A New Development in Ocean Wave Research[1]

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APPLICATION OF THE PRINCIPLES of generalized harmonic analysis to oceanic wave observations is here briefly reported. This type of analysis is new to oceanography and has rarely been used in other geophysical practices (*2, 3*). In our case it was applied initially to observed oceanic wave data as a means of overcoming ambiguities and uncertainties arising from the fitting of Fourier series and from apparent misleading indications of periods from periodogram analyses of finite amounts of data.

The hypothesis of generalized harmonic analysis provides a realistic approach to the subject of oceanic wave investigations. As will be shown, the outcome of the applications far exceeded initial expectations. The results obtained identify the method as a powerful tool for this type of investigation and, provided the hypothesis is applicable to the problem, it serves two distinct purposes:

(1) To separate the dynamic and random components of oceanic wave patterns, and to evaluate the physical characteristics of each.

(2) To employ the above results in investigations of the corresponding properties of the physical generating mechanisms.

The dynamic components of ocean wave patterns are presumed to be those generated by systematic action of winds on the sea surface. On the other hand, random components result from variables not considered, such as random motions of the water and local intermittent wind action. Separation of dynamic and random effects in this sense is paralleled in electrical communication, wherein the initial signal frequency is distorted to a varying degree, and distortion of a purely random nature is called noise. On arrival, a message contains the original signal frequency plus a random noise, and the problem is to isolate the initial signal from the message. The analogy to be drawn for ocean waves is that the ocean wave record is comparable to the message, the dynamic portion of the record to the signal, and the superimposed randomness to the noise.

The oceanic wave observations subjected to analyses were obtained in the usual manner, by photographic recordings of sea surface wave heights and by automatic recordings of wave pressure variations at the sea bottom (*1*). Such observations comprise geophysical time series from which discrete values of wave heights are scaled at equidistant (1 second) time intervals. Sequences of values so obtained contain certain inherent dynamic properties, together with certain superimposed random components.

Separation of the dynamic and random components and evaluation of their cyclical and oscillatory characteristics are possible by the hypothesis of generalized harmonic analysis. In this manner we eliminate the objectionable a priori assumptions of Fourier series applications to geophysical time series, and obtain results of dynamic significance. Thus, application of the Fourier series to a finite stationary time series permits representation by an infinite number of sums of sine waves of different periods and amplitudes, whereas the correlogram of generalized harmonic analysis is, like the original series, the sum of a series of harmonics with different coefficients, all in phase. The correlogram brings to light a more realistic and more simplified picture of the physical characteristics of the sea surface pattern. The nature of this information is briefly discussed in connection with an analysis of Wave Record 53-X.

A detailed report by the authors on the method of generalized harmonic analysis, together with a rigorous mathematical treatment of the specific applications and representative examples, will be published shortly. Essentially, the method consists of analyzing the correlogram of auto and cross correlations of the data. Thus, as an example, in the series

$$u(t) = A_1 \sin(\lambda_1 t + \alpha_1) + A_2 \sin(\lambda_2 t + \alpha_2) + \ldots\ldots A_m \sin(\lambda_m t + \alpha_m),$$

the variance is

$$\mathrm{Lim}\ \frac{1}{2h}\int_{-h}^{h} u^2(t)\,dt = \tfrac{1}{2}\Sigma\,(A_j{}^2),$$

and the auto correlation function is

$$r(k) = \frac{\Sigma\{A_j{}^2 \cos(\lambda_j k)\}}{\Sigma\{H_j{}^2\}}.$$

Record 53-X (Fig. 1) designates a 20-min underwater (70′ depth) pressure record from a location one and one quarter miles south of Cuttyhunk Island, Massachusetts, beginning at 0650, September 15, 1946.

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Previous periodogram analysis of this data, by means of the wave frequency analyzer in use at this Institution (*1*), produced the result illustrated in Fig. 2. The periodogram is a complicated picture with numerous periods. This represents the customary type of oceanic wave record analysis, and in the usual interpretation, two distinct frequency bands would be defined as characterizing the data. The first, not too well represented, lies between 6.5 and 8.00 seconds, and the second, well-defined, between 10.5 and 16.0 seconds. Maximum peaking occurs at 12.5 seconds, with two additional, well defined peaks at 11.5 and 15.0 seconds. Hence, the usual conclusions drawn from this analysis would be that the sea surface roughness pattern consists of two defined spectral bands with a dominating period in the vicinity of 12.5 seconds.

