node as in the previously described experiments, but since these plants have no leaf sheaths, the graft was supported with waxed paper tubes slipped over the stump and scion, or with bamboo splints, which served to hold the broken edges in contact until union occurred. This suggests that other monocotyledonous species may also be grafted, even though they may lack an intercalary meristem.

It is doubtful that these techniques will soon find practical application, unless a greater percentage of success can be achieved, but they should be useful for certain types of investigation.

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Tables for Use in Fourfold Contingency Tests[1]

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Statistical tests are becoming more and more commonly used by professional research workers, technologists, and "occasional investigators" such as medical practitioners. Many of these workers, however, find the arithmetic irksome, and, when dealing with small samples, they are perturbed by the possibility that the simple familiar tests may be misleading. For some tests both these problems can be solved by tables that demand little or no computation by the user. The possible objection to such tables, that they will tend to reduce the investigator to a statistical automaton, does not, we think, apply to tables more than to tests that require calculation. Repetition of arithmetic does not increase insight into the meaning of tests, and reduction of arithmetic may allow emphasis to be placed where it should always be placed, on sampling and inference.

In order to solve the two problems reliability of verdicts and reduction of arithmetic, tables for use in contingency tests and in the estimation of binomial confidence limits were published in an article on statistical methods for medical research workers by the National Research Council of Canada in 1948 (*1*). That article is now out of print, but copies are still requested by workers in many branches of applied science. Revision and extension of the tables have therefore been undertaken, and the first two of the new series are presented here.

Purpose. These two tables are primarily for the comparison of equal samples, with individuals classified as A and not-A, and arranged in a fourfold contingency table; for instance: animals wounded by the same method and randomly allocated to two diets for comparison of fatality rates; two successive differential blood counts from the same patient to show a change in the neutrophil leukocyte percentage; samples of an industrial product made by two slightly different machines and examined for proportions defective; samples of tagged fish or birds liberated in different localities or under different conditions, for comparison of proportions subsequently recovered.

In each instance, if the investigator wishes to make allowance solely for random sampling (chance) variation he will ordinarily apply the chi-square contingency test, introducing for greater accuracy Yates' correction for continuity; or he may use an equivalent form of the standard error of the difference between two proportions. Owing to the smallness of samples or skewness of distributions, one or more of the expected values in the χ^2 calculation may be small, and the smaller the expected value the less dependable is the χ^2 test, even with Yates' correction. The investigator can apply certain empirical rules (*2*) to determine the safety of the test, and if it is unsafe he can do a further calculation and use Table VIII of Fisher and Yates (*3*). If that is insufficient he can calculate the exact probabilities (Fisher [*4*], Sec. 21.02).[2]

Our tables can be substituted for all these methods, even the initial χ^2 calculation, when the samples are equal and when, as is usually the case, the investigator requires only an assessment of significance at the conventional 5% and 1% levels—i.e., when the standards are $P = 0.05$ and $P = 0.01$, where P is the probability for χ^2 or the corresponding (two-tailed) exact probability.

Method of using the tables. Let us imagine two samples, V and W, each containing 30 individuals. V is composed of 17 Xs and 13 Ys; W, 20 Xs and 10 Ys. For ease in entering our tables, we form a contingency table, with the order of the samples changed, thus:

Sample	Y(As)	X(not-As)	Total (N)
(1) W	10	20	**30**
(2) V	13	17	30

The W sample, having the greater discrepancy between X and Y, is placed in the upper line and becomes Sample (1); and in that sample the smaller value, 10,

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[2] Sometimes this method is incorrectly referred to as "the exact chi-square" method. The distinction can be illustrated in an elementary way by a sampling experiment (*2*).

