A Half-Century of Quantum Physics

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THE presidency of the American Association for the Advancement of Science is one of the greatest honors that American scientists can confer on one of their colleagues, and I am grateful for the opportunity to have served in this post. This is a particularly happy occasion* for me, for the setting is the auditorium in Wheeler Hall, where as an undergraduate I heard many a lecture, and this is the 30th anniversary of the awarding of my A.B. degree in physics by the University of California, our host institution for this meeting of the AAAS.

Before and during my undergraduate student days, I used to work this campus as a reporter for San Francisco and Oakland newspapers. Thus began a long and pleasurable acquaintance with the local chairman of our meeting, President Robert Gordon Sproul.

Of course I cannot talk to you entirely from first-hand knowledge of a half-century of quantum physics. I was born in Alamogordo, New Mexico, in 1902, which was 2 years after Max Planck introduced the quantum idea into physics in Germany. Aside from the fact that I was pretty young at the time, I do not think Planck's theory attracted much attention at the time in Alamogordo. Now, of course, my home town is quite conscious of atomic physics, and its chamber of commerce has placed signs on the highways leading into town which proclaim that the town is the birthplace of atomic energy. They refer, of course, to the fact that the first atomic bomb was exploded near here in the summer of 1945.

I first heard of physics when I was 12 years old and bought a high-school textbook by Carhart and Chute for 15 cents in the old DeWitt and Snelling bookstore in Oakland. The following year I became pretty deeply involved in what we now call the atomic age. I had been reading *The New Knowledge* by Robert Kennedy Duncan, which was a popular book on atoms and radioactivity by the man who founded the Mellon Institute for Industrial Research. That was in 1915, the year in which San Francisco celebrated the opening of the Panama Canal by holding the Panama Pacific International Exposition. Another boy and I discovered that the state of Colorado had as part of its exhibit a large pile of 10 or 20 tons of raw carnotite ore. This is a brilliant yellow sandstone, which today is being sought all over the mountain states by prospectors who rent their Geiger counters from local drugstores.

* This paper is based on the address of the retiring president of the American Association for the Advancement of Science, at the annual meeting, 28 Dec. 1954, in Berkeley, Calif.

My friend and I managed, as boys will, to acquire enough carnotite from that pile to fill a shallow cigar box. With it we could take, using overnight exposures, shadow pictures of keys and other metal objects which were made by the gamma rays emitted by the radium and uranium content of the ore.

Once a boy of 13 has become this deeply involved with modern atomic physics, there is likely to be no hope that he is good for anything else. From then on it is impossibly hard for his teachers to get him interested in reading Gayley's *Classic Myths.*

Practically all the important progress in physics in this century is bound up with quantum ideas. Moreover, it has been a half-century in which physics has developed at a revolutionary pace that is totally unprecedented in the world's history. Therefore all that I can do here is to pass the main ideas in rapid review, perhaps lightening the story with an anecdote here and there, and hope to stimulate a wider interest in this exciting subject. Everything I say is well known to the physicists However, the ideas are complicated, and they may experience the academic delight of catching me in a mistake or two.

Quantum Ideas

By quantum physics we mean all parts of the science that involve a peculiar universal constant, known as Planck's constant, h, where

$$h = 6.55 \times 10^{-27} \text{ erg sec.}$$

So defined, quantum physics involves nearly all of physics and chemistry. It also involves a good share of astrophysics. Moreover, quantum ideas have required a good deal of searching into the philosophic foundations of physics.

The quantum idea was first introduced into physics in 1900 and 1901 by Max Planck in connection with the study of the radiations emitted by hot solid bodies. Throughout most of the 19th century, such radiation, including visible light, had been regarded as a wave motion. But, in developing the theory of radiation from hot bodies, Planck found it necessary to assume that light energy is not emitted and absorbed continuously by atoms. Rather he supposed that it was emitted and absorbed in definite little bundles of energy, or quanta.

Many experimental properties of light pointed to its being propagated as a wave motion. There is nothing remote or esoteric about these experiments. Take a silk umbrella and look through the fabric at a distant street light. In addition to a central white image, you will see a series of colored images extending out from the central image in two mutually perpendicular series in directions related to the warp and woof of the fabric. These are caused by interference of light waves which go through different interstices between the evenly spaced threads of the fabric.

