Cycles in Economics and Nature

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THE EXTREME COMPLEXITY OF ECONOMIC FACTS appears to have concealed or obscured the possibility that the economic relations of human beings may not be wholly *sui generis*. Civilization has resulted in the sublimation of man's wants and in an elaboration and proliferation of his equipment, but it has not yet freed him from his fundamental needs as a living being. Basic principles developed in biometrics may be just as significant in econometrics. It is the purpose here to examine those two sciences, to see whether the cycles that have ruptured economies and generated bloody conflicts may represent merely the natural course of growth revealed in the relations among species.

For this purpose let us take two species, y and v, in loose mutualistic association with each other. Species v has, within the range of our study, an inexhaustible source of food. It prefers to live with y because of y's superior protective equipment. Species y, on the other hand, is greatly aided by v's mobility in reaching the sources of food. The rate of population increase for y is E, assuming maximum cooperation from v; v's rate of increase, however, is k, and $k < E$. Because of y's greater rate of increase and v's ample supply of food, the limiting effect of environment on v is equal to zero. In any case, for the purpose of simplicity at this point we shall assume that it is zero.

From the logistic, placing the limiting coefficient equal to zero, it follows that:

$$v = v_0 e^{kt} \tag{1}$$

and, except for the effect of a shortage of v,

$$y = y_0 e^{Et}. \tag{2}$$

Beginning at a point of equilibrium, $v_0 = y_0$, it follows, since $k < E$, that beyond that point $v < y$.

Here we must note an additional but not uncommon factor, that of time lag. Partly because of y's period of gestation, its rate of increase is not immediately affected by a shortage in reproduction of v. The lapse of time involved in those factors is c, a period which, for all practical purposes, is constant. After c time, however, the y individuals begin to attack each other in the competition for v's cooperation, and their fertility also falls off. As a result, the rate of increase of y falls to E_1, and $E_1 < k$. As a result of the same lag elements, the rate E_1 continues until a period c has elapsed after the y curve has crossed the v curve. Then the adequacy of the supply of v becomes effective, and the rate E goes into effect again.

The form of the curves is shown in Fig. 1. Let t_1 denote the value of t at the first intersection of the y and v curves, t_2, its value at the second such intersection, and so forth, as shown in the figure. It can be seen that

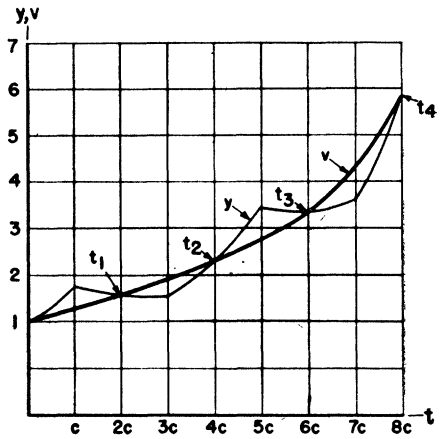

FIG. 1.

From $t = 0$ to $t = c$, $y = y_0 e^{Et}$;
From $t = c$ to $t = t_1 + c$, $y = y_{(t=c)} e^{E_1(t-c)}$;
From $t = t_1 + c$ to $t = t_2 + c$, $y = y_{(t=t_1+c)} e^{E[t-(t_1+c)]}$;

and so on, *ad infinitum*. The period of the y curve from each maximum to the following intersection with the v curve may be derived as follows:

$$\begin{aligned} y_0 e^{kt_1} &= y_0 e^{Ec} e^{E_1(t_1-c)}. \\ kt_1 &= Ec + E_1 t_1 - E_1 c, \\ &= E_1 t_1 + c(E - E_1); \\ kt_1 - E_1 t_1 &= c(E - E_1) \\ t_1 &= c\frac{E - E_1}{k - E_1}. \\ t_1 - c &= \frac{cE - cE_1 - ck + cE_1}{k - E_1}, \\ &= c\frac{E - k}{k - E_1}. \end{aligned}$$

Let us denote the value $\frac{E-k}{k-E_1}$ by d. The period of the y curve from the first maximum to the following intersection with the v curve is therefore cd. Therefore, also, $t_1 = cd + c$. In the same manner it can be shown that the period of the y curve from the first minimum to the following intersection with the v curve is c/d, and that $t_2 = 2c + cd + c/d$.

