besides amino acids are being investigated, for they may play an important role in tumor formation.

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The Einthoven Triangle: An Observation Regarding the Validity of the Originally Proposed Triangular Representation of the Human Body[1]

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The Einthoven triangle, either by direct statement or implication, is usually treated as an equilateral triangular frame of reference inscribed within a homogeneous volume conductor in the form of (*a*) a plane circular slab or (*b*) a sphere (*1*). In some analyses the frame of reference is considered to be located within a homogeneous volume conductor of infinite dimensions (*1*). The potential difference between any two apices of the triangle may be regarded as the projection of an electrical vector, the so-called manifest potential, on a line joining the two apices. It may be shown that when the source of potentials in the volume conductor is an electric doublet situated in the plane of the frame of reference at its center, the manifest potential and the axis of the doublet are codirectional. This is of some importance, since the more nearly the human body approximates the conditions of idealization, the more nearly will manifest potentials, plotted from electrocardiographic potentials or recorded by means of the vectorcardiograph, represent *true* electrical vectors of the heart.

In contradistinction to the electrical models described above, the idealization originally proposed by Einthoven (*2*) consists of a thin slab of homogeneous volume conductor limited by an equilateral triangular boundary. The heart is represented by a centrally located doublet. Whether or not such a model is superior to the more commonly employed forms, it is of interest to ascertain by rigorous analytic methods whether the manifest potential calculated from the apex potentials of the Einthoven model also indicates accurately the direction of the axis of the doublet. We have been able to demonstrate both theoretically and experimentally that such an identity of orientation does exist.

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The theoretical analysis was accomplished by converting the well-known case of potential distribution due to a centric doublet within a circle, by means of a Schwarz-Christoffel transformation, into the obscure situation of a centric doublet within an equilateral triangle. The circular case (Fig. 1) is that of a circle

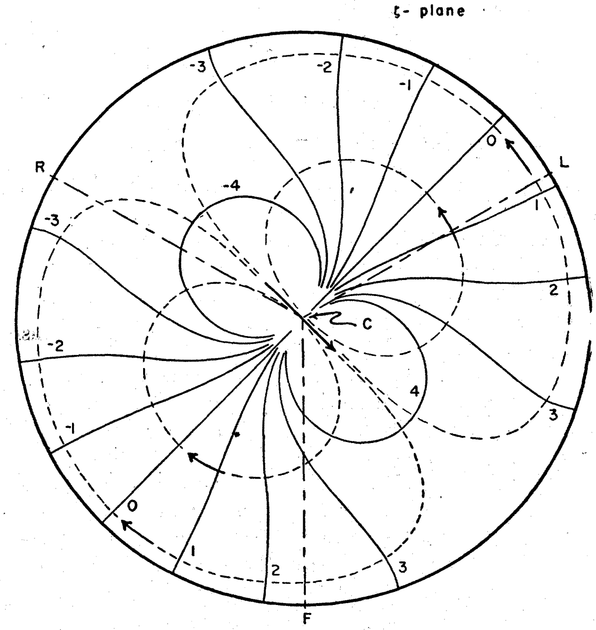

Fig. 1. Centric doublet in a circular, electrically homogeneous slab. Solid curves are isopotential lines; dashed curves are streamlines. Plotted on the basis of $V + iS = Me^{i\alpha}(1/\zeta + \zeta/R^2)$ where V is the potential function, S is the streamline function, M is the "strength" of the doublet, and α is the direction of its axis, ζ is a complex number representing any point in the plane, and R is the radius of the circle. In this figure $\alpha = -45°$. (The equation may be expressed in polar coordinates (r, θ) as a pair of conjugate functions, $V = M(1/r + r/R^2)\cos(\theta - \alpha)$ and $S = M(r/R^2 - 1/r)\sin(\theta - \alpha)$. These functions satisfy the Laplace equation $\nabla^2 V = \nabla^2 S = 0$. The boundary condition, $\delta V/\delta n = 0$, where n is the unit normal at the boundary, is also satisfied. These properties are not altered by the Schwarz-Christoffel transformation described in the text [*3*]).

of unit radius in the complex ζ-plane with three circumferential points equidistant from each other, $L(\sqrt{3}/2, i/2)$, $R(\sqrt{3}/2, i/2)$, and $F(0, -i)$. By substitution in an appropriate formula (*3*), a mapping function is determined which transforms the circle into an equilateral triangle in the complex *w*-plane, the coordinates of *L, R,* and *F* remaining unchanged. The mapping function is defined by the relation

$$dw = K(\zeta^3 - i)^{-\frac{2}{3}} d\zeta \qquad (1)$$

where K is a complex constant arbitrarily chosen so that the points *L, R,* and *F* will not be rotated by the transformation, or the distance between them altered, and $i = \sqrt{-1}$. Eq (1) can be solved for w by a binomial expansion of the radical expression followed by term-by-term integration. However, the desired information can be obtained without the necessity of resorting to such lengthy manipulations: The radii *CL, CR,* and *CF* can be expressed as