TABLE 1

MINIMUM CONTRASTS REQUIRED IN FOURFOLD CONTINGENCY TABLES FOR SIGNIFICANCE AT THE 5% LEVEL

N	As in Sample (1)/As in Sample (2)
4	0/4 1/– 2/–
5	0/4 1/5 2/– 3/–
6	0/5 1/6 2/– 3/–
7	0/5 1/6 2/7 3/–
8	0/5 1/6 2/7 3/8 4/–
9	0/5 1/6 2/8 3/8 4/9
10	0/5 1/7 2/8 3/9 4/10 5/10
11	0/5 1/7 2/8 3/9 4/10 5/11
12	0/5 1/7 2/8 3/9 4/10 5/11 6/12
13	0/5 1/7 2/8 3/9 4/10 5/11 6/12
14	0/5 1/7 2/8 3/10 4/11 5/12 6/12 7/13
15	0/5 1/7 2/9 3/10 4/11 5/12 6/13 7/14
16	0/5 1/7 2/9 3/10 4/11 5/12 6/13 7/14 8/15
17	0/5 1/7 2/9 3/10 4/11 5/12 6/13 7/14 8/15
18	0/5 1/7 2/9 3/10 4/11 5/12 6/13 7/14 8/15 9/16
19	0/5 1/7 2/9 3/10 4/11 5/12 6/14 7/14 8/15 9/16
20	0/5 1/7 2/9 3/10 4/11 5/13 6/14 7/15 8/16 9/16 10/17
30	0/6 1/8 2/9 3/11 4/12 5/13 6/15 7/16 8/17 9/18 10/19 15/24
40	0/6 1/8 2/9 3/11 4/12 5/14 6/15 7/16 8/18 9/19 10/20 20/30
50	0/6 1/8 2/10 3/11 4/13 5/14 6/15 7/17 8/18 9/19 10/20 11/22 25/36
60	0/6 1/8 2/10 3/11 4/13 5/14 6/16 7/17 8/18 9/20 10/21 11/22 12/23 13/24 14/26 15/27 30/42
70	0/6 1/8 2/10 3/11 4/13 5/14 6/16 7/17 8/18 9/20 10/21 11/22 12/23 13/25 18/30 19/32 20/33 35/48
80	0/6 1/8 2/10 3/11 4/13 5/14 6/16 7/17 8/19 9/20 10/21 11/22 12/24 13/25 14/26 15/27 16/29 23/36 24/38 40/54
90	0/6 1/8 2/10 3/11 4/13 5/14 6/16 7/17 8/19 9/20 10/21 11/23 12/24 13/25 14/26 15/28 20/33 21/35 31/45 32/47 44/59 45/59
100	0/6 1/8 2/10 3/11 4/13 5/15 6/16 7/17 8/19 9/20 10/21 11/23 12/24 13/25 14/27 18/31 19/33 25/39 26/41 50/65
150	0/6 1/8 2/10 3/12 4/13 5/15 6/16 7/18 8/19 9/20 10/22 11/23 12/24 13/26 14/27 15/28 16/30 19/33 20/35 25/40 26/42 32/48 33/50 41/58 42/60 75/93
200	0/6 1/8 2/10 3/12 4/13 5/15 6/16 7/18 8/19 9/21 10/22 11/23 12/25 13/26 14/27 15/29 18/32 19/34 22/37 23/39 27/43 28/45 33/50 34/52 41/59 42/61 51/70 52/72 65/85 66/87 100/121
300	0/6 1/8 2/10 3/12 4/13 5/15 6/16 7/18 8/19 9/21 10/22 11/24 12/25 13/26 14/28 15/29 16/30 17/31 18/33 19/34 20/35 21/37 24/40 25/42 29/46 30/48 35/53 36/55 41/60 42/62 48/68 49/70 56/77 57/79 66/88 67/90 78/101 79/103 95/119 96/121 150/175
400	0/6 1/8 2/10 3/12 4/13 5/15 6/17 7/18 8/19 9/21 10/22 11/24 12/25 13/26 14/28 15/29 16/30 17/32 20/35 21/37 24/40 25/42 28/45 29/47 33/51 34/53 38/57 39/59 44/64 45/66 51/72 52/74 58/80 59/82 67/90 68/92 76/100 77/102 87/112 88/114 100/126 101/128 117/144 118/146 141/169 142/171 200/229
500	0/6 1/8 2/10 3/12 4/13 5/15 6/17 7/18 8/19 9/21 10/22 11/24 12/25 13/26 14/28 15/29 16/30 17/32 18/33 19/34 20/36 23/39 24/41 27/44 28/46 32/50 33/52 37/56 38/58 42/62 43/64 48/69 49/71 55/77 56/79 62/85 63/87 70/94 71/96 79/104 80/106 89/115 90/117 100/127 101/129 113/141 114/143 128/157 129/159 147/177 148/179 172/203 173/205 250/282

is placed on the left and classified as "*A*." We now take from this table the pair of figures 10/13 and enter Table 1 at $N = 30$. There we find 10/19, which means that, corresponding to 10 *A*s in Sample (1), we require in Sample (2) at least 19 *A*s for significance at the 5% level. Our observed difference, therefore, is not significant.