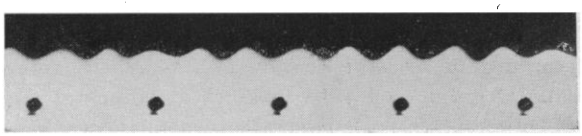

FIG. 1. Portion of Wave Record 53-X. On the original record the upper 30-sec time marks are approximately 1¾" apart.

Although conclusions like these are not readily disproved, the accepted explanations of geophysical phenomena are those that provide the most realistic and simplified hypothesis. In this case, the fallacy in the analysis appears to lie in the fact that the large number of periods shown for consideration is misleading. The mere fact that the data can be mathematically represented in this fashion does not lend significance to any identical period or group of periods. The component periods are not identifiable, and the period bands which may be construed as constituting the oceanic wave pattern do not possess a realistic physical significance. Rather, this type of analysis suggests that the large number of periods in the periodogram may be indicative of some one disturbed period in the series. Thus, it can be demonstrated by a generalized harmonic analysis of the original record (53-X) that the wave pattern is not an interference pattern composed of the numerous periods of Fig. 2, but one that is apparently controlled by a single cyclical component on which is superimposed an oscillatory component of the same frequency. Combination of the two components results in the apparent complex basic wave pattern of the record (Fig. 1). This latter, and more realistic explanation, has been derived for Record 53-X in the following manner.

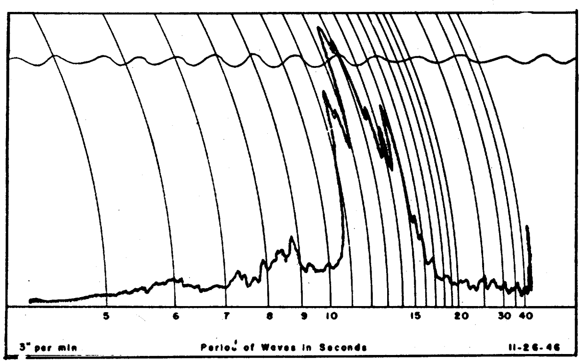

FIG. 2. Results of periodogram analysis of Wave Record 53-X, as given by the wave frequency analyzer.

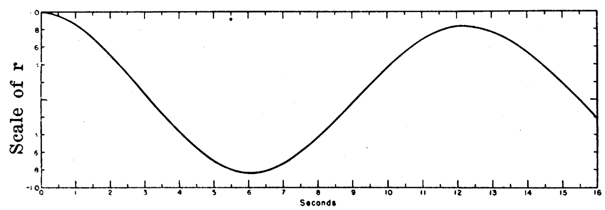

FIG. 3. Correlogram of Wave Record 53-X; 0- to 16-sec lags.

Correlograms of Wave Record 53-X for lag correlations of 0 to 16 seconds, 100 to 111 seconds, and 200 to 211 seconds are illustrated by Figs. 3 and 4. The sinusoidal symmetry of the curves and the time locations of valleys (6.13, 104.13, 202.13 seconds) and crests (12.25, 110.25, 208.25 seconds) indicate a persistent period very near to 12.25 seconds. The variance of the primary data is 0.1824 ft².

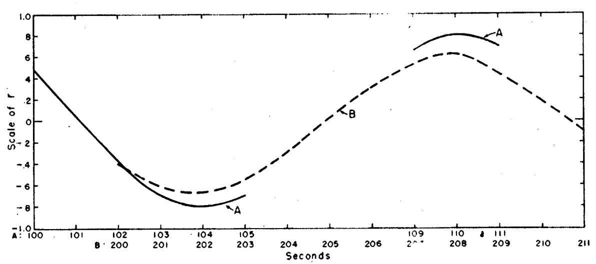

FIG. 4. Sections of correlogram of Wave Record 53-X for the 100- to 111-sec and 200- to 211-sec lags.

The correlogram damps slowly. At the 202nd lag the wave valley has an amplitude of $r_k = 0.66$, and near the 208th lag the crest amplitude is $r_k = 0.62$. The slight asymmetry appears to result from small errors in estimation of the mean value. The mean amplitude for the 16th cycle is taken as $r_k = 0.64$.

