A Federal Program in Applied Mathematics

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THE GROWING IMPORTANCE OF MATHEMATICS, particularly applied mathematics, is one of the most significant trends in science today. This trend is part of a broader and necessary development in the physical sciences. Thus, the Steelman report (*1*) notes that the national research and development budget for the fiscal year 1947 in the physical and biological sciences alone was approximately $1,200,000,000, excluding expenditures for atomic energy. The Federal Government's share of this total was $625,000,000 of which $200,000,000 was spent in government laboratories. Even larger allocations for research and development are recommended for the future.

Because of the growing mathematical complexity of current problems in the physical sciences, the national research and development program carries with it substantial requirements for research and services in applied mathematics and related numerical techniques. This is particularly true in the case of the Government's part of the program, for at present a large fraction of it is concerned with the problems of national security, many of which are highly mathematical. Considerations of economy in Federal expenditures, on which so much emphasis is now properly being placed, add impetus to the full exploitation of the mathematical approach. Mathematical analysis prior to expensive laboratory experimentation or the construction of elaborate models is economical, both in actual costs and in the utilization of scientists.

Thus, even if no very remarkable developments were in sight in applied mathematics and related numerical techniques, there would be few arguments as to the importance of their place in the national scientific effort. As it happens, remarkable developments really are in sight in the field of numerical analysis. Automatic computing machinery is now being designed and constructed which will obtain numerical answers at amazing speeds. This machinery will not only greatly increase the effectiveness of the mathematical approach to standard problems, but will also permit successful attacks to be made on problems which have hitherto been considered to be inaccessible by mathematical methods. Examples of problems of the latter type arise in weather forecasting, in the rational planning of military and business operations, and in the automatic control of industrial processes.

The National Applied Mathematics Laboratories of the National Bureau of Standards have been recently established with the aim of strengthening the national applied mathematics effort. A particular concern of the new organization consists in promoting the rapid development of automatic computing machines and in taking steps to insure their most effective use when they become generally available.

Background and Motivation

"Applied mathematics" is not easy to define, and the term has different meanings for different persons. Work in applied mathematics is carried on largely at one or the other of two quite different levels, identified here as the research level and the level of applications of mathematics.[1]

The research level is characterized by complicated chains of original mathematical reasoning quite similar to those which characterize creative research in pure mathematics; but in applied mathematics the hypotheses and initial mathematical relationships are always intended to be approximate and abstract representations of some situation in another science, and conclusions are at once translated back into the language of that science. The main interest in the conclusions lies sometimes in the direction of predicting further experimental phenomena and sometimes in the direction of evaluating the practical validity of the hypotheses.

At the level of application of mathematics, the work consists in fitting already established (or trivially provable) mathematical propositions to situations in other sciences. The conclusions are again utilized at the research level.

In work at either level it is customary to require that conclusions be stated quantitatively, rather than qualitatively, so that a check against experimental data will be possible. Thus, the science of numerical analysis is frequently drawn upon by workers in applied mathematics. Accordingly, it has been customary to consider numerical computation, together

[1] This terminology was suggested by Mina Rees, head of the Mathematics Branch, Office of Naval Research.

with its theory and the related technology, as a companion-field to applied mathematics.[2]

In the United States, research in both pure and applied mathematics has been traditionally concentrated in educational institutions. Fortunately, there are now a number of lively centers of research in applied mathematics in such institutions. Most of them have been established within the last 10 years, and nearly all of them are able and willing to participate in the Federal scientific program through the medium of contracts. For example, centers for research in classical applied mathematics now exist at Brown University, Carnegie Institute of Technology, Massachusetts Institute of Technology, New York University, and a number of other universities. Additional centers for research in mathematical statistics have been established at Princeton University, Columbia University, the University of North Carolina, Iowa State College, and the University of California, among others. In addition, teams of research workers in applied mathematics have been organized at several of the government laboratories, notably at the Naval Ordnance Laboratory and the Aberdeen Proving Ground. The total effect has been to maintain research activities in this field at an intensity quite comparable to that of the wartime period.

At the level of applications, as contrasted with the level of research, the particular requirements of the Federal scientific program have been more difficult to meet. Some laboratories have successfully coped with their routine requirements in the application of mathematics by setting up jobs for one or two full-time consultants in applied mathematics. But the work load at any one laboratory is apt to be rather irregular, and, unfortunately, there are just not enough competent consultants to go around. There is, in particular, a scarcity of qualified consultants in the applications of mathematical statistics.