If, instead of 10/13, the observed contrast had been 10/20, the difference would have been significant at the 5% level; but Table 2 shows that it would not have been significant at the 1% level, which requires at least 21 *A*s in Sample (2) for contrast with 10 in Sample (1). (Entries such as "2/–" at $N = 4$ in Table 1 indicate that, however much the number of *A*s in Sample (2) was increased, even as far as N itself, the contrast could not reach the level of significance. For N less than 4, no entries are given, because with samples of 3 or less the 5% level cannot be reached. Likewise, in Table 2 no entries are given for N less than 5.)

Interpolation. To save space, many entries have been omitted, but they are easily supplied. Thus, at $N = 30$ in Table 1 there is a gap between 10/19 and 15/24, but it will be noted that the difference between 10 and 19 is 9, the same as the difference between 15 and 24. The missing items are therefore: 11/20, 12/21, 13/22, and 14/23.

Interpolation between values of N will seldom cause serious doubt. If two samples, each with 320 individuals, contained, respectively, 92 and 117 *A*s, we should find from Table 1 that the contrast for $N = 300$ is 92/116, and for $N = 400$, it is 92/118. We should conclude that the difference between our samples was in the neighborhood of significance at the 5% level, with P more likely to be less than 0.05 rather than greater, because 320 is nearer to 300 than to 400. This would be correct, because χ^2 (with Yates' correction) for the observed samples is 4.092, which Table VIII of Fisher and Yates shows to be significant.

Again to save space, the values of A in Sample (1) are not carried beyond half the sample size. Let us consider, therefore, samples with $N = 190$. One contains 80 As and 110 not-As; the other, 100 As and 90 not-As. For $N = 200$ the minimum contrast in Table 1 is found by interpolation between 66/87 and 100/121. In both of these the difference is 21; therefore, the required contrast is 80/101, which is greater than the observed contrast, 80/100. This might suffice for our purpose, for it would indicate that the observed difference was in the neighborhood of significance. We can, however, obtain a closer estimate by the following procedure, which, although at first sight apparently somewhat complicated, soon becomes automatic.

Looking at $N = 150$, we find that the last entry is 75/93, but we require a value of the form $80/x$, where

TABLE 2

Minimum Contrasts Required in Fourfold Contingency Tables for Significance at the 1% Level