A diffraction grating is an accurately made device for observing these spectra more accurately. By measuring the angle of spread between them and the central image, one can find the wavelength of the waves, and, by knowing the velocity of the waves, one can find the frequency or number of oscillations per second that occur as the wave passes a fixed point.

In this way, one finds that the wavelength for violet light is about 3×10^{-5} cm and that the wavelength for red light is about twice as great, or 6×10^{-5} cm. Thousands upon thousands of these wavelengths have been measured to at least 6 decimal places. These form the largest and most precise body of experimental data in all physics. Since the velocity of light is 3×10^{10} cm/sec, it turns out that the frequency of violet light is about 3×10^{14} cy/sec.

On Planck's view, light of frequency n cy/sec is emitted and absorbed in quanta of energy equal to hn, which is therefore about 6.5×10^{-12} erg for violet light. For x-rays the frequencies are some 10,000 times greater, and the quanta are therefore some 10,000 times greater.

The reasoning that led to this result was so complicated that Planck himself was not fully convinced of its validity. Physicists are all an extremely conservative group of people, at least in matters having to do with their own science, and they were reluctant to accept the radical quantum idea on such slender evidence.

In 1905 Einstein showed how clearly and neatly the main facts regarding the photoelectric effect could be undestood if the quantum view of light were favored over the wave view. In the photoelectric effect, electrons are emitted from a metal when light shines on it.

Early experiments showed that increasing the brightness of the light caused more electrons to be emitted but did not increase the energy of motion with which the emitted electrons came out. On the wave view, one would think that a bigger wave would shake the electrons harder and make them come out with more energy.

Experiment also showed that the energy with which the electrons were emitted increased linearly as the frequency of the light was increased. This result was not at all understood in terms of the wave theory of light.

Einstein pointed out that on the quantum view, if 1 light quantum goes to 1 electron, then greater brightness means more quanta and therefore more emitted electrons. Planck had already found it necessary to suppose the energy content of a quantum to be proportional to the frequency of the light wave, and thus a natural explanation is provided of why the energy of the emitted electrons increases linearly with the frequency.

Wave-Particle Duality

Thus was born the wave-particle duality or dilemma of modern physics. Light, on going through a series of closely spaced slits, behaves in ways that have found only quantitative explanation on the wave theory. Light, on falling on a metal, liberates electrons in ways that have found satisfactory explanation only in terms of the quantum or corpuscular theory. From here on, the subject began to develop at an ever-increasing rate.

When atoms are excited in a gaseous discharge tube, such as is used for advertising signs, the kinds of light emitted consist of sharply defined frequencies characteristic of the gas atoms in the discharge tube. If light is emitted in quanta of definite amounts, this must mean that the atoms are capable of existing only in states of definite energy values. The differences in these allowed, or quantized, energy values are the energies of the light quanta emitted by an atom in passing from a state of higher total energy to one of lower total energy. In 1913 Niels Bohr built his successful theory of the hydrogen atom on a combination of this quantum idea with the general picture of the nuclear atom that had been developed experimentally by Ernest Rutherford. Soon afterward, James Franck and Gustav Hertz performed experiments in which they showed the reality of these quantized energy levels in atoms by finding that electrons can give up quantized amounts of energy to atoms only on colliding with them, and that these quantized amounts are closely correlated with the sizes of the emitted light quanta.

In 1912 another discovery of major importance was made. Since the discovery of x-rays in 1896 by Wilhelm Roentgen, there had been speculation on whether these were a wave motion or a stream of corpuscles. Attempts at diffraction experiments gave negative results with a sensitivity indicating that, if they are a wave motion, the wavelength cannot be more than about 10^{-8} cm. This is just about the distance apart of layers of atoms in a crystal, which gave Max von Laue the idea that perhaps the regular arrangement of atoms in a crystal would diffract x-rays in the way that the rulings of a diffraction grating diffract light. The experiment was successful. Thus two new branches of physics were born. By use of a crystal of known structure, it was now possible to measure the wavelengths of the characteristic x-rays emitted by various atoms, so spectroscopy was extended to the x-ray region. By use of x-rays of known wavelength, it was possible to infer from the nature of the diffraction pattern how the atoms are arranged in crystals of unknown structure. Thus a powerful tool was provided for the study of the structure of solid matter.

