several for which no careful treatment at the beginning graduate level has been available previously. The typography is excellent (boldface and italics are used freely), and there are numerous exercises, many of which serve to amplify and extend the text. All through the book there are informal notes, cautions, and pointed queries, which serve the double purpose of easing the sometimes heavy going and of keeping the reader alert.

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Functional analysis and semi-groups. Einar Hille. (American Mathematical Society Colloquium Publications, Vol. XXXI.) New York: American Mathematical Soc., 1948. Pp. xi + 528. $7.50.

A semigroup is an associative (but not necessarily commutative) multiplicative system. It is more general than a group because neither the existence of the unit element (and *a fortiori* of the inverse element) nor the rule of cancellation (rule of division) is assumed. This book is devoted to the study of such systems and their representation by linear operators in a Hilbert or a Banach space (for example, a one-parameter semigroup of operators T_t, $t > 0$, which satisfy $T_s T_t = T_{s+t}$ for any positive s and t).

Investigation of semigroups and their representations is not only interesting as a mathematical theory by itself (Laplace transforms and binomial series, Fourier series and integrals, summability and Tauberian theorems, operator calculus and spectral theory), but it is also important in view of its application to probability and statistical mechanics where the notion of irreversibility and tendency toward equilibrium comes into question (ergodic theory, Markoff processes, stochastic processes, diffusion problem, conduction of heat, and related partial differential equations).

The book consists of three parts and an appendix. The first part, which is a preparation for the rest of the book, is itself a self-contained, up-to-date introduction to the theory of functional spaces. Part II develops an analytical theory of general semigroups and their representations, which is applied in Part III to special cases to obtain important results concerning groups and their representations in a unified way. The book also contains many interesting results of the author which have not been published elsewhere.

The appendix is an exposition of the theory of Banach algebras (normed rings), a field which promises to become one of the main centers of mathematical interest. Here the stress is shifted to the algebraical aspects of the problem, while in the main body of the book emphasis is always placed on the analysis. Readers who have witnessed the achievements of topological and operational methods in functional analysis (differential and integral equations, theory of Hilbert and Banach spaces) will find the recent success of the algebraic method (as in the case of Tauberian theorems) very interesting. Professor Hille's book constitutes an important addition to mathematical literature, coming as it does at a moment when systematic rearrangement and clear exposition of the various results of this already large and ever-growing subject seem to be necessary for the purpose of stimulating further progress in research.

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Theory of equations. J. V. Uspensky. New York-London: McGraw-Hill, 1948. Pp. vii + 353. $4.50.

The late Prof. Uspensky's new volume will be welcomed by all who teach or use the theory of equations. It is considerably longer than the well-known texts of Profs. Dickson, Weisner, and J. M. Thomas, and presents a wealth of ideas in easily available form.

The discussion of the numerical solution of real polynomial equations is masterly, and includes valuable appendices on Routh's rule, iterative methods, Graeffe's method, and a new method for separating roots (Vincent's method). This discussion will make the book a valuable text and reference book for engineers.

On the other hand, no flavor of the modern abstract point of view is given: thus "fields" of complex numbers are nowhere defined, and matrices are treated after determinants. Moreover, the classical prescription offered for solving simultaneous linear equations by Cramer's Rule (determinants), however elegant theoretically, is notoriously ineffective. For these reasons, many teachers will wish to use the book not as a basic text but as a supplementary text or reference. It should be nearly indispensable for that purpose.

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