Low-Energy Physics from a High-Energy Standpoint

L. I. Schiff
Stanford University, Stanford, California

THE title of this article was suggested by the title of a three-volume work first published nearly 50 years ago by Felix Klein: *Elementary Mathematics from an Advanced Standpoint.* The object of Klein's work was to point out the relationships among problems that arise in different fields of mathematics and, in particular, to show how more advanced mathematical concepts could bring added insight to the teaching of elementary mathematics in the secondary schools. Needless to say, I do not mean to imply by the similarity of titles that low-energy physics is elementary in character or that high-energy physics is advanced; I mean simply that there are differences in approach. More important, since both types of physics deal with the same basic material, atomic nuclei and their constituents, relationships can be expected between problems that arise in the two fields. In short, experiments in high-energy physics can throw a good deal of light on the low-energy properties of nuclei: their ground and low-lying excited states.

High-energy experiments generally make use of nucleons, pions, muons, electrons, or photons. For the purpose of the present discussion (*1*), such experiments can be divided into two classes: those in which a large amount of energy is transferred to the target nucleus, and those in which the nucleus is left in its ground state or a low-lying excited state. It is possible in principle to learn a good deal about the nuclear ground state from the first class of experiments, but it is usually difficult in practice, for a transition from the ground state to a highly excited state is involved, and the properties of the final state are generally not well understood. In this class belong strongly inelastic scattering experiments and absorption experiments in which pions or high-energy photons disappear.

Perhaps the simplest of these experiments to interpret is photodisintegration without meson production. A reasonable model for this process has been proposed by Levinger (*2*), according to which the photon is absorbed by a pair of nucleons within the nucleus. In this model, the ground state of the nucleus is characterized in first approximation by a nucleon momentum distribution, and the electromagnetic transition carries two interacting nucleons from a low-energy state to a high-energy state in which they may escape from the rest of the nucleus. The interaction of the nucleons with each other is important, for this means that the wave function for the relative motion of the two nucleons appears in an essential way. Another way of putting this is to say that not only does the single-nucleon distribution function enter, but the two-nucleon correlation function is also of importance. A quantitative calculation along these lines, made by Dedrick (*3*), shows surprisingly good agreement with recent experiments of Johansson (*4*). Perhaps the main conclusion that can be drawn from this agreement is that correlations of three or more nucleons are not of dominant importance in the ground-state wave function. The same conclusion has been reached by Levinson (*5*) from a study of configuration mixing in nuclei with doubly magic plus three nucleons.

Most other experiments of the first class involve additional unknown factors that complicate their interpretation, such as pion-nucleon scattering cross sections or pion production cross sections. However, some high-energy processes that involve only nucleons can be handled in similar fashion to the photoeffect. An example is the pickup process, first discussed by Chew and Goldberger (*6*).

Experiments in the second class, those in which a relatively small amount of energy is transferred to the target nucleus, are easiest to interpret in terms of the properties of the nuclear states when the interaction with the incident particle is known and is weak. The reason for the first qualification is apparent, for the number of unknown parameters is thereby reduced. It means that the situation is more favorable for muons, electrons, and photons than for nucleons and pions, since for practical purposes the interaction is then entirely electromagnetic and hence known. The existence of an anomalous muon-nucleon interaction comparable in strength with the electromagnetic interaction would probably be inconsistent with the measurements of Fitch and Rainwater (*7*) on the spectra of mu-mesic atoms. Experiments on the nuclear scattering of cosmic-ray muons, by Amaldi and Fidecaro (*8*) and by Rochester and Wolfendale (*9*), and measurements of muon pair production by photons, performed by Masek, Lazarus, and Panofsky (*10*), are in general agreement with this conclusion.

It is also desirable that the interaction between incident particle and target nucleus be weak. This greatly simplifies any calculations that are made, for perturbation methods can then be used. It also means that higher-order processes, which involve virtual intermediate states of the nucleus, do not play an important part. Since some of these states will have high energy, their appearance would make it impossible to interpret the experimental results entirely in terms of the low-lying initial and final nuclear states. This criterion of weak interaction also favors muons, electrons, and photons as compared with nucleons and pions.