All this served to point up the disturbing puzzle of the dilemma on whether x-rays and light were really a wave motion or really a stream of corpuscles, for it seemed to be something like both and yet no one could see how it was possible for it to be both in any sense.

Only W. H. Bragg, writing in *Nature* in late 1912, hinted at a combined outlook. He wrote:

> The problem then becomes, it seems to me, not to decide between the two theories of x-rays, but to find . . . one theory which possesses the capacities of both.

On Bohr's model of the atom, the electrons revolve around the nucleus like planets going around the sun in the solar system. Although the theory was immensely successful in correlating spectroscopic facts, it threw no light on the fundamental nature of the valence forces that hold atoms together in molecules. In Berkeley, G. N. Lewis developed a rival theory based on a static model of the atom in which electrons had favored locations at the corners of a series of cubes surrounding the nucleus, the eight corners corresponding to the length of the short periods in the periodic system of the elements.

When I entered the University of California as a freshman in 1921, the Bohr atom was being taught in the physics department and the Lewis atom was orthodox doctrine in the chemistry department. Now both departments are preaching the same kind of atom, which resembles neither of its forerunners and combines the best features of both. The things I am talking about are so old that if they are mentioned anywhere it is probably in the history department.

The early 1920's were an exciting time to be studying physics. We had these rival atomic theories, each with its inadequacies and uncertainties. Some things were lacking. In Livermore, California, there was only the rodeo and on Charter Hill nothing but the Big C and a few grazing cows.

In 1923 the wave-particle dilemma became even more acute. Arthur Compton, in St. Louis, discovered that x-ray quanta have momentum as well as energy. When x-rays are scattered by matter of light atoms, it is found that some of them are scattered, but that the scattered x-rays consist of smaller quanta than those which struck, and the shift toward smaller quanta is greater, the larger the angle of deflection through which the x-rays are scattered. All this was exactly in accord with the idea that the x-ray quanta were scattered by colliding with electrons by exactly the same rules of conservation of energy and momentum that are applicable to the collision of two material particles, such as billiard balls.

In that same year, 1923, Louis de Broglie in Paris published his now-famous doctor's thesis, in which he suggested that the wave-particle duality might extend to the behavior of electrons as well as to light and x-ray quanta. Up until this time physicists felt sure that a beam of cathode rays was simply a corpuscular stream of electrons moving in accordance with Newton's laws of motion, as corrected in the high-energy region for relativistic effects.

De Broglie suggested that the relationship between the wavelength of the wave aspect of an electron and the momentum of the particle aspect of the electron ought to be the same as that already found to hold for x-ray quanta, namely, that wavelength equals Planck's constant divided by momentum. This suggestion made possible a simple interpretation of the existence of discrete energy levels in atoms, which in Bohr's theory was simply postulated in order to get agreement with spectroscopic facts.

We are all familiar with the fact that a stretched string in a musical instrument vibrates freely at a particular frequency such that the length of the string is just equal to half a wavelength of the wave of that frequency which might travel on the string. Then it can vibrate also at double this frequency, so the length equals two half-wavelengths, or at triple the fundamental frequency so the string's length equals three half-wavelengths, and so on. Similar rules apply to the modes of vibration of other continuous bodies such as the stretched membrane of a drum. De Broglie argued by analogy that, if the motion of electrons was somehow governed by an associated wave motion, then the allowed orbits in an atom must be governed by mathematical restrictions similar to those which determine that vibrating bodies can vibrate only in a certain discrete set of modes of vibration.

It turns out, on these views, that the de Broglie wavelengths of electrons which have been accelerated by a potential drop of a few hundred volts will be of the same order as that of x-rays. This suggests that electrons, too, ought to show diffractive scattering by the regularly spaced layers of atoms in a crystal. In 1927 electron diffraction was discovered in New York by C. J. Davisson and L. H. Germer, working with the scattering of low-energy electrons by a single crystal of nickel, and independently that same year by G. P. Thomson in England, who worked with the scattering of higher energy electron beams by polycrystalline materials. These experiments fully confirmed the idea that electrons are scattered from crystals like a wave motion having the wavelength that was predicted by de Broglie. At the same time a new tool for crystallographic studies, supplementing that of x-ray diffraction, was made available.

A few years later it was shown experimentally that beams of hydrogen molecules and of helium atoms were also governed by de Broglie wave principles when scattered by crystals. This was done by Otto Stern, now a distinguished resident of Berkeley, who was then professor of physics in Hamburg, Germany.

