Comments and Communications

Angular Correlation of Scattered Annihilation Radiation

Under the above title, Snyder, Pasternack, and Hornbostel (quoted in this note as SPH; *Phys. Rev.*, 1948, **73**, 440) published a calculation of the cross section for coincidence measurements of the scattering of two photons, emitted in the annihilation of a positron and a negatron. The same result was independently derived by Pryce and Ward (*Nature, Lond.*, 1947, **160**, 435), who did not give their method for obtaining this result. It has recently been checked experimentally by Bleuler and Bradt (*Phys. Rev.*, 1948, **73**, June 1).

SPH derive the expression for the cross section in two different ways. One is by the usual perturbation theory methods; the second, by what they call the partial polarization analysis method. We wish to give here an admittedly crude and far from rigorous derivation of the final formula which seems to us to show more clearly what is done and which physical assumptions were implied in the partial polarization method. Although there is no essential difference between this method and that given in the paper of SPH, we believe that the method given here is easier to understand.

As has been emphasized often, the two photons produced at the annihilation of the positron and the negatron (in a singulet state) will be polarized in such a manner that if one of them shows a right-hand circular polarization, the other will show a left-hand circular polarization,[1] since the total angular momentum after the annihilation has to be zero as it was before the annihilation. This situation is equivalent to the situation where the two photons are linearly polarized perpendicular to each other. In order to show this often-quoted result, we may consider the wave function describing the two photons, given by

$$\varphi = \frac{1}{\sqrt{2}}\,[\varphi_1(l)\varphi_2(r) - \varphi_1(r)\varphi_2(l)],^2$$

where $\varphi_i(l)$ is the wave function of the i-th photon, with a left-hand circular polarization. If we now change from circular polarization to linear polarization, we have to use the transformation (cf. W. Pauli. *Handbuch der Physik.* (Vol. 24, Pt. 1), Berlin: Springer, 1933).

$$\varphi\,(l) = \psi\,(\alpha)\, cos\,\omega\, t + \psi\left(\alpha + \frac{\pi}{2}\right) \sin\omega\, t,$$

$$\varphi\,(r) = -\psi\,(\alpha)\, sin\,\omega\, t + \psi\left(\alpha + \frac{\pi}{2}\right) \cos\omega\, t,$$

where $\psi_i\,(\alpha)$ is the wave function of the i-th photon which is polarized in a direction making an angle, α, with a fixed plane. In this way, we get

$$\psi\,(\alpha) = \psi_1\,(\alpha)\,\psi_2\left(\alpha + \frac{\pi}{2}\right) - \psi_1\left(\alpha + \frac{\pi}{2}\right)\psi_2\,(\alpha).$$

Since the direction of the polarization of one of the photons is arbitrary, we can write for the wave function of the system:

$$\psi = \frac{1}{2\pi\sqrt{2}}\int_0^{2\pi} d\alpha\left[\psi_1(\alpha)\psi_2\left(\alpha + \frac{\pi}{2}\right) - \psi_1\left(\alpha + \frac{\pi}{2}\right)\psi_2(\alpha)\right].$$

We see here that ψ describes a situation where both photons are unpolarized, but are polarized perpendicularly to each other (compare SPH). We wish now to discuss first the case of the (nonrelativistic) photo effect, *i.e.* the case where the photons eject electrons in the direction of their polarization. We are especially interested in the probability that the two photoelectrons are ejected in directions which make an angle, ϕ, with each other. That is, we have to look for the amplitude of the function $\psi_1(0)\,\psi_2\,(\phi)$ in ψ, if we take the fixed plane, mentioned above, through one of the counters which are counting the photoelectrons. In order to determine that amplitude, we write

$$\psi_1(\beta) = cos\,\beta\,\psi_1(0) + sin\,\beta\,\psi_1\left(\frac{\pi}{2}\right),$$

$$\psi_2(\beta) = cos\,(\beta - \phi)\,\psi_2(\phi) + sin\,(\beta - \phi)\,\psi_1\left(\phi + \frac{\pi}{2}\right),$$