$$\left.\begin{aligned} CL&: \zeta = e^{i\frac{\pi}{6}}\lambda \\ CR&: \zeta = e^{i\frac{5\pi}{6}}\lambda \\ CF&: \zeta = e^{i\frac{3\pi}{2}}\lambda \end{aligned}\right\} \qquad (2)$$

where e = the base of natural logarithms and λ is a real parametric variable equal to $|\zeta|$. Substitution of Eq (2) in Eq (1) yields

$$\left.\begin{aligned} CL&: dw = |K|(\lambda^3-1)^{-\frac{2}{3}}\, e^{i\frac{\pi}{6}}\, d\lambda \\ CR&: dw = |K|(\lambda^3-1)^{-\frac{2}{3}}\, e^{i\frac{5\pi}{6}}\, d\lambda \\ CF&: dw = |K|(\lambda^3-1)^{-\frac{2}{3}}\, e^{i\frac{3\pi}{2}}\, d\lambda \end{aligned}\right\} \qquad (3)$$

the real value of the radical being intended in each case.

Certain conclusions are warranted from an inspection of Eq (3): (*a*) the radii in question remain rectilinear after the transformation, (*b*) the 120° angular separation between them is not altered by the transformation, and (*c*) the transformed radii are all of equal length. Therefore we may conclude that the point C remains centric in the triangle and that the transformation does not alter the angular relation of any line, as it passes through the center, with respect to the points L, R, and F. Since Schwarzian transformations do not alter the topology of isopotential distributions, and the present transformation in particular does not translocate the electric doublet or rotate its axis, it is evident that the manifest potential calculated from the potentials at the apices of the triangle will have the same direction as the axis of the doublet.

The transformation was demonstrated experimentally by preparing an equilateral triangular model 32.5 cm on a side (Fig. 2) from a sheet of conducting material of intermediate resistivity.[2] A centric doublet consisting of two circular poles 2.5 mm in diameter and separated by a distance of 15 mm between centers was drawn on the model with silver ink. The axis of the doublet was inclined at 45° to the horizontal axis of the model. The poles were energized with an alternating current of approximately 125 c/sec. Isopotential lines were mapped out with an exploring electrode by a null technique in which the model constituted two arms of an impedance bridge. A high sensitivity vacuum tube voltmeter was employed as an indicating device. Since no significant amount of phase shift occurred, the mapping procedure was accomplished with considerable precision.

The relative potentials at the apices of the model were measured to three significant figures. The manifest potential calculated from these values was directed 44.9° clockwise to the horizontal axis. Because the deviation between this value and the anticipated value of 45° was so small, the experiment appeared to confirm the theory that in the Einthoven model the manifest potential accurately indicates the direction of the axis of the doublet.

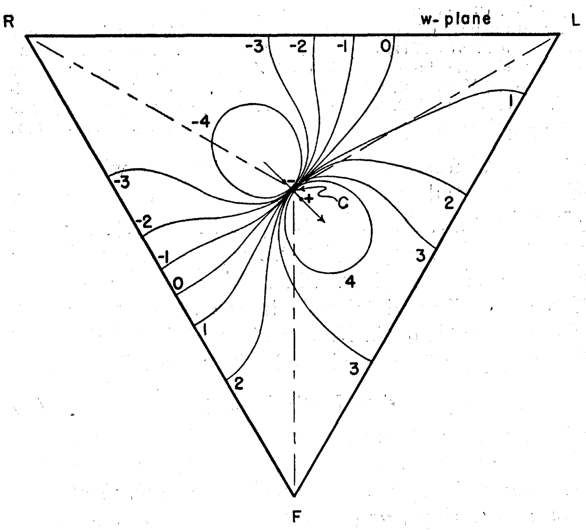

FIG. 2. Isopotential distribution in an Einthoven model of the human body, experimentally determined. The direction of the axis of the doublet is the same as the theoretical case in Fig. 1. Note that the topology of the isopotential distribution in the two figures appears to be the same to the extent that the point R remains in the area bounded by the -3 and -4 lines, etc.

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[2] Type 1 Teledeltos paper, General Electric Company.

Surface Activity of Naturally Occurring Emulsifiers

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Recently the addition of emulsifiers or other surface-active agents to foods has led to a discussion of the possible effects of surface activity on normal digestive processes. Before the effects of added emulsifiers can be properly assessed it is necessary to know the level of surface activity resulting from the emulsifiers naturally present. The surface activity of an emulsifier[1] is measured by the change in the tension at an oil-water phase boundary as the concentration of that emulsifier is changed. As far as is known, no systematic study has been made of the boundary tension relationships in systems resembling those appearing during the digestion of fats. In particular, no study has been made of the effects of monoglycerides, bile salts, or fatty acids on oil-water boundary tensions in such systems. Research in this latter field has been started in these laboratories, and the first results of the study are reported here.

[1] The surface activity of any material is measured by the extent to which it is adsorbed at a phase boundary. Highly surface-active materials are strongly adsorbed.