SCIENCE

Vol. 101 FRIDAY, MARCH 30, 1945 No. 2622

SCIENCE: A Weekly Journal, since 1900 the official organ of the American Association for the Advancement of Science. Published by the American Association for the Advancement of Science every Friday at Lancaster, Pennsylvania.

Editors: JOSEPHINE OWEN CATTELL and JAQUES CATTELL.

Policy Committee: MALCOLM H. SOULE, ROGER ADAMS and WALTER R. MILES.

Advertising Manager: THEO. J. CHRISTENSEN.

Communications relative to articles offered for publication should be addressed to Editors of Science, 34 Gramercy Park, New York 3, N. Y.

Communications relative to advertising should be addressed to THEO. CHRISTENSEN, Advertising Manager, Smithsonian Institution Building, Washington 25, D. C.

Communications relative to membership in the Association and to all matters of business of the Association should be addressed to the Permanent Secretary, A.A.A.S., Smithsonian Institution Building, Washington 25, D. C.

Annual subscription, $6.00 Single copies, 15 cents

NEW FRONTIERS[1]

By Professor RICHARD STEVENS BURINGTON

DEPARTMENT OF MATHEMATICS, THE CASE SCHOOL OF APPLIED SCIENCE (ON LEAVE)
HEAD MATHEMATICIAN, BUREAU OF ORDNANCE, NAVY DEPARTMENT[2]

(1) FOR the past several years it has been my privilege to have intimate contact with a large amount of work concerned with an array of investigations in engineering, physical research and related activities, and with work in which all sorts of mathematical and physical fields appear as fundamental items in the larger problems under study. The nature of this work has been such that much of it, as met, would not be suitable for discussion here. Nevertheless, there are certain important facts and observations which I have noted from time to time that I hope will be of interest to many persons—to engineers, physicists, chemists, investigators, businessmen, industrialists, as well as to pure and applied mathematicians, and university people. Some of these items may well be made the subject of serious study, and such action as may be indicated should then be pushed vigorously by responsible individuals and organizations.

In discussing this topic, "New Frontiers"—some new, some old, some always new—I have in mind the vast store of potentially powerful and valuable mathematical knowledge and talent which has scarcely been tapped by industry and business. I should like to

[1] Address delivered to Section A—Mathematics, American Association for the Advancement of Science, Cleveland, Ohio, September 12, 1944. The author wishes to express his appreciation for the many constructive suggestions and critical comments made in connection with the preparation of this paper by Dr. Arthur Bennett, astronomer and engineer, and by Dr. C. C. Torrance, mathematician.

[2] The opinions expressed in this paper are those of the author and are not necessarily those of the Navy Department.

point out various directions in which some types of mathematicians might well—and should—direct their efforts. I should not presume to indicate the direction which pure research should take, for I doubt that any one should try to do that. I wish, nevertheless, to record certain specific suggestions for further development in mathematics, its applications and its uses in industry. In this connection I shall point out certain items which I believe should be considered by educators in developing their plans for the training of students for useful and fruitful careers, and, particularly, for the training of men and women in the mathematical and physical sciences.

Traditionally, mathematicians have found employment principally as teachers of mathematics. Some of them, along with their teaching, have developed into researchers in pure mathematics. Since the beginning of this century, very few mathematicians in this country have directed their investigations to applied mathematics and a still smaller number have found their way into industrial development and research. That more of these mathematicians have not followed the applied fields can undoubtedly be traced in no small measure to economic factors and to the influence of some of the outstanding schools of mathematics.

Although I have no doubt that this influence has been good so far as it went, I firmly believe that some of these schools should have undertaken vigorous programs years ago for the training of mathematicians for careers in industry and business. Only a few mathematicians recognized this need before the present war; those of us who did were often severely criticized by our colleagues who preached and sought for the crystal pure in their teaching and in their pursuit of pure research.

(2) *The Growth of Analytical Methods.* The expansion of the use of analytical methods in industry received a tremendous impetus at the beginning of this century from the introduction of steel structures and from the birth and growth of the electrical industry. This development of methods spread rapidly to the chemical and motor industries and, more recently, to the aircraft business. The present trend will undoubtedly continue for some time to come. Analytical activities in connection with many urgent development and production programs have been accelerated by the war; although some reduction in this pace can be looked for after the war, a considerable demand will continue, I feel, for a high level of analytical work in industry and for persons competent to handle such work economically and adequately.

