

sary to give inhalations, as many as four to five times daily, to maintain satisfactory quantities of penicillin in the lungs. Moreover, when Penicillin Aerosol is employed radiographic control of the location of the agent is not possible.

The primary purpose of this study has been to illustrate that after a single instillation of penicillin iodized oil, penicillin is present locally in the lungs for a minimum of 24 hours, as indicated by its excretion in the urine for that period of time. Further study is necessary relative to the frequency and interval of instillation of penicillin iodized oil as well as the therapeutic possibilities.

#### SUMMARY

(1) A suspension of calcium penicillin in 40 per cent. iodized oil produces a stable mixture which has been instilled in the lungs of 12 patients without adverse effect and has maintained penicillin in the lung for a minimum of 24 hours, after a single instillation.

(2) The penicillin iodized oil has maintained its potency for 60 days at ice box, room and 37° C. temperatures.

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#### CHROMOSOME MOUNTS FOR TEMPORARY STUDY

ZIRKLE'S<sup>1</sup> solution, or a modification of this, as a sealing agent for the temporary preservation of chromosome mounts has been found to be more effective

and more convenient than any of the various paraffin or wax and gum mastic mixtures. With a pipette a small amount of Zirkle's solution—80 cc aceto-carmin, 10 cc Karo corn syrup (Dextrose), and 10 cc Certo (Pectin)—is placed around the edges of the cover slip and allowed to dry. By this method smears of leaves,<sup>2</sup> root tips and anthers have been preserved in good condition for periods varying from several weeks to several months. The procedure is equally effective with propio-carmin, Feulgen, and aceto-carmin smears.

The solution has been modified by substituting 45 per cent. acetic acid for the aceto-carmin and the resulting solution used as a sealing agent. While slides so sealed do not seem to remain in good condition as long as those prepared by the first method, excellent results have been obtained with slides kept for periods up to a week or ten days. Since the possibility of over-staining which might result from the presence of Zirkle's solution around the edges of the cover slip is thereby avoided, this modified solution is suggested for slides which are to be kept for short periods.

In addition to the effectiveness of these solutions as sealing agents and the ease with which they are applied, they are also recommended, for slides sealed by them can be made into permanent mounts by removing the solution from the edges of the cover slip with a moistened cloth, by inverting the slide in acetic alcohol (1:1) until the cover slip floats off, and by following from here Meyer's<sup>3</sup> procedure for making smears permanent, or by using any desirable modification of McClintock's<sup>4</sup> permanent aceto-carmin method.

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## DISCUSSION

#### THE LAPLACE EQUATION

THE most important partial differential equation is

$$(1) \quad \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0,$$

known as the Laplace equation. The applications embrace gravitation, electricity, magnetism, hydrodynamics, conduction of heat, stream lines, isothermal families, conformal mapping.

In three dimensions, the corresponding equation is

$$(2) \quad \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0.$$

This equation is much more complicated analytically; in particular the powerful method of functions of a

complex variable successful for (1) is no longer available for (2).

Although the analytic difficulties are well known, we shall point out new geometric aspects. Sophus Lie found the first geometric property of isothermal families of curves in the plane. We prove that this result is no longer valid in three dimensions. From Lie's work, Kasner and De Cicco found purely intrinsic geometric properties of isothermal families and isothermal networks. These have applications to stream lines in two dimensions.

Our object is to find analogues in three dimensions.

<sup>2</sup> J. T. Baldwin, Jr., *SCIENCE*, 90: 240, 1939.

<sup>3</sup> James R. Meyer, *Stain Tech.*, 18: 53-56, 1943.

<sup>4</sup> B. McClintock, *Stain Tech.*, 4: 53-56, 1929.

<sup>1</sup> C. Zirkle, *SCIENCE*, 85: 528, 1937.

The discussion is very complicated and not complete, but we report progress. Lie's theorem refers to the single inclination of the curve; but in space we have to introduce two slopes, or three direction angles of the surface. We find that these slopes or angles obey a fundamental set of three partial differential equations of second order. (In Lie's case, he found a single equation of second order which was identical in form with Laplace's equation). Our new equations are not of the Laplace form.