In consequence of these experimental discoveries and many associated theoretical developments, physicists now believe that the wave-particle duality applies to all things in nature, be they light quanta, electrons, protons, or entire atoms and molecules. With larger things, the wavelength becomes so small that the wave aspect escapes observation, which is why all ordinary motions appear to be governed entirely by the particle formulation originating in Newton's laws of motion.

Matrix Mechanics

In 1925, Werner Heisenberg in Göttingen discovered a new mathematical way of treating problems in atomic physics. It was called matrix mechanics because quantities which in Newtonian mechanics are represented by ordinary numbers are represented in this theory by an abstract kind of mathematical entity known as a Hermitian matrix.

This theory caused physicists a lot of trouble. Up to then practically none of them had ever studied matrix algebra. It is true that the mathematicians knew about matrices but, under pressure from the physicists to teach them only what they needed to know, the mathematicians had not talked about matrices when physicists were around. Max Born, the 1954 Nobel prize winner in physics, was in Berkeley from Göttingen in 1925 as a visiting professor, and he lectured on matrix mechanics. What a rough time he gave us as we tried to grasp the strange new ideas of matrix mechanics.

Then, in the spring of 1926, what a relief it was when Erwin Schrödinger's rival wave mechanics came on the scene, and we could avoid the difficulties of matrix algebra. And what a surprise it was in the summer of 1926 when Carl Eckart, in Pasadena, and also Schrödinger himself discovered that the two theories were identically the same. They were simply dressed up in such totally different mathematical costumes that it took some time before their identity was recognized.

In the early fall of 1926 I left Berkeley to study the new quantum mechanics with Born in Göttingen. There the great mathematician, David Hilbert, used to delight to tell us how he had told the Göttingen theoretical physicists of the close relationship between matrix algebra and certain boundary value problems of differential equations. If they had followed up this lead they might have discovered wave mechanics before Schrödinger.

In those days Hilbert used to say, "Die Physik wird zu schwer für die Physikern"—physics is becoming too difficult for the physicists.

In 1927 the pace of discovery in theoretical physics was probably greater than in any other year in the history of the science. Every issue of the leading journals had at least one paper of great importance. There was the more general formulation of the laws of quantum mechanics that was made principally by P. A. M. Dirac in England and John von Neumann in Germany. There was the development of the quantum theory of the radiation field by Dirac and the relativistic form of the quantum theory of the electron, which led to the prediction of the existence of the positively charged electron or positron, that was discovered a few years later by Carl Anderson in Pasadena.

Arnold Sommerfeld laid the foundations for the whole modern theory of metals and semiconductors by applying the quantum mechanical methods to the treatment of the free electrons in a conductor. W. Heitler and F. London applied quantum mechanics to the theory of the covalent chemical bond between two hydrogen atoms and showed that this atomic theory could at last meet the needs of the chemists. This gave rise to a wide program of developments, which resulted in the award of the 1954 Nobel prize in chemistry to Linus Pauling of the California Institute of Technology.

Heisenberg showed how the new quantum theory could account for the extremely strong interactions between the electrons in iron, cobalt, and nickel, which give rise to the strong magnetic effects shown by these elements. Many other discoveries of great importance were made among which may be mentioned the final clarification of the low-temperature heat capacity of gaseous hydrogen. It had long been known that this had something to do with quantum restrictions on the rotation of hydrogen molecules, but David M. Dennison showed the solution of this problem leading to the discovery of two stable forms of hydrogen gas known as orthohydrogen and parahydrogen.

Things were happening at such a pace that all the physicists, young and old, were suffering from acute mental indigestion. In the spring of 1928 when I taught a course in quantum mechanics for the first time at Columbia University, I remember that the late Bergen Davis summed it all up by saying,

> I don't believe you young fellows understand it any better than I do—but you all stick together and say the same thing.

Statistical Theories

Going back a bit, it was in the fall of 1926 that Max Born took a decisive step in supplying the hypothesis that provided a general basis for interpretation of the mathematical formalism of quantum mechanics. We had a mathematics of wave motion that was somehow associated with the motion of the electrons or other atomic particles. The big question was What is the basic relationship between the associated wave motion and the behavior of the atomic particles?