Since precisely the same relations repeat themselves from each subsequent intersection following a minimum, the time interval between maximum and crossover is always cd, and between minimum and crossover it is always c/d. There is a clear periodicity, the period of each cycle being $2c + cd + c/d$. In the special case in which $E - k = k - E_1$, so that $d = 1$, the period

is $4c$. The deviations of the y curve from the v curve in terms of their ratio then form a symmetrical series. In order to state such a deviation curve more simply, let $z = (k-E)c + (E-k)t$, where t is measured from the beginning of the particular segment. Then each complete cycle of the deviation curve from a point where y is a maximum has a downward segment according to the equation

$$\frac{y}{v} = e^{-z}, \tag{3}$$

and an upward segment according to the equation

$$\frac{y}{v} = e^{z}. \tag{4}$$

The time interval from maximum to crossover, from crossover to minimum, from minimum to crossover, and from crossover to maximum again, is in each case c, the total period of the cycle being, as above stated, $4c$.

The sharp discontinuity of the curve results from the assumed perfection of the lag period, c. If one takes the more realistic view that the impact of the shortage of v is gradual, and that not all members of y are reached by it at the same time, the curve evolves into one lending itself to sinusoidal description,

$$\frac{y}{v} = e^{(E-k)c \sin \frac{\pi}{2c} t^1} \tag{5}$$

or, in differential form,

$$\frac{dy}{dv} = \frac{k + (E-k)\frac{\pi}{2}\cos\frac{\pi}{2c}t}{k} e^{(E-k)c \sin \frac{\pi}{2c} t}, \tag{6}$$

with t measured in each case from any point $y = v$ and $dy > dv$. It must be clear, however, that Equations (5) and (6) are purely descriptive approximations and are not intended to represent the precise mathematical effect of the forces stated.

Nor, of course, is the situation as simple as Equations (3) and (4) would indicate. Some argument might be made, on the basis of Newton's third law of motion, that $k - E_1$ does tend to equal $E - k$. Considering the complexity of the actual forces, however, it would be idle to belabor that point. For example, the v curve is not wholly unaffected by the y curve. When, as a result of the shortage of v, the mortality of y increases and its fertility falls off, and this continues beyond the point of intersection with the v curve, the v species will in turn suffer because of its dependence in small or great degree on the protection of y. The effect of this is to introduce a secondary cyclicity into the v curve. Even this would not be an adequate description, however. Consideration would also have to be given to any lag factors affecting v, as a result of which the cyclicity of the two curves would be mutual and opposite, and to the extent of the dependence of y on v, and v on y.

A very loose association would produce one result, and a tighter association another. Then, again, one species might be less dependent on the other than the other is on it. In that case there would be an approach to the predator-prey relation, in which a voracious species might multiply rapidly because of an ample food supply, as a result completely consume the supply, and then starve to death for lack of food. These additional factors and variations, however, do not subserve the purpose of this study. Suffice it to say that the population effects of interdependence of species have been tested out and the oscillatory pattern in fact observed.[2]

[1] Derived from the value of $\frac{y}{v}$ at $t = c$ as a maximum.

Symbiosis in the Human Economy

As it was stated at the outset, the things we learn from biometrics may help us along with econometrics. That human beings have reached a stage of almost rigid interdependence is no longer subject to challenge. It would be difficult, moreover, to classify it as pure mutualism. There are many phases and situations in which the economic efforts of some human beings are harmful to others, where the relations, therefore, are of the predator-prey or exploitation type; and there are even cases in which the efforts of individuals or groups are harmful each to the other.

By and large, however, the relations are mutually beneficial. The essence of the situation is specialization and its necessary accompaniment, exchange. Not only is there specialization in classes of work and classes of products. There is specialization as between ownership and work. The extent to which that specialization is beneficial depends upon the extent to which concentration of ownership advances the need of the economy for capital. No clear case has yet been made that it is also beneficial from the standpoint of the quality of management. Whether the concentration of ownership, to the extent that it does not advance the need for capital, is harmful to the economy seems to depend upon other factors.

Among those other factors is the specialization of industry as between capital and consumers' goods, not that the variations in the production of capital goods would be eliminated if such specialization did not exist, but rather that such specialization concentrates the incidence of those variations so as to give greater force to the interdependence of products and thus of the groups of capital and labor employed in their production. It is simply the operation in the economy of the divide-and-conquer principle of war and politics—the law of mass action, as it is known in the physical sciences.

[2] *Principles of Animal Ecology*, by W. C. Allee, A. E. Emerson, Orlando Park, Thomas Park, and K. P. Schmidt (Philadelphia and London: Saunders, 326 [1949]), summarizing the experiments of G. F. Gause, A. J. Lotka, Vito Volterra, Paul Debach, and H. S. Smith. In regard to mutualism generally, see the same work, p. 710, and *Mathematical Biology*, by V. A. Kostitzin (Paris: Armand Colin., 145 [1937]). In regard to the cycle developed by the predator-prey relationship, see the precise exposition of Volterra's laws in Kostitzin, *op. cit.*, p. 131. See also "Mathematical Analysis of Growth of Mixed Populations," by C. P. Winsor, *Cold Spring Harbor Symposia on Quantitative Biology*. Cold Spring Harbor, N. Y.: Biological Laboratory, Vol. II, 181 [1934].

