain, who made frequency analyses of tsunami records; and to the many people who helped with the seagoing work.

3. E. Gossard and W. H. Munk, *J. Meteorol.* 11, 259 (1954).
4. ———, *Quart. J. Roy. Meteorol. Soc.* 81, 484 (1955).
5. W. H. Munk, H. V. Iglesias, T. R. Folsom, *Rev. Sci. Instr.* 19, 654 (1948).
6. F. Ursell, *Proc. Roy. Soc. London* A214, 79 (1952).
7. J. S. Winston, *Monthly Weather Rev.* 82, 228 and Chart X (1954).
8. A. C. Redfield and A. R. Miller, unpublished manuscript.
9. W. Malkin and G. C. Holzworth, *Monthly Weather Rev.* 82, 267 and Chart X (1954); H. C. Sumner, *ibid.* 72, 187 (1944).
10. T. Shimizu, *Bull. Geograph. Survey Instr.* 4, 69 (1954).
11. M. Ewing, F. Press, W. Donn, *Science* 120, 684 (1954).
12. M. J. Tucker, *Proc. Roy. Soc. London* A202, 565 (1950).
13. J. D. Isaacs, E. A. Williams, C. Eckart, *Trans. Am. Geophys. Union* 32, 37 (1951).
14. W. H. Munk, *ibid.* 30, 849 (1949).
15. C. K. Green, *ibid.* 27, 490 (1946).
16. C. Eckart, *Revs. Mod. Phys.* 20, 399 (1948).
17. W. H. Munk, *Bull. Seismol. Soc. Amer.* 43, 219 (1953).
18. J. D. Cochrane and R. S. Arthur, *J. Marine Research Sears Foundation* 7, 239 (1948).
19. H. E. Tatel, *J. Geophys. Research* 59, 289 (1954).

Experiments have two great uses—a use in discovery and verification, and a use in tuition. They were long ago defined as the investigator's language addressed to Nature, to which she sends intelligible replies. These replies, however, usually reach the questioner in whispers too feeble for the public ear. But after the discoverer comes the teacher, whose function is so to exalt and modify the experiments of his predecessor as to render them fit for public presentation.—JOHN TYNDALL, in *Six Lectures on Light,* Lecture 1.