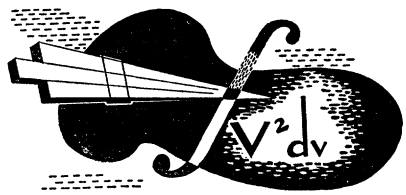
those explaining the irregularities in physiology and nutrition of animals and man.

E. R. GRAHAM and WM. A. ALBRECHT
University of Missouri



Rings and ideals. Neal H. McCoy. (Carus Mathematical Monographs of The Mathematical Association of America.) LaSalle, Ill.: Open Court Publ., 1948. Pp. xii + 216. $2.00.

This attractive little volume, well written and neatly printed, offers an excellent and almost entirely self-contained introduction to some of the most interesting topics in elementary modern algebra. Chapter I discusses the idea of a ring, with the help of some carefully thought out examples; chapter II further illustrates the same concept by a more detailed discussion of polynomial rings. Chapter III introduces the all-important notions of the ideal, of homomorphism, and of the residue class ring. After some auxiliary results in chapter IV, chapters V and VI offer an excellent, thoroughly modern, and lucid treatment, by means of Zorn's principle (without any finiteness assumptions), of the basic properties of the radical and of the Jacobson radical, and their application to the representation of rings as direct or subdirect sums. This is illustrated, in chapter VII, by a discussion of Boolean rings and of p-rings. The last two chapters (somewhat loosely connected with the rest of the book) give a concise exposition of matrices and determinants over arbitrary commutative rings (knowledge of the theory of determinants over a field being assumed), and of the basic facts concerning primary ideals and Noetherian rings.

Utmost simplicity has been reached in nearly all proofs; this, together with an easy and readable style, and a skillful use of examples, should put the book well within reach of beginners in the field, without rigor being sacrificed in any way to this object. Both the simplicity of the exposition and the generality of the results are further enhanced by the fact that the author carefully avoids bringing in any assumption before it really becomes necessary; when he does so, he takes into account the results of the latest research, including some of his own. Thus, chain-conditions are not even mentioned before the last chapter. Similarly, commutativity is introduced only when it is needed, even though the emphasis is mostly on commutative problems and methods. All this is in full agreement with the lastest developments in algebraic theory and practice, and should prove most helpful to readers who wish to acquaint themselves with such developments; further help and guidance is provided by means of a good bibliography and a brief discussion of the main source material at the end of each chapter.

The author deserves to be congratulated for this valuable addition to the literature; and the book may be warmly commended, both to newcomers to the subject and to those who have lost contact with it and may wish to bring their knowledge of its basic principles up to date.

ANDRÉ WEIL
University of Chicago

Introduction to applied mathematics. Francis D. Murnaghan. New York: John Wiley; London: Chapman & Hall, 1948. Pp. ix + 389. $5.00.

This is a time when applied mathematics is being cultivated very vigorously. Problems of gas dynamics, elasticity, atomic physics, and nonlinear mechanics, to name but a few fields, require the most powerful and advanced tools available to the mathematician, physicist, or engineer. Coincidentally, there is a need for books which will present in a clear, connected, and reasonably complete fashion the known facts about eigenvalue problems, partial differential equations, integral equations, calculus of variations, etc. Prof. Murnaghan, who has done outstanding work in applied mathematics, has drawn on his twenty years' experience in presenting such material to graduate students and has written a good and helpful introduction to the field.

The first two chapters are devoted to vectors and matrices. The treatment is vigorous and proceeds from the special to the general, a plan followed wherever possible throughout the book. Included are n-dimensional complex space and an introduction to the eigenvalue problem. Chapter 3, one of the most important, carries the discussion into function space; here the student will find orthonormal sets of functions, generalized Fourier expansions, and linear integral operators. Chapters 4 and 5 discuss curvilinear coordinates and Laplace's equation. Considerable attention is paid in the latter chapter to the useful "method of images." Chapter 6 discusses separation of variables for partial differential equations, power-series methods for ordinary differential equations, and special functions. In Chapter 7, the student will find self-adjoint linear differential operators, boundary conditions of various types, and a very complete discussion of Green's functions—including the central theorem concerning the equivalence of linear second-order boundary-value problems and Fredholm integral equations. Chapter 8 gives the theory of Fredholm integral equations and concludes with a careful discussion of Rayleigh's principle. Chapter 9 discusses the calculus of variations, with applications to dynamics. The last chapter is devoted to operational calculus, based on the unilateral Laplace transformation. The discussion is restricted to ordinary differential equations of general order and systems of such, but these are treated completely.