A somewhat similar situation exists in the field of numerical analysis. Many laboratories have set up computing facilities to meet routine requirements. However, if a computing group is small, the occasional large (and sometimes important) problem must be by-passed. If the group is a large one, there is usually a good deal of idle time due to irregularities of load. If the computing group is very large and has expensive automatic equipment, then the difficulty of recruiting competent programming personnel at the higher grades may seriously cut down the true maximum capacity of the group.

[2] It should be noted, however, that certain branches of pure mathematics, notably number theory, rely heavily on numerical computation, and most of the theory of numerical analysis itself is quite clearly a branch of pure mathematics. It therefore seems hardly possible to classify numerical computation simply as a subregion of applied mathematics.

This suggests that a limited centralization of computing facilities and consulting services at the level of the applications of mathematics would be profitable. The case for centralizing in a limited way is further strengthened when certain aspects of the new automatic computing machinery are considered. The present cost of a large-scale automatic digital computing machine is between $300,000 and $500,000. This is clearly beyond the resources of most of the university centers of applied mathematics, and every effort should be made to insure the widest possible availability of such costly machinery through centralization. Furthermore, the design and construction of such machines involve coordination problems of considerable magnitude. Large-scale computing machines have become the urgent concern of a number of different Federal agencies, and a central point of reference is needed to prevent unnecessary duplication of effort in the development program and to insure uniform quality. Since the maximum exploitation of such machinery will be heavily dependent on further research work in certain fields of pure and applied mathematics, it is the responsibility of any coordinating agency to promote such research to the utmost.

It seems apparent that there is a definite need for a central mathematical laboratory in the Federal scientific program. The organization should carry on a rather specialized set of activities chosen with the view of supplementing and supporting the existing research work in applied mathematics. The activities of the center could profitably include consulting services at the level of the applications and the production of aids for such consulting services (such as technical manuals), computing services and the production of aids to computations (such as mathematical tables), and a coordinated program of automatic computing machine development and research in associated branches of mathematics.

This need was the basis for the establishment of the National Applied Mathematics Laboratories as a division of the National Bureau of Standards.

Evolution of the Organization

The National Bureau of Standards had previously been active in the computing field for some years (1938–43) through its scientific sponsorship and administration of the Mathematical Tables Project in New York City. The Mathematical Tables Project started as a WPA project with a program of computing large tables of the basic mathematical functions. In 1943 its financial support was assumed by the OSRD, and it thereafter worked on some of the war problems of the Applied Mathematics Panel.

When OSRD support was withdrawn in the fall of 1946, the Office of Research and Invention of the Navy Department (now the Office of Naval Research) undertook temporarily to provide funds to resume the tabulations and continue the problem-solving work of the Tables Project. Some idea of the success of the tabulation work can be gained from the fact that in the period from 1940 to 1946 about 28,000 volumes of the tables (some of which contain several hundred pages of tabular material) were sold to the public, in addition to the free distribution to government laboratories and depositories. Several of the original editions of about 2,000 copies are now exhausted. The director of the Project since its inception has been A. N. Lowan.

Two other mathematical activities were initiated by the Bureau in the calendar year 1946. The writer was appointed as assistant to the director in April of 1946 and authorized to introduce mathematical statistical methodology into the routine work of the Bureau. The operating phases of the job were assigned later in the year to a group headed by Churchill Eisenhart. Shortly thereafter, the Bureau of the Census requested the Bureau to undertake the construction of a large-scale automatic digital computing machine suitable for the preparation of census reports and the performance of mathematical calculations arising in sampling surveys. Soon after this contract was accepted, the Army Ordnance Department transferred to the Bureau a substantial sum of money for research on electronic computer components, and the Office of Naval Research did likewise for the construction of a second large-scale automatic digital computing machine. Thus, the nucleus for a mathematics center of the type under discussion existed at the National Bureau of Standards by the end of the calendar year 1946.

Paralleling these developments was a movement within the Navy Department to foster the establishment of a national computing center. The idea seems to have originated in the Planning Division of the Office of Research and Invention. From the outset the recommendations of the Planning Division envisioned a center which not only would be equipped with high-speed automatic machinery (with emphasis on digital types) but also would assume leadership in the development of such machinery. Early in 1946 Rear Adm. H. G. Bowen, then chief of Naval Research, approached E. U. Condon, the director of the National Bureau of Standards, with a suggestion that the Office of Naval Research and the Bureau should jointly undertake to establish such a facility. A year of cooperative study and of consultation with various possible "clients" and applied mathematical groups ensued. The study clearly revealed the need for a Federal center of applied mathematics specializing in the types of activity identified in the preceding section as appropriate. Accordingly, the plans which finally emerged proposed that a facility with a mission considerably broader than that of a central computing laboratory should be established; further, that it should take the form of a new division of the National Bureau of Standards, but with an administration guided by a committee of representatives of various Federal agencies interested in the work of the center. The plan was presented in a widely distributed report (*2*), which received formal or informal concurrence from all officially interested groups.