and we get

$$\begin{aligned}\psi = \frac{1}{2\pi\sqrt{2}}\Big[&\Big(-\int cos\,\alpha\, sin\,(\alpha - \phi)\,d\alpha\Big)\,\psi_1(0)\psi_2(\phi) + \\ &\Big(\int cos\,\alpha\, cos(\alpha - \phi)\,d\alpha\Big)\psi_1(0)\psi_2\left(\phi + \frac{\pi}{2}\right) + \\ &\Big(-\int sin\,\alpha\, sin(\alpha - \phi)\,d\alpha\Big)\psi_1\left(\frac{\pi}{2}\right)\psi_2(\phi) + \\ &\Big(\int \sin\alpha\cos\,(\alpha - \phi)\,d\alpha\Big)\psi_1\left(\frac{\pi}{2}\right)\psi_2\left(\phi + \frac{\pi}{2}\right) - \{2, 1\}\Big]\end{aligned}$$

or

$$\begin{aligned}\psi = \frac{1}{\sqrt{2}}\Big[& sin\,\phi\,\psi_1(0)\psi_2(\phi) + cos\,\phi\,\psi_1(0)\psi_2\left(\phi + \frac{\pi}{2}\right) \\ & - cos\,\phi\,\psi_1\left(\frac{\pi}{2}\right)\psi_2(\phi) + sin\,\phi\,\psi_1\left(\frac{\pi}{2}\right)\psi_2\left(\phi + \frac{\pi}{2}\right)\Big].\end{aligned} \qquad (1)$$

Since the four functions, $\psi_1(0)\psi_2(\phi)$, $\psi_1(0)\psi_2\left(\phi + \frac{\pi}{2}\right)$, $\psi_1\left(\frac{\pi}{2}\right)\psi_2(\phi)$, $\psi_1\left(\frac{\pi}{2}\right)\psi_2\left(\phi + \frac{\pi}{2}\right)$, are orthogonal wave functions, we immediately obtain from equation (1) the probability for the above-mentioned situation. That probability is $\frac{1}{2}\, sin^2\,\phi$, already mentioned by Pryce and Ward.

We may express equation (1) in the following way. The situation from which we start is fourfold degenerate, and four orthonormal wave functions pertaining to this

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[1] This definition of right-hand and left-hand circular polarization is different from the usual one. We have followed the description of SPH, and others. If one, however, uses right- and left-hand polarization in the way it is defined in most textbooks, *i.e.* referred to the direction of propagation of the photon and not to a fixed system of reference, as is done here, one should say that the two photons are either both right- or both left-hand polarized.

[2] The negative sign is necessary because φ describes the polarization for the case where the circular polarizations are of opposite sign. There ought to be included an also antisymmetric part describing the movement of the two photons in opposite directions. The fact that φ is antisymmetric is analogous to the fact that for a singulet state of a two-electron system, the spin function is antisymmetric (see, for instance: H. A. Kramers. *Hand- und Jahrbuch der Chemischen Physik.* (Bd. I.) Leipzig: Akademische Verlaggesellschaft, 1938; and G. Wentzel. *Quantentheorie der Wellenfelder.* Vienna: Deuticke, 1942).

state are $\psi_1(0)\psi_2(\phi)$, etc. Since only one of the four functions contributes to the phenomenon we want to observe, we obtain the probability for that phenomenon immediately from the square of the amplitude.

The process mentioned in the beginning of this note is, however, not so simple, since, also, photons polarized perpendicularly to the plane of scattering contribute to the coincidences. If and *only if* the four functions used in the expansion in equation (1) represent an orthonormal set for the fourfold degenerate state, we can use equation (1) to calculate the cross section for the above-mentioned process. For this cross section, we can write:

$$\sigma = \Sigma a_k^2 \, \sigma_k, \tag{2}$$

where the a_k represent the amplitudes of the functions $\psi_1(0)\psi_2(\phi)$, etc., and σ_k are the cross sections for the photons in the directions considered. For the function $\psi_1(\alpha)\psi_2(\beta)$, the cross section is, apart from a normalizing factor,

$$\sigma_k = (\gamma_1 - 2 \sin^2 \theta_1 \cos^2 \alpha) \; (\gamma_2 - 2 \sin^2 \theta_2 \cos^2 \beta), \tag{3}$$

where the θ_i and γ_i have the same meaning as in the paper by SPH. From equations (1) to (3) we finally get, again apart from a normalizing factor

$$\sigma = \gamma_1 \gamma_2 - \gamma_1 \sin^2 \theta_2 - \gamma_2 \sin^2 \theta_1 + 2 \sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \phi, \tag{4}$$

which is identical with the result of SPH or Pryce and Ward.