Altogether, the reviewer considers this a good though demanding introduction to applied mathematics. The word *introduction* is to be emphasized; in its 389 pages, the author has been forced to omit mention of several important topics, notably initial value problems. However, there are good discussions elsewhere of most of the omitted questions—while of the topics included, there are

several for which no careful treatment at the beginning graduate level has been available previously. The typography is excellent (boldface and italics are used freely), and there are numerous exercises, many of which serve to amplify and extend the text. All through the book there are informal notes, cautions, and pointed queries, which serve the double purpose of easing the sometimes heavy going and of keeping the reader alert.

A. WEINSTEIN

U. S. Naval Ordnance Laboratory

Functional analysis and semi-groups. Einar Hille. (American Mathematical Society Colloquium Publications, Vol. XXXI.) New York: American Mathematical Soc., 1948. Pp. xi + 528. $7.50.

A semigroup is an associative (but not necessarily commutative) multiplicative system. It is more general than a group because neither the existence of the unit element (and *a fortiori* of the inverse element) nor the rule of cancellation (rule of division) is assumed. This book is devoted to the study of such systems and their representation by linear operators in a Hilbert or a Banach space (for example, a one-parameter semigroup of operators T_t, $t > 0$, which satisfy $T_s T_t = T_{s+t}$ for any positive s and t).

Investigation of semigroups and their representations is not only interesting as a mathematical theory by itself (Laplace transforms and binomial series, Fourier series and integrals, summability and Tauberian theorems, operator calculus and spectral theory), but it is also important in view of its application to probability and statistical mechanics where the notion of irreversibility and tendency toward equilibrium comes into question (ergodic theory, Markoff processes, stochastic processes, diffusion problem, conduction of heat, and related partial differential equations).

The book consists of three parts and an appendix. The first part, which is a preparation for the rest of the book, is itself a self-contained, up-to-date introduction to the theory of functional spaces. Part II develops an analytical theory of general semigroups and their representations, which is applied in Part III to special cases to obtain important results concerning groups and their representations in a unified way. The book also contains many interesting results of the author which have not been published elsewhere.

The appendix is an exposition of the theory of Banach algebras (normed rings), a field which promises to become one of the main centers of mathematical interest. Here the stress is shifted to the algebraical aspects of the problem, while in the main body of the book emphasis is always placed on the analysis. Readers who have witnessed the achievements of topological and operational methods in functional analysis (differential and integral equations, theory of Hilbert and Banach spaces) will find the recent success of the algebraic method (as in the case of Tauberian theorems) very interesting. Professor Hille's book constitutes an important addition to mathematical literature, coming as it does at a moment when systematic rearrangement and clear exposition of the various results of this already large and ever-growing subject seem to be necessary for the purpose of stimulating further progress in research.

SHIZUO KAKUTANI

Institute for Advanced Study

Theory of equations. J. V. Uspensky. New York-London: McGraw-Hill, 1948. Pp. vii + 353. $4.50.

The late Prof. Uspensky's new volume will be welcomed by all who teach or use the theory of equations. It is considerably longer than the well-known texts of Profs. Dickson, Weisner, and J. M. Thomas, and presents a wealth of ideas in easily available form.

The discussion of the numerical solution of real polynomial equations is masterly, and includes valuable appendices on Routh's rule, iterative methods, Graeffe's method, and a new method for separating roots (Vincent's method). This discussion will make the book a valuable text and reference book for engineers.

On the other hand, no flavor of the modern abstract point of view is given: thus "fields" of complex numbers are nowhere defined, and matrices are treated after determinants. Moreover, the classical prescription offered for solving simultaneous linear equations by Cramer's Rule (determinants), however elegant theoretically, is notoriously ineffective. For these reasons, many teachers will wish to use the book not as a basic text but as a supplementary text or reference. It should be nearly indispensable for that purpose.

GARRETT BIRKHOFF

Harvard University

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