In the case where the series consists of a sine term, A sin θt, plus a random residual, we have as a theoretical value of the terminal amplitude of the correlogram

$$r_k = \frac{\text{variance cosine}}{\text{total variance}} = \frac{A^2 \cos \theta k}{2\sigma_y{}^2} .$$

Thus,

$$A = \sqrt{2}\,\sqrt{r_k}\,\sigma_y .$$

If the value of $r_k = 0.64$ and that for the variance of the primary data (0.1824 ft^2) is substituted, the theoretical amplitude (A) of the 12.25-second cyclical component in Wave Record 53-X is computed to be: $A = 0.49$ ft.

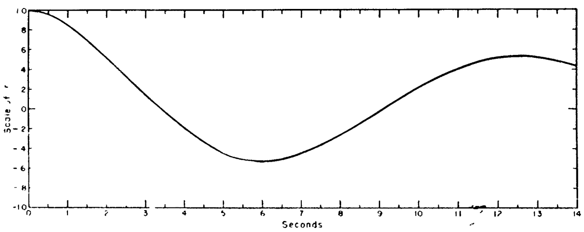

FIG. 5. Correlogram of residuals of Wave Record 53-X; 0- to 14-sec lags.

This value is in close agreement with the amplitude of 0.54 ft computed for the 12.25-second cyclical component by a least-squares fit of Fourier series to the primary data of 53-X. Subtracting this cyclical component from the primary data and computing the autocorrelation functions of the residuals, we obtain the correlogram of Fig. 5. The correlogram damps rapidly to an amplitude of $r = 0.54$ at the end of the first cycle, and thereafter continues to damp. The variance of the residuals is 0.0636 ft^2.

Substituting above values in the relation

$$\frac{\text{variance of residuals}}{\text{variance of primary data}} = 1 - r_k$$

where r_k is the terminal amplitude of the primary correlogram, we obtain $r_k = 0.656$; a value in good agreement with that of $r_k = 0.64$ for the terminal amplitude of the correlogram of the primary data (Fig. 4).

We may now proceed on the basis that, after extraction of the cyclical component from a finite amount of primary data, the residuals represent the autoregressive part of the primary data. Hence, the total or primary data may be considered to consist of a cyclical component and an oscillatory component, the latter of which is autoregressive. The correlogram of the residuals has a period of 12.25 seconds, identical with that for the primary data. From the Fourier transform of the autocorrelation coefficient, the intensity (S^2) in the periodogram of the residual series is computed. Fig. 6 illustrates the results which show that the energy of this spectrum attains its maximum value ($S^2 = 7.2$) at about 12.25 seconds.

Further numerical computation provides items of verification interest. Thus, multiplying the residual variance by the maximum intensity of the periodogram (Fig. 6) we obtain $A = 0.67$ ft as the theoretical amplitude of the oscillatory component.

The total amplitude of the sea surface oscillation (C) may now be computed as the square root of the sum of the squares of the cyclical and oscillatory component amplitudes. Thus

$$c = \sqrt{(0.54)^2 + (0.67)^2} = 0.86 \text{ ft.}$$

The mean deviation of the primary data is 0.362 ft; that for the residuals, after subtraction of the cyclical component, is 0.194 ft. Thus, the ratios of mean deviation to standard deviation for each case is: primary data = 0.84 and residuals = 0.77; or the mean deviation is approximately $\sqrt{2/\pi}$ of the standard deviation, a relationship which holds approximately for unimodal curves approaching symmetry.

Application of the hypothesis of generalized harmonic analysis permits the following geophysical interpretation of Wave Record 53-X. The wave pattern represented consists of a well-developed cyclical component (12.25-second period) on which is superimposed an oscillatory component of the same period. The former, which accounts for approximately 64 percent of the sea surface roughness, proceeds through the data as the predictable component, that is, as long as the generating mechanism remains unchanged. The oscillatory component represents the autoregressive part of the data. The cyclical component is presumed to be generated by a prominent oceanic meteorological situation, and the superimposed oscillatory component from local winds and other local disturbances which tend to change the basic wave pattern. The cyclical is the dynamic component, whereas the oscillatory component is random, although it may possess dynamic characteristics.