It was agreed that the new unit would be called the National Applied Mathematics Laboratories, and the Laboratories were formally established as Division 11 of the National Bureau of Standards on July 1, 1947.

Description of the Laboratories

The scientific work of the National Applied Mathematics Laboratories is performed in the following four operating units: (1) *The Institute for Numerical Analysis,* at the University of California, Los Angeles; (2) *The Computation Laboratory,* at present in New York City; to be moved to the National Bureau of Standards, Washington, D. C., during the calendar year 1948; (3) *The Statistical Engineering Laboratory,* at the National Bureau of Standards, Washington, D. C.; (4) *The Machine Development Laboratory,* at the National Bureau of Standards, Washington, D. C.

In addition, there is an administrative office, located at the Bureau in Washington, which is responsible for program planning, the assigning of priorities, budgets, contracts, etc. Its decisions are subject to review by the director of the Bureau and are made with the guidance of a committee known as the Applied Mathematics Executive Council.

The Council is a committee of representatives of various operating agencies of the Government. The agencies initially represented on the Council were the Navy Department, the Department of the Army, the U. S. Air Force, the National Advisory Committee for Aeronautics, the Bureau of Agricultural Economics, the Bureau of the Census, and the Weather Bureau.

The total personnel complement of the National Applied Mathematics Laboratories is to be between 115 and 140. Annual operating costs, exclusive of capital costs and depreciation of equipment, are estimated at around $500,000. It might be mentioned here that this is only one-fourth of 1% of the total amount which, according to the previously cited report of the President's Scientific Research Board, is being spent during the fiscal year 1947 in government laboratories, and less than one-twelfth of 1% of the total Federal research and development budget as given by that report.[3]

It will be noticed that the locations chosen for the units of the Laboratories are restricted to Washington and Los Angeles. In making this choice, the three determining considerations were proximity to immediate sources of problems, ease in recruiting personnel, and continuity of the already existing mathematical activities of the Bureau. The problems worked on by the Mathematical Tables Project have originated almost entirely in the East, particularly in and around Washington, D. C. Of all the areas west of Washington, the Southern California area, with its extensive contract-supported, guided-missile development work and its large government laboratories, seemed to be the region most likely to make good use of a branch of the Laboratories. The Air Materiel Command of the U. S. Air Force particularly favored the Los Angeles location, and the highly cooperative attitude of the personnel of the University of California at Los Angeles (in particular, that of the provost, C. A. Dykstra) was another decisive factor. If further expansion of the Laboratories is justified at some future date, the desirability of placing further branches in the Middle West and elsewhere will be given serious consideration.

The Institute for Numerical Analysis

Section 1, the Institute for Numerical Analysis, is the focal point in the organization for basic research and training in the types of mathematics which are pertinent to the efficient exploitation and further development of high-speed automatic digital computing equipment. A secondary function is to provide a computing service for the southern California area and to give assistance in the formulation and analytical solution of problems in applied mathematics. It will be equipped with one general-purpose automatic electronic digital computing machine and with the usual desk calculators and punch-card equipment. The automatic computing machine will probably be supplied by the Air Materiel Command of the U. S. Air Force. It is hoped that this machine, which is now being developed in Section 4, the Machine Development Laboratory, will be delivered on or about January 1, 1949.

The research program will depend in part on the personnel at the Institute. The intention is to maintain a small, permanent staff at the top research levels and to carry on most of the work through temporary appointments of strong research men on leave of absence from their regular places of employment. In addition, the Institute will provide facilities for visiting scholars to investigate and develop in their own special fields of research the mathematical techniques studied by the Laboratories. A percentage of the time of the automatic computing machine will be reserved for purposes of mathematical research, as opposed to service computations. The original plans for the Institute called for a personnel complement of about 40 persons, including part-time workers. (It is expected that some of the training functions will be discharged by employing graduate students part time.)