Born's answer, which was largely the basis of the award of the 1954 Nobel prize in physics to him, was that the theory does not and cannot make precise predictions about the motion of the particles, but that it can make only predictions about the relative probability of appearances or motions of different kinds. In particular he postulated that the square of the amplitude of the de Broglie waves at a particular place gives the relative probability of finding a particle in that place. This is a radical and revolutionary idea in its implications, and fundamental disputes among physicists still rage regarding its basic meaning. Nevertheless, it must be realized that this idea of a statistical interpretation of the waves as describing probabilities of behavior of the particles has now stood the test of time for more than a quarter-century and lies at the foundation of all modern atomic physics.

Physical science got its start with the precise astronomical predictions resulting from the dynamical theory of the solar system. These many quantitatively verified results exercised a dominating influence on physical thinking. All physics was assumed to be reducible to a fully deterministic description of motions, such that, given a full description of the situation as of now and sufficient calculating skill, one could calculate precisely what will happen at all times in the future.

Prior to 1926 statistical theories had been used in physics. Statistical methods were used to give an over-all average description of the heat motions that give rise to the thermal properties of matter. But in all such theories it was supposed that there really exists an underlying fully deterministic reality, and that statistical methods are used by choice for simpler descriptions rather than by fundamental necessity.

The questions now arise: Is there really an underlying fully deterministic description of the phenomena of atomic physics that has so far eluded our observations and theory-making because of some basic incompleteness that may be remedied in the future? Or, on the other hand, is there some inherent limitation in the world and our possible ways of observing it such that our knowledge of events is fundamentally restricted to observations and conclusions of a statistical character?

In the fall of 1927, Heisenberg provided an analysis of the processes of measurement that strongly favors an affirmative answer to the second question. Later analysis by Bohr in 1928 extended these ideas. The essence is that on an atomic scale the processes of observation necessarily introduce uncontrolled disturbances, and it is these which give rise to the over-all uncertainties that make fully deterministic knowledge impossible. If one refrains from observing, he makes no disturbance but remains ignorant of the data needed for deterministic calculations. Observations can be arranged in ways that increase the precision of knowledge of one variable but only at the price of introducing more uncertainty into the knowledge of a complementary variable.

The analysis of Heisenberg and Bohr provides a deep insight into the nature of limitations on knowledge of deterministic behavior, which seem to be truly fundamental. Most physicists today accept these views and regard the statistical element of the theory as an intrinsic feature of the world in which we live. Classical determinism on this view is an ideal limit toward which our knowledge can approach in large-scale phenomena where the quantum limitations become unimportant corrections.

But one physicist of outstanding importance steadfastly thinks otherwise. He is Albert Einstein. At the very outset he expressed himself by saying "Der lieber Gott würfelt nicht." In American vernacular we would say "the good Lord doesn't shoot craps."

Born's book, *Natural Philosophy of Choice and Chance* quotes a letter Einstein wrote in 1947 in which he says, "the statistical interpretation . . . has a considerable content of truth." However, he goes on to say

> I am absolutely convinced that one will eventually arrive at a theory in which the objects connected by law are not probabilities, but conceived facts as one took for granted only a short time ago.

With characteristic modesty he concludes then by saying

> Zur Begründung deiser Überzeugung kann ich aber nicht logische Gründe, sondern nur meiner kleinen Finger als Zeuger beibringen, also keine Autorität, die ausserhalb mainer Haut irgendwlechen Respect einflössen kann. [I cannot provide logical arguments for my conviction but can only call on my little finger as witness, which cannot claim any authority to be respected outside my own skin.]

Whether all the data of experience can be codified in terms of fully deterministic relations I do not know, of course, but unquestionably it is useful to have such organization of knowledge carried to its furthest limits. The history of science is filled with facile generalizations and the kind of oversimplification that fails to qualify what would be true if properly qualified. Think of the many pages of disputatious writings on free will and determinism!

In my view physics has nothing to say on this one way or the other as an issue related to human conduct. It was an unwarranted extrapolation in the first place to pass from the planetary successes of classical mechanics to extreme mechanistic determinism for human actions. It is equally incorrect to argue from the statistical determinism of quantum mechanics any support for the idea of free will in human behavior.