The great depletion of material resources which this country has recently suffered demands a reconsideration of our spendthrift methods. Costly, wasteful processes and cut-and-dried methods of development which heretofore have often been the simplest and cheapest and sometimes, perhaps, the only methods available, must be bettered in the future. A much wider use of analytical methods than has been employed in the past will be forced upon many businesses by economic necessity. This need may be filled by the services of competent mathematicians of a new type, provided the men employed are able, by use of their abilities and knowledge, to reduce the need for and cost of experimentation and to aid in eliminating wasteful production. The ever-increasing need for more precise and exacting products and devices entails an ever-increasing need for a high degree of skill on the part of those engaged in research, development and production. With this trend, there is no question in my mind that the properly prepared mathematician should be able, and is able, to cooperate with scientists, engineers and business men at a high level in a most helpful, efficient, economical and satisfactory manner.

(3) *Mathematicians in Industry and Business.* Mathematicians have not been used to any great extent in most businesses. True, actuaries have long been used by insurance companies, and various types of statisticians have been used in most businesses. True, a highly efficient research mathematics department under a director of mathematical research, and staffed by mathematicians of high caliber, has been maintained at a few outstanding organizations such as the Bell Telephone Laboratories. Mathematicians and mathematical physicists have been employed by some aircraft, motor and electrical companies. But, on the whole, the wealth of available mathematical talent is not widely used. Typical situations in many concerns might be something like the following:

(a) No mathematicians are employed and very little mathematical work is done even when a goodly amount of it would yield dividends. Such statistical work as is carried out, if anything beyond a mere compilation of records, is usually done by clerks, accountants, draftsmen, economists, or inadequately trained statistical help. A remarkably good job is done by these persons when one considers their background and the fact that this statistical work is often only a minor part of their many responsibilities. In some cases analytical work is recognized as necessary, but even then, the work is done by engineers and draftsmen who have rather limited knowledge of mathematical technique.

(b) Even in those companies where a very considerable amount of development and research work is carried out, neither the need for nor the value of analytical studies is always appreciated. When the need is recognized the analytical work is carried out by engineers, physicists, chemists, draftsmen, accountants or statisticians who simply haven't the time to make the most effective use of mathematical methods; are not primarily interested or trained in the mathematical methods; and do not engage the cooperation, services or advice of men whose prime knowledge and abilities are in mathematics. Because these analytical problems arise within the jurisdiction of men in physical science it is natural, but sometimes unfortunate, that these men undertake the studies themselves.

I have seen many instances of these procedures; while the individuals involved do the very best they can, the methods used are frequently crude, and the results often take the form of costly design procedures, excessive use of material and labor and an inferior product. I have seen instances when there existed a real problem requiring a solution, but when there was no appreciation of the existence of this problem on the part of those responsible. There have been instances when the presence of a problem needing an improved solution was appreciated, yet an understanding of the sort of talent needed to produce the desired result was completely lacking. What is needed in these cases is a recognition of the caliber of assistance required and the sound, effective use of mathematicians—men whose perspective and training permits the ready use of methods of which engineers and physicists have some knowledge, but not enough to enable them to choose the most economical and best method. Such mathematicians must be broadly trained in mathematical methods, must be resourceful, and especially must be gifted with an abundance of good sense. Properly trained mathematicians should be able to do for a dollar what others who know something of mathematics, but do not specialize in mathematics, can do for ten dollars. Mathematicians of this sort must retain a high professional standing and should, and do, have the same prestige as other highly qualified professional men.

(4) *Why Mathematicians Have Not Been Utilized More in Business.* One of the reasons for the fact that mathematicians have not been used more extensively in business is probably economic. The attitude of business in general is that a mathematician is a luxury. But as I have stated above the growing scarcity of materials and the high cost of experiment tends to weaken this attitude and to produce a demand for more mathematical work. Another reason lies in the type of training that has been given to mathematicians and the atmosphere in which mathematicians have been reared. Many men who should naturally be in mathematics have been drawn into the physical sciences by better wages, by the pressure of teachers of the physical sciences, or by a complete lack of interest in the physical sciences on the part of their mathematics professors. Furthermore, the experiences of some industrial concerns which have employed a few pure mathematicians have not always been of the best, either because the company did not know how to make the best use of the mathematicians, or because the intuition or the background in the physical sciences of the mathematicians was inadequate. Among the common mistakes made by the mathematician in connection with practical problems are the following:

(a) He is unable to select a sensible set of physical assumptions upon which to formulate his problem.

(b) He exhibits various forms of lack of good sense, either because of lack of physical background or because he has had little acquaintance with men and the ways of the world. Some mathematicians take too many courses or stay in school too long and consequently are considered to be crackpots, geniuses or adolescents.