In spite of this it can be seen that photons are not suitable for experiments of the second class. Either a high-energy photon is absorbed, in which case the final nuclear state has high energy, or it is scattered, in which case some intermediate states of high energy appear, because of the virtual absorption and reemission of the photon. If these intermediate states correspond to high excitation of the nucleus, the experiments again cannot be interpreted entirely in terms of low-lying states; this represents the effect of nuclear dispersion. If on the other hand the intermediate states correspond to virtual electron-positron pairs, only the static charge distribution of the nucleus enters. This is Delbrück scattering, which therefore can in principle furnish information about the nuclear ground state, but in practice is too weak to be useful in this connection (*11*).

The foregoing discussion leaves muons and electrons as the two products of high-energy physics that are most likely to be useful in the study of the low-energy properties of nuclei. Thus far, muons have been available at rather low energies as decay products of pions produced with high-energy accelerators, and at high energies in cosmic radiation. The low intensity and wide energy distribution of the latter have so far made precision scattering experiments with cosmic-ray muons impossible. The accurate mumesic atom spectra obtained by Fitch and Rainwater (*7*) have on the other hand provided reliable values for the mean-square radius of the nuclear charge distribution. The possibility of obtaining such information was pointed out by Wheeler (*12*), and experiments of this type were first performed by Chang (*13*). Nuclear radii obtained in this way rest on interpretations of the experimental data by Cooper and Henley (*14*) and by Hill and Ford (*15*). More detailed information concerning the shape of the charge distribution cannot be obtained without high-energy, well-collimated, monoergic muon beams, which so far do not exist. However, the fine structure of the mumesic atom spectra can be used to infer the structure of the low-lying nuclear rotational states, as has been pointed out by Wilets (*16*) and by Jacobsohn (17). Further, the capture of negative muons by nuclei, measured by Keuffel (*18*), does not leave the final nucleus with very high energy, for most of the energy is carried off by the neutrino. Then, as is shown by Primakoff (*19*), the rate of capture can be expressed in terms of the proton density and correlation function for the ground state.

Electrons are now available in well-collimated, monoergic beams with energies up to 630 Mev, almost completely free of background radiation of other kinds (*20*). Their interaction with nuclear matter is known to be electromagnetic and, hence, is reasonably weak. Elastic scattering of electrons provides direct information on the ground-state nuclear charge distribution, and inelastic scattering data can be analyzed to yield information on nuclear excited states. Since the effects produced by high-energy electrons are often discussed in terms of the method of virtual quanta, it is worth pointing out the limitations of this concept. A charged particle in the extreme relativistic region has its coulomb field contracted to a narrow sheet that is perpendicular to the line of motion. The electric field vector is radially out from this line, and the magnetic field vector associated with the motion of the charge is also in the sheet and tangential to a circle centered on the charge. The two field vectors are nearly equal in magnitude, so that together they are represented to good approximation by a transverse light pulse that travels along with the charge. This pulse may then be Fourier-analyzed into virtual quanta, which cause effects like any other photons in the same frequency range. The principal limitation of this picture is that it represents undisturbed motion of the electron. As long as the electron is scattered through a very small angle with very little energy loss, the method of virtual quanta can be used. Such events comprise the overwhelming bulk of the scattering processes but give little more information than can be obtained from experiments with low-energy photons.

More distinctive data are obtained from the relatively few events in which the electron is scattered through a large angle. It is better to think of these processes in terms of the Møller fields (*21*), which do not have the transverse character of the virtual quanta and, hence, are not even approximately representable as photons. Unlike photons, for which the energy is always equal to the momentum multiplied by the speed of light, the Møller fields can transfer large amounts of momentum and small amounts of energy from the electron to the nucleus. The momentum is simply the vector difference between the initial and final electron momenta and, hence, is large when a high-energy electron is scattered through a large angle. The energy depends on the particular nuclear transition and can be anything from zero, for elastic scattering, up to the initial electron energy. The inherent advantage of large momentum transfer is that the wavelength associated with the Møller fields is short, so that small-scale details of the nuclear structure can be mapped out. At the same time the energy transfer can be small, so that only low-lying nuclear states are involved.