If the four functions used in equation (1) had not been an orthonormal set, equation (2) should have included cross terms with $a_k a_m$, and we ought first to orthonormalize the set of functions used before applying an equation such as equation (2).

The fact that our choice, which is the natural, obvious choice makes it possible to use equation (2), corresponds to the remark of SPH that one *has* to resolve the polarizations of the photons in this particular way. It also corresponds to the fact that, only for this particular choice of the system of reference, the cross terms in SPH's equation (13) obtained by the usual perturbation theory vanish, as was also remarked by SPH.

The final cross section (4) may also be obtained by treating the scattering as consisting of two parts, an isotropic and an anisotropic part. One can then again use a formula, analogous to equation (2). If one chooses the right frame of reference, the result is the same.

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Formation of Itacolumite

Itacolumite, a stratified flexible sandstone, consists of symmetrically arranged rows of interlocking quartz crystals. In the study of its mode of formation, previous workers limited themselves in the main to general observations on field characteristics without considering the conditions essential to its formation. Again, extensive work has been carried out on the effect of fluxes upon the transition temperatures of quartz, though without reference to the problem under review.

A survey of the field observations shows that specimens taken from different localities vary both in their mineral associations and their degree of metamorphism. The rock matrix may, for example, comprise mica flakes along the bedding planes and as a joint between the quartz crystals (Itacolumi, Minas Geraes, Brazil—Schulz. *Bull. Soc. Geol. France,* 1834, 416; Gergens. *Neues Jahrb.,* 1841, 566; Zerrener and Von Humbolt. *Z. dtsch. geol. Ges.,* 1849, 484); it may consist of traces of muscovite and unevenly distributed clay (Delhi—Mügge. *Neues Jahrb.,* 1887, **1**, 195), sometimes soft felspathic materials form the matrix (Kaliàna, India—Falconer. *J. Soc. Bengal,* 1847, **6**, 240; Medlicott. *Rec. geol. Surv. India,* 1874, **7**, 30), and, again, calcium carbonate may be the cementing agent (Chàrli, Berar—Hughes, who quotes Fedden. *Mem. geol. Surv. India,* 1877, 16). These, together with Derby's observations (*Amer. J. Sci.,* 1884, **28**, 205) of weathered itacolumite as a purely quartz entity, illustrate the wide variety in composition.

The range in degree of metamorphism is equally marked, for the Brazilian and Kaliàna specimens have their mica flakes arranged with their longer axes parallel, suggesting considerable pressure and squeezing. Buschendorf (*Fort. Min. Krist. Pet.,* 1933, **17**, 407) has described the compression of individual quartz grains in the Kaliàna material—again signifying intense metamorphism, while the flexible sandstone discovered by Fedden, though exhibiting all the characteristics of itacolumite, contains calcium carbonate and shows virtually no metamorphism.

A study of the influence of fluxes upon the transition of amorphous silica to quartz goes to show that, though the mineral associations are diverse in type, they are of definite importance in the formation of itacolumite.

For quartz crystals to be formed, molecular mobility is essential and is, no doubt, attained either by fusion or by solvent action. With regard to fusion, the high temperatures involved may lead to nondifferentiation between the lower and higher forms of quartz, the additional formation of tridymite and cristobalite, and possibly the twisting and twinning of the quartz crystals. It has been shown, furthermore, that by this pyrogenic method, there is a tendency for silica to separate as a metastable, solid, vitreous mass when concentrations are high, cooling is rapid, and nuclei are absent. In contrast, the hydrothermal process, involving a variety of fluxes, actually facilitates the transition to quartz at much lower temperatures. It thus eliminates most of the anomalies cited.

Confining our attention to the latter, a number of experimenters have demonstrated this facility in the case of particular fluxes. The transition of amorphous silica to quartz occurs in the region of 870°. The temperature is depressed and the time factor lowered in the presence of water and solutions of lithium chloride-oxychloride. It is reduced to 750° with sodium or lithium tungstate, to 350°–380° with hydrofluoboric acid, and to 300° in the presence of sodium metasilicate and sodium chloride. Again, quartz has been obtained by heating silicic acid gel with potassium or sodium carbonate at 350°–390° and by the action of sodium carbonate and bicarbonate on aluminosilicate gels at 255°–355°.

The rhythmic deposition of quartz crystals would be brought about in a system specific to the Liesegang phe-