N	As in Sample (1)/As in Sample (2)
5	0/5 1/– 2/–
6	0/6 1/– 2/– 3/–
7	0/6 1/7 2/– 3/–
8	0/6 1/8 2/8 3/– 4/–
9	0/6 1/8 2/9 3/9 4/–
10	0/7 1/8 2/9 3/10 4/– 5/–
11	0/7 1/8 2/9 3/10 4/11 5/–
12	0/7 1/8 2/10 3/11 4/11 5/12 6/–
13	0/7 1/9 5/13 6/13
14	0/7 1/9 6/14 7/14
15	0/7 1/9 7/15
16	0/7 1/9 2/10 3/12 4/13 5/14 6/14 8/16
17	0/7 1/9 2/11 7/16 8/16
18	0/7 1/9 2/11 8/17 9/17
19	0/7 1/9 2/11 9/18
20	0/7 1/9 2/11 4/13 5/15 6/16 7/16 10/19
30	0/8 1/10 2/12 3/13 4/15 10/21 15/26
40	0/8 1/10 2/12 3/14 4/15 5/17 8/20 9/22 19/32 20/32
50	0/8 1/10 2/12 3/14 4/15 5/17 6/18 7/20 9/22 10/24 25/39
60	0/8 1/10 2/12 3/14 4/16 5/17 6/19 8/21 9/23 11/25 12/27 19/34 20/36 24/40 25/41 26/41 30/45
70	0/8 1/10 2/12 3/14 4/16 5/17 6/19 7/20 8/22 10/24 11/26 14/29 15/31 21/37 22/39 32/49 33/49 34/50 35/51
80	0/8 1/10 2/12 3/14 4/16 5/18 6/19 7/21 9/23 10/25 12/27 13/29 16/32 17/34 24/41 25/43 38/56 39/56 40/57
90	0/8 1/10 2/12 3/14 4/16 5/18 6/19 7/21 8/22 9/24 11/26 12/28 15/31 16/33 19/36 20/38 28/46 29/48 43/62 44/62 45/63
100	0/8 1/10 2/13 3/14 4/16 5/18 6/19 7/21 8/22 9/24 10/25 11/27 14/30 15/32 18/35 19/37 23/41 24/43 33/52 34/54 47/67 48/67 49/68 50/69
150	0/8 1/11 2/13 3/15 4/16 5/18 6/20 7/21 8/23 9/24 10/26 11/27 12/29 14/31 15/33 17/35 18/37 21/40 22/42 26/46 27/48 31/52 32/54 39/61 40/63 51/74 52/76 75/99
200	0/8 1/11 2/13 3/15 4/16 5/18 6/20 7/21 8/23 9/24 10/26 11/27 12/29 13/30 14/32 16/34 17/36 19/38 20/40 23/43 24/45 26/47 27/49 31/53 32/55 36/59 37/61 43/67 44/69 51/76 52/78 63/89 64/91 100/127
300	0/8 1/11 2/13 3/15 4/17 5/18 6/20 7/22 8/23 9/25 10/26 11/28 12/29 13/31 15/33 16/35 17/36 18/38 20/40 21/42 23/44 24/46 27/49 28/51 31/54 32/56 35/59 36/61 40/65 41/67 45/71 46/73 51/78 52/80 58/86 59/88 66/95 67/97 76/106 77/108 88/119 89/121 107/139 108/141 150/183
400	0/8 1/11 2/13 3/15 4/17 5/18 6/20 7/22 8/23 9/25 10/26 11/28 12/29 13/31 14/32 15/34 17/36 18/38 19/39 20/41 22/43 23/45 26/48 27/50 29/52 30/54 33/57 34/59 37/62 38/64 41/67 42/69 46/73 47/75 52/80 53/82 57/86 58/88 64/94 65/96 71/102 72/104 79/111 80/113 88/121 89/123 98/132 99/134 111/146 112/148 127/163 128/165 152/189 153/191 200/238
500	0/8 1/11 2/13 3/15 4/17 5/18 6/20 7/22 8/24 9/25 10/27 11/28 12/30 14/32 15/34 16/35 17/37 19/39 20/41 22/43 23/45 25/47 26/49 28/51 29/53 32/56 33/58 35/60 36/62 40/66 41/68 44/71 45/73 49/77 50/79 54/83 55/85 59/89 60/91 65/96 66/98 72/104 73/106 79/112 80/114 86/120 87/122 95/130 96/132 104/140 105/142 115/152 116/154 127/165 128/167 141/180 142/182 159/199 160/201 184/225 185/227 250/292

x is greater than 80. We therefore visualize a fourfold table of the form:

80	70	150
> 80	< 70	150

Here the lower line contains the greater discrepancy, and so, rearranging the samples as in our first example, we look under $N = 150$ for a contrast of the form $< 70/70$. Obviously it must lie between the last two entries, and the difference there—e.g., in 42/60—is 18. Therefore the required contrast is 52/70. The second line of the above fourfold table thus becomes 98; 52; 150. Summarizing, we have for $N = 150$: 80/98; for $N = 200$: 80/101. Interpolating linearly, we should estimate that for 80/100 the corresponding value of N should be about 184. Our actual N is 190;

therefore we should doubt if the contrast 80/100 would be great enough for significance. The exact probability, P, derived from the observed pair of samples is 0.0508. Therefore our conclusion that the difference did not quite reach the 5% level of significance would be correct. (Anyone who expects to use our tables frequently will find it helpful not only to fill in the gaps, but to write out the second half of the series for each value of N by the method just indicated.)