Elastic electron scattering experiments of Lyman, Hanson, and Scott (*22*) at 16 Mev were the first to give some indication that the nuclear charge distribution has a somewhat smaller radius than the specifically nuclear force field. Other experiments at moderate energies, mainly by Pidd, Hammer, and Raka (*23*), and the high-energy experiments between 80 and 200 Mev, by Hofstadter and collaborators (*24, 25*), have abundantly confirmed this indication. In order to take full advantage of high-energy electrons, it is essential to have good energy and angle resolution of both the incident and scattered electrons: well-collimated and monoergic incident beams and careful magnetic analysis of the electrons scattered at several well-defined angles. Elastic scattering experiments by Hofstadter, McIntyre, Fechter, Knudsen, and Hahn (*24, 25*) on a number of elements have been interpreted in terms of static, spherically symmetric charge distributions by Yennie, Ravenhall, and Wilson (*26*), by Brown and Elton (*27*), and by Hill, Freeman, and Ford (*28*). Experimental results for a wide range of energies and angles and for the larger atomic numbers lead to nuclear charge distributions that are uniform in the central region and taper to zero over a surface layer about 2×10^{-13} cm thick and that have the same root-mean-square radius as a uniformly charged sphere of approximate radius 1.20×10^{-13} $A^{1/3}$ cm. Similar experiments in hydrogen by Hofstadter and McAllister (*29*) indicate a finite electromagnetic size for the proton. It is expected that with sufficient experimental accuracy it will be possible to assign sizes separately to the distributions of charge and of magnetic moment of the proton.

There are several respects in which a nucleus differs from the static, spherically symmetric charge distribution mentioned previously. Nuclei that possess electromagnetic multipole moments are not spherically symmetric. In the heavier nuclei, the most important of these moments is the electric quadrupole moment. According to the theory of Bohr and Mottelson (*30*), a nucleus can have an intrinsic angular deformation of shape and still have zero angular momentum in its ground state, and indeed this occurs for many even-even nuclei. Then the nucleus has no quadrupole moment in its ground state and therefore gives no indication of its deformation in its optical atomic spectrum. In such a case, the deformation does not show up in the truly elastic electron scattering either. However, it gives rise to a set of low-lying nuclear rotational states with excitation energies of the order of a few hundreds of kilovolts. With energy resolution in the scattering experiments of the order of ¼ percent—500 kev out of 200 Mev—inelastic scattering to these levels often cannot be distinguished from elastic scattering.

A general theory of inelastic scattering caused by arbitrary nuclear multipole transitions has been given (*31*). An interesting special result of this theory concerns the total scattering caused by the nuclear deformation, in addition to that caused by the spherical charge distribution. This includes additional elastic scattering when the ground state has spin one or greater and also inelastic scattering to the lowest rotational states. The sum of these depends only on the intrinsic deformation and not on the ground-state spin. A specific calculation of this effect has been made by Downs, Ravenhall, and Yennie (*32*) for 180-Mev electrons scattered by tantalum, for comparison with the experiments of Hahn and Hofstadter (*33*). The experimental scattering curve is much smoother than that for gold or lead, both of which have small deformations. It cannot be fitted with a spherically symmetric charge distribution of the type mentioned previously. The fitting procedure is as follows. The general slope of the experimental curve fixes the root-mean-square radius of the charge distribution. The surface layer thickness is then chosen to give the best detailed fit, in which case the theoretical curve is too low in the neighborhood of 70°. This minimum, as well as another near 110° that is beyond the range of the experiments, is then filled in by the deformation contribution to the scattering; this gives an estimate for the deformation that is in good agreement with that obtained from Coulomb excitation experiments. Finally, it is recognized that the surface thickness obtained in this way is greater than the true surface thickness, because of the averaging of the deformed nucleus over angles to obtain the spherically symmetric part of the charge distribution. The corrected root-mean-square radius and surface thickness are then in good agreement with the corresponding values for gold.