It is well known that the only transformations which convert (1) into itself form the conformal group. This is obvious from the theory of functions of a complex variable. Therefore isothermal families are converted into isothermal families by conformal transformations. The authors have proved that no other point transformations are legitimate. This is difficult because we can no longer take advantage of the isothermal parameter, and therefore we have to use an equation of third order, related to the Laplace equation, but not identical with it.

Now in three dimensions it is shown that the only legitimate point transformations which send (2) into itself are those of the similitude seven-parameter group. Thus we have merely arbitrary constants but no arbitrary functions. For isothermal families of surfaces, our discussion yields the same group.

If, instead of point transformations, we use the larger body of contact transformations, no new auto-transformations are possible. However, by using general element transformations, we have discovered a larger set of possibilities in the plane. Studying the analogous situation in space, we prove that no larger group can exist.

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### THE RATE OF DEVELOPMENT OF RED CELL PRECURSORS

SEVERAL attempts have recently been made to answer the question as to whether there are sufficient cells in the early developmental categories to maintain the number of mature red cells in the circulating blood. The solution of the general problem is simple when one approaches it from the standpoint of the steady state, and in the adult rat there are figures of sufficient accuracy to enable one to carry out the calculations.

Assuming that the cells of the red cell series develop irreversibly from hemocytoblasts to mature erythro-

cytes by passing through the successive stages of erythroblast, normoblast and reticulocyte, the number of cells  $N$  in any one class per unit volume at any one time will depend on  $P$ , the number of cells which enter the class per unit time, and  $Q$ , the number which leave it in unit time to enter the next class of the consecutive development. In the steady state in which  $N$  for the class remains constant, being fed by cells from the class before it as it delivers cells into the class beyond it,

$$t = N/Q,$$

where  $t$  is the average duration of the life of the cell in the class. When there is no cell division,  $P = Q$ , and the  $P$  for any class is the  $Q$  for the class before it.<sup>1</sup> If some of the cells in the class undergo mitotic division, the fractional number of cells which divide is

$$f = mt/d$$

where  $m$  is the number of the cells in the class observed in mitosis at any one time and expressed as a fraction of unity, and  $d$  is the duration of a mitosis in the same units of time as used to express  $t$ . For such a class in which a fraction of the cells divide,  $P$  is smaller than  $Q$ , and is

$$P = Q(1 - f/2).$$

The values assumed for the adult rat are shown in Table 1.

TABLE 1

Class	$N$ per $\text{mm}^3$	Explanation
Mature red cell	$18.8 \times 10^6$	$9.0 \times 10^6 \times 6.27/3.02^*$
Reticulocyte	$11.6 \times 10^6$	$5.4 \times 10^6 \times 6.27/3.02$
Normoblast	$7.2 \times 10^6$	36 p.c. of $2 \times 10^6$ <sup>†</sup>
Erythroblast	$8.0 \times 10^4$	4 p.c. of $2 \times 10^6$
Hemocytoblast	$5.0 \times 10^4$	2.5 p.c. of $2 \times 10^6$ ; 50 p.c. of all hemocytoblasts present.

\* Ratio of circulating blood to active marrow, Fairman and Corner.<sup>7</sup> The multiplication makes 1  $\text{mm}^3$  of marrow equivalent to 1  $\text{mm}^3$  of blood.

† Total number of cells per  $\text{mm}^3$  of marrow, Kindred,<sup>5</sup> Farrar.<sup>8</sup>

Starting the calculations with  $t = 60$  days for the mature red cell, and assuming (a) that the fraction of cells observed in mitosis in the normoblast, erythroblast and hemocytoblast classes is  $m = 0.005$ ,<sup>2</sup> and (b) that  $d$  is 0.5 hour, we get the values shown in Table 2.

The number of cells which must be produced by the cells of the erythrocytic capillaries or other cells of origin in order to maintain the steady state is accordingly  $2.12 \times 10^5$  per  $\text{mm}^3$  of marrow per day. This is not a remarkable rate of production; if, for

<sup>1</sup> E. Ponder, *Quart. Jour. Exp. Physiol.*, 16: 241, 1926.

<sup>2</sup> Kindred,<sup>5</sup> Table 2. The value 0.005 is applied to all three classes as an approximation, but the number of mitoses is small, and even considerable variation in the value would not affect the final result much. No account is taken of amitotic divisions.