Nuclear Physics

By 1927 the principles of quantum theory as we know them today were pretty well developed. In the 27 years since then the ideas of quantum physics have been so closely identified with all the progress that has been made in physics and chemistry that it is not possible to discuss quantum physics separately from progress as a whole in these sciences.

The entire theoretical structure of nuclear physics is cast in quantum mechanical terms. This new branch of physics has never been handled in any other way. The application of quantum mechanics to problems of the internal structure of the nucleus was initiated in 1928 with the discovery of the theory of alpha-particle radioactivity by George Gamow in Göttingen and independently by the late Ronald Gurney and myself in Princeton.

This theory provides one of the most extreme examples of the use of probability ideas. According to classical mechanics, it is not possible for a particle to be in places where its total energy is less than its potential energy. In quantum mechanics this impossibility is changed into an improbability. An alpha particle in a uranium nucleus collides with the wall surrounding the nucleus some 10^{20} times a second. According to quantum mechanics it has a very slight chance of getting through the wall, even though it does not have energy enough to get over it. This chance is extremely small, amounting to only about one chance in 10^{36}. In consequence, the alpha particle remains in the nucleus on an average about 10^{9}

years before the spontaneous disintegration occurs. Nevertheless, the statistical feature of the theory shows up in that some uranium atoms disintegrate in a very short time, whereas others have lasted for many thousands of years without disintegrating.

This same theory of barrier leakage applied in reverse indicated that light elements could be made to undergo artificial transmutations using particles accelerated with voltages much lower than had been estimated to be necessary. This gave a strong stimulus to the experimental investigation of nuclear reactions which began in the early 1930's.

Quantum mechanics has also provided the concept of saturable exchange forces between fundamental particles, an idea that is foreign to classical ideas but appears to be essential in the further development of the theory of nuclear structure. Relativistic quantum mechanics, as I have already mentioned, provided the prediction of the existence of the positron and provides the theoretical basis for calculations of many of the basic processes that occur in the region of high-energy physics—that is, the physics of particles having energies of several hundred million to billions of volts, a branch of physics that is extensively studied here in Berkeley.

From quantum mechanical theories concerning exchange forces between protons and neutrons, H. Yukawa in 1936 was led to postulate the existence of a hitherto undiscovered kind of particle, called the meson, intermediate in mass between the electron and the proton. Experiments in recent years have shown that there are in fact many kinds of mesons, with complicated interrelationships, whose study is today one of the most important topics in fundamental research in physics.

In spite of all these successes and many others too numerous to mention here, the record is not one of complete success. Very early in the modern period, namely in 1927, Dirac took the decisive steps toward the development of a quantum theory of the electromagnetic field and had a number of significant successes with the theory as he developed it. Heisenberg and W. Pauli extended the theory, and many others have worked on it.

This theory, or rather this family of theories, in various forms, however, suffers from a fatal defect that many of the important problems of physics have no solution. When the solution is carried out, they lead to divergent integrals that give infinity for a formal answer to a problem that ought to have a finite solution. A large amount of study has gone into efforts to remove this difficulty but with little success. Therefore the quantum theory of the electromagnetic field remains today in an unsatisfactory state. Probably the difficulties can be overcome only by some radical revision of the fundamental ideas that is as revolutionary in its nature as the ideas of the present theory seemed when they were first developed in 1927.

The past half-century has been an exciting period of enormous fruitfulness in the development of physics and chemistry. Today a greater effort, measured both by adequacy of the equipment and numbers of well-trained men, is going into the investigation of the fundamental nature of matter than ever before in the world's history. We may expect therefore that the next 50 years will bring a development of our knowledge and our ideas that is even greater than has occurred in the first half of the present century. If this happens, the physics of the year 2000 will be as strange and unforeseeable by us today as the physics of today would have seemed to the physicists of 1900.

Some scientists regard an interest in the history of their subject as mere antiquarianism, and it may be that the very remote past consists largely of mistakes to be avoided. But it deserves to be remembered that the history of any scientific discipline intimately determines the current modes of investigation. The frames of reference which appear eligible at any given epoch, the instruments accepted as respectable, and the types of "fact" taken to have evidential value are historically conditioned. To pretend otherwise is to claim for human reason, as manifested in scientific progress, a universality and fixity it has never manifested.—MAX BLACK, "The Definition of Scientific Method," *Science and Civilization*, Edited by Robert C. Stauffer (The University of Wisconsin Press, Madison, 1949).