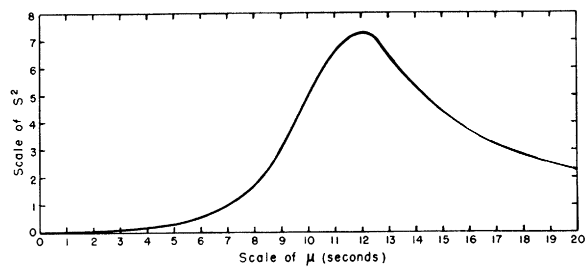

FIG. 6. Energy distribution (S^2) in periodogram of the residual services obtained from the Fourier transform of the autocorrelation coefficient.

Most of the wave records[2] analyzed to date conform to the general pattern of Record 53-X; that is, a well developed cyclical component on which is superimposed an oscillatory component, plus, possibly, other random factors. In a few instances, the cyclical component has been absent, or so weakly developed as to be indistinct. In rare cases, two distinct cyclical components were evaluated from the primary data.

[2] These records comprise both sea surface and sea bottom observations. For observing methods see reference 1.

The conclusions to be drawn from the foregoing analyses may be summarized briefly as follows:

(1) Periodogram analyses performed on oceanic wave records do not appear to give correct geophysical information. The numerous wave periods, and bands of periods, indicated by this type of analysis do not necessarily possess physical significance.

(2) Application of the hypothesis of generalized harmonic analyses to western North Atlantic wave records indicates that ocean wave patterns are not complex interference patterns resulting from combinations of many wave frequencies, but frequently consist of a single sinusoidal wave on which is superimposed an oscillatory component.

(3) The cyclical component appears to be that generated under the influence of a dominating oceanic meteorological situation, and the oscillatory component by local winds and other local disturbances tending to change the basic ocean wave pattern.

(4) Separation of the cyclical and oscillatory components and determination of their physical properties is possible by generalized harmonic analysis of finite portions of the primary data.

(5) In the case of Record 53-X, the wave pattern is indicated to be composed of a cyclical and an oscillatory component. The former proceeds throughout the data in regular sinusoidal fashion with an amplitude of 0.54 ft and accounts for approximately 64 percent of the variability in the wave pattern. The oscillatory component, with a theoretical amplitude of 0.67 ft is autoregressive, and strongly damped.

References

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Comparative Electron and Light Microscopic Investigations of Tactoid Structures in V_2O_5-sols

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THE SPINDLE–SHAPED TACTOIDS in Fig. 1, whose biological interest was recognized at an early date (*2*), have been investigated repeatedly with the polarizing microscope (*15*) and by X-rays (*1*). Electron microscopy has not been applied extensively to tactoids, although it has been used by various authors to study aqueous dispersions of tobacco mosaic virus (*10*) and of V_2O_5 (*3*), in both of which tactoids do form under suitable conditions.

In this investigation we studied the development and transformations with time of tactoid structures in a 2% V_2O_5-sol prepared according to the recipe of Zocher and Jacobsohn (*15*). Immediately after preparation a solution of this concentration shows a fine particle dispersion in the electron microscope, without structure or apparent orientation (Fig. 2). These colloidal particles have a mean diameter less than 50 A. In the polarizing microscope a solution of this concentration is optically empty.

In a liquid medium, structures of low rigidity are liable to deform and alter during the drying of speci-

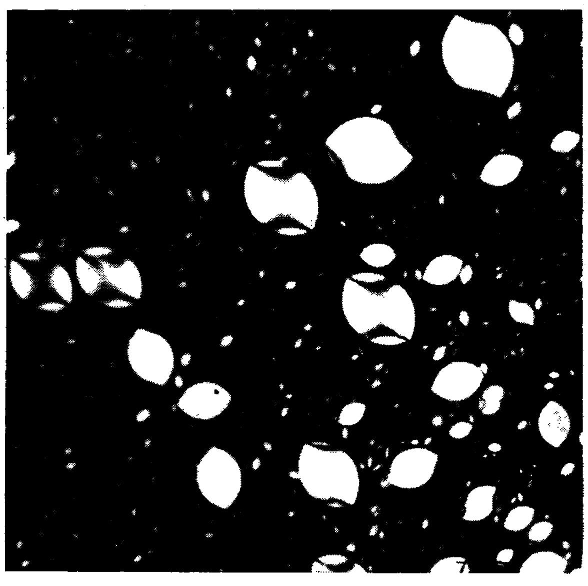

FIG. 1. Photomicrograph between crossed nicols of V_2O_5 tactoids, × 100.