As I have just pointed out, a nucleus departs from the simple model of a static, spherically symmetric charge distribution in that it is not always spherically symmetric. It is also not static, for it has internal degrees of freedom. These manifest themselves in an infinite set of excited states, which appear as virtual intermediate states in any scattering process, even though the initial and final nuclear states have low energy. This nuclear dispersion contribution to electron scattering would, of course, be very small in comparison with the first-order scattering calculated from the Møller fields if the interaction between electron and nucleus were very small. The actual electromagnetic interaction is of such magnitude that the dispersion contribution is relatively small in a number of interesting cases but is not negligible. A calculation indicates that most of the effect comes from terms in which the intermediate state is either the initial or the final nuclear state (*34*). When this is the case, it is expected that the bulk of the dispersion contribution can be included by using initial and final electron wave functions that are exact solutions in the static Coulomb field of the nucleus.

Still another way in which the real nucleus departs from its simple model is in the expected correlations between positions of nucleons. This short-range order within the nucleus, at least insofar as it refers to the protons, should give the charge distribution a granular structure. Because of the small dimensions of this granularity, short wavelengths are required for its investigation, probably such as those that correspond to the large-angle scattering of 400-Mev electrons.

The three examples just discussed—deformation of shape, dispersion effects, and granularity—point to the importance of having available exact solutions of the Dirac equation for static, spherically symmetric nuclei. In this way the principal effect of the nucleus on the scattered electron is taken into account, and one can hope to treat the other effects as relatively small perturbations. However, an exact solution in the form of a series of a large number of partial waves is a very cumbersome thing with which to work. It is therefore of great importance for further work along these lines that Yennie found that the exact solution can be represented to good approximation, in the immediate neighborhood of the nucleus, by a plane wave with altered parameters (*35*). This arises because the scattering phase shifts are nearly constant; moreover, the plane wave can be modified to take account of a linear dependence of the phase shifts on angular momentum, which is a better representation of the actual situation. It is in this way that the afore-mentioned quadrupole effects in tantalum were calculated.

One of the side effects that can complicate the interpretation of electron-scattering data is the emission of radiation, or *bremsstrahlung*. This is particularly important for elastic or nearly elastic scattering, since large numbers of soft photons are expected. This radiative correction to scattering was first calculated by Schwinger (*36*) under the assumption that the scattering potential is weak enough to be treated by first Born approximation. Recently, Suura (*37*) has shown that the fractional radiative correction at high energy—the ratio of the actual cross section to that computed with neglect of radiation—is the same as that found by Schwinger. This turns out to be rather small in comparison with the absolute accuracy of the experiments and also not to be a rapidly varying function of energy or angle, so that it does not distort the experimental results appreciably.

As a final example of a bit of low-energy physics information that has been obtained only from experiments at high energies, I shall discuss briefly the electric monopole transition in carbon-12 (*38*). The ground state of this nucleus has zero total angular momentum and even parity, and it is believed that the second excited state, at 7.68 Mev, is also 0^+. This state has been excited by Fregeau and Hofstadter (*39*) by means of inelastic scattering of 190-Mev electrons, and an angular distribution has been obtained.

The electric monopole matrix element that corresponds to this transition between 0^+ states can be inferred from suitable plots of the experimental results against the momentum transfer. These plots are consistent with an assumed matrix element of 4.0×10^{-26} cm^2. It is interesting that this value happens to be about the same as that found by Devons, Goldring, and Lindsey (*40*) for the transition between the ground state and the first excited state at 6.06 Mev in oxygen-16, which is also an electric monopole transition. The latter experiment measured the half-life of the excited state for pair emission, which is possible only if the upper state is the first excited state, since otherwise the rate of cascade gamma radiation greatly exceeds the rate of pair emission. This low-energy physics approach cannot, therefore, be used in the case of carbon, where the second excited state is involved.

Further, Coulomb excitation by positive ions, which cannot in any event be used on a monopole transition, is not very effective in dealing with light elements that have widely spaced energy levels, because of barrier penetration. In such situations, high-energy physics not only makes possible determination of the rates of low-energy processes but can in principle provide detailed information on the radial dependence of the pertinent matrix elements.