Application to unequal samples. For samples that are almost equal the tables are clearly of use; but even with more gross inequality they can give adequate verdicts regarding significance in two types of case:

1) When the smaller sample is enlarged to the size of the larger (with the original proportions of As and not-As unchanged) and Table 1 shows that the difference is not significant at the 5% level, it must be even farther from the significance level in the original samples—i.e., P must be even farther above 0.05. Thus, if a sample of 20 contains 5 As and a sample of 40 contains 15 As, we can imagine that the smaller sample is enlarged to contain 40 individuals of which 10 are As. With 10/15 we enter Table 1 at $N=40$ and find that the minimum contrast necessary for significance is 10/20. The difference in the original samples is therefore not significant.

This method was applied to the data from an experiment in which 19 rabbits with a certain skin tumor had been treated by injection of a tumor extract. Eight rabbits with the same tumor had not been injected, and the results were:

	Tumors disappeared	Tumors persisted	Total
Injected	3	16	19
Not injected	0	8	8

Since it was known that spontaneous disappearance was possible, it was asked: Is there a significant difference between the two groups in the incidence of disappearance of tumors? Table 1 shows that, even if the second sample contained 19 rabbits instead of 8, the contrast 0/3 under $N=19$ is far from significant at the 5% level.

2) When the larger sample is reduced to the size of the smaller (again with the original proportions unchanged) and the tables show that the difference is significant, it must a fortiori be significant in the original samples—i.e., P must be still farther below 0.05 (or 0.01).

It is of interest to apply this rule to the data on which Fisher (*4*) demonstrates the exact test. Among 13 criminals who were monozygotic twins, 10 had twin brothers or sisters who had been convicted. Among 17 criminals who were dizygotic twins, only 2 had twin brothers or sisters with records of conviction. In tabular form we have:

	Convicted	Not convicted	Total
Monozygotic	10	3	13
Dizygotic	2	15	17

To apply our tables by reducing 17 to 13, we should reduce 2 and 15 each by about one quarter of its original value; but here even this is unnecessary, for on entering Table 2 at $N=13$ we find that 2/10 is significant at the 1% level. Therefore, for the actual samples, P must be much less than 0.01. The exact probability is in fact less than 0.001.

It is possible that, when a difference is on the verge of significance and there is considerable disproportion between the As and not-As in one or both samples, these two methods of approximate assessment might lead one astray; and so, where the verdict from them is not quite obvious, the full analysis (χ^2 or the exact probability test, as required) should be used. Even in these doubtful cases, however, the tables will enable one to decide whether calculation is necessary to reach a verdict.

Information on required sample sizes. The tables help, also, to indicate sample sizes that may be required to establish significance when the observed difference is not significant, but we assume, nevertheless, that a real (population) difference exists. With samples of 20 containing, respectively, 4 As and 7 As, the difference is far from significant at the 5% level. If, on increasing the sample size, we found that the proportions remained the same as in the samples of 20, we should need two samples of about 100 to give a significant difference, for the contrast then would be 20/35, and at $N=100$ the contrast 20/34 is sufficient.

Preparation and reliability of the tables. For N up to 20, the entries, obtained from exact probabilities, have been extracted from the tables published previously (*1*). For samples of 30 onward, χ^2 with Yates' correction was commonly used, and its significance was assessed by Table VIII of Fisher and Yates (about 1850 χ^2 values); but where there was any doubt the exact probabilities were calculated (300 computations). In certain parts of the series it was found that the differences between adjacent entries, as exemplified above, were so constant that χ^2 (or probability) calculations were required only at intervals. (A large number of the entries omitted from the tables were, however, actually calculated, and interpolation can be considered safe.)

When exact probabilities were obtained we were impressed by the accuracy of the Fisher and Yates table. For this reason and also because of the methods, direct and indirect, employed in checking the computations, we believe that incorrect assessments in our tables must be rare, and, as they will be borderline cases, the resulting error of judgment will be negligible.

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