The research program has been underwritten for 1947–48 and 1948–49 by the Office of Naval Research. The Air Materiel Command of the Air Force is at present the principal supporter of the computing service. Operations commenced in January 1948 in a temporary building. Permanent quarters will later be furnished on a rental basis by the University in a new engineering quadrangle now under construction.

The Computation Laboratory

Section 2, the Computation Laboratory, consists chiefly of the old Mathematical Tables Project with some strengthening at the top. The chief mission of the Computation Laboratory is to provide a general computing service of high quality and large capacity. A function of almost equal importance is the continuation of the series of tables of the Mathematical Tables Project. Services are also offered in analytical applied mathematics at the level of the applications, and a considerable amount of research is performed on the classical theory of numerical analysis. The Computation Laboratory will be equipped with at least one, and perhaps two, general-purpose, automatic-sequenced computing machines. The machine now earmarked for the Computation Laboratory is the one being developed by Section 4 for the Office of Naval Research. It may be ready for operation sometime on or about January 1, 1949.

The general scope of the work of the Computation Laboratory can be indicated by a brief résumé of the work performed during the three-month period from July 1 to September 30, 1947.

A table (3,000 entries) of mutual impedances of metal detector head coils from Wheatstone bridge arm data was prepared for the laboratories of the Engineer Corps at Fort Belvoir, Virginia. Tables of a two-parameter integral arising in antenna theory were prepared by mechanical quadrature for the Naval Research Laboratory, Anacostia, Maryland. The difference in the lattice sums of the third order Van der Waals interaction for the two closest packed lattices

[3] It should be re-emphasized here that the report excludes expenditures for atomic energy and pertains only to the physical and biological sciences.

(the hexagonal and face-centered cubic) was computed for a research project in the physics of solids under way at the Bureau. A long series of computations relating to 9 given matrices arising in the theory of atomic spectra were carried out for another Bureau project; the computations involved solution of the secular equations and of systems of related linear equations. Third-degree, two-variable surfaces of constant barometric pressure were fitted by least squares to climatological data arising in the meteorological study under way at New York University and the Institute for Advanced Study in Princeton. Work was continued on a large, two-volume table of the Mathieu Functions. A nearly complete table of Bessel Functions of fractional orders was checked by differencing. A nonlinear partial differential equation arising in heat conduction theory was solved for various boundary conditions. Work was performed on large tables of the Coulomb Wave Functions, of the exponential integral, of the Bessel Functions $Y_0(z)$ and $Y_1(z)$, of trigonometric functions to a hundredth of a degree, of 10^x to 10 decimal places, of spheroidal wave functions, of Jacobi Elliptic Functions, and of Gamma Functions for complex arguments. Ordinary differential equations arising in guided missile work and in the study of the effect of centrifugal forces on the human body were solved. Systems of simultaneous equations containing 46 unknowns arising in the theory of logistics were solved. Various integrals involved in the theory of atomic spectra and electronics were evaluated by numerical methods.[4]

The personnel complement of the Computation Laboratory is between 60 and 70. The Office of Naval Research is currently supplying the major part of the operating funds, the Hydrographic Office of the Navy Department and the National Bureau of Standards also contributing a share.

The Statistical Engineering Laboratory

Section 3, the Statistical Engineering Laboratory, provides a general consulting service on methods of modern statistical inference as applied to the engineering and physical sciences. Basic research in statistical theory and the formulation of requirements for new statistical tables, which are then computed in Section 2, are also undertaken.

Until the present, this Laboratory has devoted its attention chiefly to other divisions of the National Bureau of Standards and to the Federal Specification Board. In the routine testing and calibration activities and engineering experiments of the Bureau there are practically endless opportunities for the application of mathematical statistics, and a much larger group than the unit of about 10 persons originally provided for in the plans for the statistical laboratory could profitably be employed on Bureau work alone. A controlling factor in the expansion of the laboratory is the difficulty of obtaining qualified personnel at the higher levels. In spite of the limited capacity of the present unit, it is expected that an increasing amount of the energies of the Laboratory will be devoted in the future to problems originating outside of the Bureau.

The Machine Development Laboratory

Section 4, the Machine Development Laboratory, consists at present of a mathematics group of 5 under the direct administration of the National Applied Mathematics Laboratories and an engineering group of 15 at present being administered by the Ordnance Development Division of the Bureau. Technically, the two groups operate as a single unit. The primary function of this Laboratory is to develop and supervise the construction of automatic computing machines which will meet performance specifications established by other sections of the National Applied Mathematics Laboratories and by outside agencies. The Laboratory also develops standards of performance and specifications for computer components, such as arithmetic units, input devices, memory devices, and electronic tubes.