References and Notes

1. Presented at the Washington meeting of the American Physical Society, 28 Apr. 1955. Work supported in part by the Office of Scientific Research, Air Research and Development Command.
2. J. S. Levinger, Phys. Rev. **84**, 43 (1951).
3. K. G. Dedrick, submitted to *Phys. Rev.*
4. S. A. E. Johansson, *Phys. Rev.* **97**, 434 (1955).
5. C. A. Levinson, quoted by K. A. Brueckner, private communication.
6. G. F. Chew and M. L. Goldberger, *Phys. Rev.* **77**, 470 (1950).
7. V. L. Fitch and J. Rainwater, *ibid.* **92**, 789 (1953).
8. E. Amaldi and G. Fidecaro, *Nuovo cimento* **7**, 535 (1950); *Phys. Rev.* **81**, 339 (1951).
9. G. D. Rochester and A. W. Wolfendale, *Phil. Mag.* **45**, 980 (1954).
10. G. E. Masek, A. J. Lazarus, W. K. H. Panofsky, *Bull. Am. Phys. Soc.* **30**, No. 3, 50 (1955).
11. See R. R. Wilson, *Phys. Rev.* **90**, 720 (1953).
12. J. A. Wheeler, *Revs. Mod. Phys.* **21**, 133 (1949).
13. W. Y. Chang, *ibid.* **21**, 166 (1949).
14. L. N. Cooper and E. M. Henley, *Phys. Rev.* **92**, 801 (1953).
15. D. L. Hill and K. W. Ford, *ibid.* **94**, 1617 (1954).
16. L. Wilets, *Kgl. Danske Videnskab. Selskab Mat.-fys. Medd.* **29**, 3 (1954).
17. B. A. Jacobsohn, *Phys. Rev.* **96**, 1637 (1954).
18. J. W. Keuffel, *Proc. 5th Ann. Rochester High-Energy Conf.* January, 1955.
19. H. Primakoff, *ibid.*
20. M. Chodorow *et al.*, *Rev. Sci. Instr.* **26**, 134 (1955).
21. C. Møller, *Z. Physik* **70**, 786 (1931).
22. E. M. Lyman, A. O. Hanson, M. B. Scott, *Phys. Rev.* **84**, 626 (1951).
23. R. W. Pidd, C. L. Hammer, E. C. Raka, *ibid.* **92**, 436 (1953).
24. R. Hofstadter, H. R. Fechter, J. A. McIntyre, *ibid.* **91**, 422 (1953): **92**, 978 (1953).
25. R. Hofstadter *et al.*, *ibid.* **95**, 512 (1954).
26. D. R. Yennie, D. G. Ravenhall, R. N. Wilson, *ibid.* **95**, 500 (1954); D. G. Ravenhall and D. R. Yennie, *ibid.* **96**, 239 (1954).
27. G. E. Brown and L. R. B. Elton, *Phil. Mag.* **46**, 164 (1955).
28. D. L. Hill, B. E. Freeman, K. W. Ford, *Bull. Am. Phys. Soc.* **30**, No. 3, 49 (1955).
29. R. Hofstadter and R. W. McAllister, *Phys. Rev.* **98**, 217 (1955).
30. A. Bohr and B. R. Mottelson, *Kgl. Danske Videnskab. Selskab Mat.-fys. Medd.* **27**, 16 (1953).
31. L. I. Schiff, *Phys. Rev.* **96**, 765 (1954).
32. B. W. Downs, D. G. Ravenhall, D. R. Yennie, *ibid.* **98**, 277 (1955).
33. B. Hahn and R. Hofstadter, *ibid.* **98**, 278 (1955); see also B. Hahn, R. Hofstadter, D. G. Ravenhall, D. R. Yennie, to be submitted to *Phys. Rev.*
34. L. I. Schiff, *Phys. Rev.* **98**, 756 (1955).
35. D. R. Yennie, D. G. Ravenhall, B. W. Downs, *ibid.* **98**, 277 (1955).
36. J. Schwinger, *ibid.* **75**, 898 (1949).
37. H. Suura, *ibid.* **98**, 278 (1955); fuller account submitted to *Phys. Rev.*
38. L. I. Schiff, *Phys. Rev.*, in press.
39. J. H. Fregeau and R. Hofstadter, *Bull. Am. Phys. Soc.* **30**, No. 1, 45 (1955); fuller account submitted to *Phys. Rev.*
40. S. Devons, G. Goldring, G. R. Lindsey, *Proc. Phys. Soc. (London)* **A67**, 134 (1954).