But that factor presents only a problem of intensity. The basic factor involved on the disadvantage side of concentration of ownership is its effect on the rate of investment. Here we return to the y and v, and E and k of the equations derived from the relations of species. In this case y would refer to total invested capital, v to the capacity of the economy to utilize capital, and E and k to the growth exponentials of those two quantities.[3] E is then the rate of net investment, as a fraction of total invested capital; and k is the rate of growth in the capacity of the economy to utilize capital, as a fraction of that capacity. Such capacity depends upon manpower, natural resources, and, as a dimension of both manpower and natural resources, the level of technology.

E is in fact distinct from k and, without regard to the fluctuations of the business cycle, may not be taken for granted. The rate of investment is in part a function of the supply of investment funds, and that supply is made up in large part of earnings. The extent to which earnings form a source of investment funds depends geometrically upon the level of individual incomes; for investment funds are not merely proportionate to income, but come more readily out of the higher brackets. The level of individual incomes, in turn, depends not only on total income; it depends in part also upon the distribution of earnings, and the distribution of earnings depends to a considerable extent upon the distribution of ownership. In summary, the higher the inequality of ownership, and the higher the level of total production, the greater is the supply of investment funds, *as a percentage of total production,* and the higher therefore is the value of E.

There is one additional point. Over a short period the inequality of earnings is a function, not only of the inequality of ownership, but of the level of total production. Although there is no evidence that on a secular basis there is any relation between the inequality of income and the level of total production, there is some evidence of such a relation in the short run, and it is an element of aggravation that must be considered.

At least this factor of the distribution of income, over both the long and the short periods, is a factor directly influencing E without corresponding effect on k. True, if there is a greater supply of investment funds, and investors as a result will take less compensation for its use, more capital can be used, so that indirectly k is also affected. But actual capital is in the main physical, and the efficiency of its use slopes rapidly either way from an optimum total of capital in use. As a result k is a delicate instrument. If there were no time gap between E and k, a continuous balance between them would no doubt obtain. But there is such a time gap—the length of the supply process, which separates investment from its fruition. We have, then, precisely the same situation here as we have in the relation of interdependent species above described. The cyclical character of production is not merely a result. It is a mathematically necessary result.

[3] The y and v thus used are the same precisely as the C and C_θ, respectively, previously used in the same connection in *Invisible Barrier,* by George T. Altman (Los Angeles: DeVorss & Co., Chap. 7 [1949]).

News and Notes

Scientists in the News

Robert J. Anderson has left the Light Metals Division of the National Production Authority, Washington, and has joined the Southwest Research Institute, San Antonio.

E. Dwight Barnett, director of Harper Hospital in Detroit since 1946, has been appointed first director of Columbia University's Institute of Administrative Medicine, which will begin active operation this month.

Otto A. Bessey has accepted appointment as professor of biochemistry and nutrition, and chairman of the department at the University of Texas Medical Branch, Galveston. Professor Bessey is serving in a similar position at the University of Illinois Medical Center, Chicago.

Francis G. Blake, wartime adviser to the President on problems of epidemic disease and former dean of the Yale Medical School, has been named civilian technical director of medical research in the Office of the Army Surgeon General. Dr. Blake will serve as expert consultant to **John R. Wood,** chairman of the Army Medical Research and Development Board, in the evaluation of current and proposed research projects conducted by Army medical installations and 81 cooperating civilian institutions. He will be responsible for ensuring close coordination between the Army's present $10,079,000 medical research effort and similar programs sponsored by the other armed forces and civilian federal agencies. Now on leave of absence from Yale, Dr. Blake retired in 1951 as chairman of the Committee on Medical Sciences of the RDB.

William F. Cassedy, Jr., has been elected president of Aircraft Radio Corporation succeeding **Lewis M. Hull,** who has become chairman of the board and remains as treasurer. Mr. Cassedy was previously vice president and general manager of Kearfott Manufacturing Co., of Newark, N. J. **Richard W. Seabury,** who has been chairman of the board of Aircraft Radio, is now chairman of the finance committee.

The National Research Council has appointed two new members to the Building Research Advisory Board: **Edmund Claxton,** director of research, Armstrong Cork Company; and **Mason C. Prichard,** special assistant, Office Chief of Engineers for Military Con-