Present emphasis is entirely on high-speed automatic electronic digital computers; no attention is being paid to the analogue types of device such as the differential analyzer. Detailed design specifications have already been established for automatic computers for the Bureau of the Census, the Office of Naval Research, and the U. S. Air Force. Several alternative designs, two of which were produced by contractors working directly for the Machine Development Laboratory, are now beng evaluated with the assistance of the Committee on High-Speed Calculating Machinery of the National Research Council. Construction of the machines themselves is expected to start early in 1948. Another important project has consisted of the research and development in components, performed for the Office of the Chief of Ordnance. This project has resulted, among other things, in the preparation of data transducers and verifiers for the EDVAC machine being built by the Moore School of Electrical Engineering for the Aberdeen Proving Ground and for the machine being developed under J. von Neumann at the Institute for

[4] The details of these projects will be forwarded upon application to the Chief, National Applied Mathematical Laboratories, National Bureau of Standards, Washington 25, D. C. The Computation Laboratory issues monthly progress reports which will be sent regularly to any interested scientist upon request.

Advanced Study in Princeton, New Jersey, with the cooperation of the Radio Corporation of America.

A very important part of the work of the Machine Development Laboratory consists in acting as a coordinating agency and an information exchange center for the Federal automatic digital computer program. To this end, bibliographies have been prepared and distributed, and an information and discussion section for the journal *Mathematical Tables and Other Aids to Computation* (published by the National Research Council) is being compiled and edited.

Much of the program of the National Applied Mathematics Laboratories (with the exception of that of the Statistical Engineering Laboratory) is not strictly classifiable as "applied mathematics" at all, since it is concentrated in the near-by field of numerical analysis. The immediate reason for this phenomenon should be apparent from the background and history of the organization. It is interesting to note in this connection that, some three years ago, a mathematical organization with a similar program and setting was established in England as the Mathematics Division of the National Physical Laboratory. Other countries are also setting up national mathematical centers with emphasis on computing an automatic computer development.

However, there is considerable reason to believe that as the National Applied Mathematics Laboratories mature and, in particular, as automatic equipment now under development comes into actual use in the Laboratories, the program will tend to conform more and more closely to a puristic interpretation of the name of the organization. Even now, plans are being made for a further strengthening of the work at the level of the applications of mathematics. Traditionally, a certain amount of basic research in applied mathematics has been carried forward in various scattered groups in the National Bureau of Standards, and it may be that later on a special section should be added to the Laboratories to consolidate and extend this activity. In the meantime, there is much to be done in the present areas of concentration of the work of the Laboratories, and it is believed that a substantial contribution to the national scientific effort will be made if the present program is effectively carried out.

References

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Obituary

Alfred North Whitehead

1861-1947

Alfred North Whitehead died in Cambridge, Massachusetts, on December 30, 1947. He was born in Ramsgate, England, on February 15, 1861.

He attended Trinity College, Cambridge, where he obtained the B.A. degree in 1884, the M.A. degree in 1887, and the D.Sc. in 1905. He also received the D.Sc. degree from the universities of Manchester, Harvard, Wisconsin, Yale, and McGill and an LL.D. from St. Andrews.

He was lecturer and later senior lecturer on mathematics in Trinity College, Cambridge, from 1885 to 1911; lecturer on applied mathematics and mechanics and later reader in geometry at University College, University of London, 1911–14; professor of applied mathematics and later chief professor of mathematics, Imperial College of Science and Technology, University of London, 1914–24; dean of the Faculty of Science, 1921; professor of philosophy, Harvard University, 1924–36; and professor emeritus, 1936 to the time of his death. He was a Fellow of the Royal Society and the British Academy and a member of the Mathematical Society, the British Association for the Advancement of Science, the Aristotelian Society, and the American Philosophical Association. He received the James Scott Prize from the Royal Society of Edinburgh in 1922, the Sylvester Medal from the Royal Society of London in 1925, and the Order of Merit in 1945.

His publications include *A treatise on universal algebra,* 1898; *Principia mathematica* (with Bertrand Russell), 1910; *An introduction to mathematics,* 1910; *The organization of thought,* 1916; *The principles of natural knowledge,* 1919; *The concept of nature,* 1920; *The principle of relativity,* 1922; *Science and the modern world,* 1925; *Religion in the making,* 1926; *Symbolism: its meaning and effect,* 1927; *The aims of education,* 1928; *Process and reality* (the Gifford Lectures) and *The function of reason,* 1929;