(c) He likes to overuse mathematics and to revel in bigger and better formulas. He is unable to simplify his results to a form which ordinary men can understand. He seems to feel that his work is finished if he exhibits a set of differential equations or some sort of abstract solution when, actually, his work is not finished until it is in a form suitable for simple computation. In many industrial situations, complete numerical results are the only useful product.

(d) He claims to have proved a physical theory when neither his assumptions nor any of his conclusions have been subjected to the proper physical tests. Very good men have made this mistake.

(e) He changes the essential assumptions of a problem in order to produce either a problem which he can solve or one that is more interesting to him than the problem actually at hand. The simplification of a problem merely to obtain an answer is not always the way out of a practical problem in industry. Such procedures lead to solutions which are often completely worthless in industry or elsewhere. Many good men in mathematics, particularly pure mathematicians, have committed these blunders.

(f) He seems to feel that he is not doing mathematics unless he is immersed in some very advanced branch of the subject, whereas actually the highest type of mathematics needed may be the use of an unusually intelligent train of thinking leading to a sound conclusion. In the end, the actual manipulative techniques used may be extremely elementary and simple—so simple in fact that, after the problem is solved, the ordinary professional man may gain from its simplicity the notion that anybody could have worked the problem in a short time and that the mathematician was not needed.

(5) *Training of Mathematicians.* I believe that mathematicians should continue to be trained for

teaching and for pure mathematical research as has been done in the past, for their work has been good and fruitful, and has contributed much to the training of men in many fields and to the discovery of new knowledge in those fields.

With an industrial demand for mathematicians, which I am sure will grow, there should be trained among the younger people an additional *new group* of mathematicians. The schooling of this new group should be somewhat different from that of the group in the teaching and pure research fields. This group should be trained for service to business and industry. The training should be at least as thorough and rigorous as that given for the doctorate to those men who plan to follow physics or chemistry in the industrial world. This group should have an extensive amount of practical industrial experience involving mathematical and physical activities. Such students should have sufficient training in the physical and engineering sciences to enable them to understand the language of the engineer, scientist and businessman, and the practical problems which they must face. They must have extensive training in the art of translating physical problems into mathematical language, training in the art of approximating a solution when a true solution appears to be impossible or too time-consuming, and training in the skills needed to translate mathematical solutions back into the language of the original physical problem. The details of such training have been outlined in a previous paper.[2] Briefly, the curriculum for such students should include about 50 per cent. pure mathematics, 30 per cent. applied mathematics and 20 per cent. engineering and physical sciences. The training in pure mathematics should be very broad. The courses should be given with the highest standards of rigor now generally in force in our better graduate schools. Comprehensive courses in the fields of analysis, geometry and algebra are required. Work in statistics, probability, graphical and numerical methods should be insisted upon.

The course in applied mathematics should be given with strong emphasis on the physical basis of the material and with greater care in reasoning than is currently manifested by those in the physical sciences who teach such courses. Comprehensive courses in elasticity, electricity and magnetism, analytical dynamics, celestial mechanics, hydro- and aero-dynamics, statistical mechanics, quantum mechanics and optics should be included. The courses in engineering and science should include a number of courses in modern physics and advanced engineering topics.

Mathematicians who are trained according to these principles and who have natural capabilities should, after a reasonable period of experience, have a mathematical consulting background of a value to business and industry which most engineers, physicists and other scientists seldom acquire, and probably should not try to acquire if they are to concentrate their energies on their main occupation of engineering, physics, or other speciality. Such mathematicians should be able to furnish industry and their scientific colleagues with a vast store of mathematical methods which are hardly dreamed of by their scientific friends, and which in the normal course of events remain buried in little seen mathematical journals. These mathematicians should exhibit an ability to abstract and formulate problems—an ability rarely possessed by their professional friends in other fields. They should know the methods of calculation and procedures which will greatly help their colleagues and the industries in which they are working.

This program will require a new ability and breadth of thought on the part of the faculties undertaking it. Various industrial concerns may have to be shown not only the need for the finished product—the mathematician—but also the facilities which they themselves have for aiding in the development of this type of mathematician.

(6) *Fields of Activity for This New Group of Mathematicians.* If mathematicians are trained in accordance with the suggestions mentioned above, then they can be profitably employed, not only in teaching and pure research positions, but in many other types of positions as well, such as:

(a) In the great research laboratories, both private and public, devoted to either practical or theoretical investigations.

(b) In a large number of industries, utilities and businesses which require planning, operational analysis, development and research.

(c) In government laboratories and related research organizations which must continue to devote their energies to the development and improvement of national defense.

(7) *Operational Analysis.* One needs only to read the financial reports of any large public utility, business or transportation system to comprehend the extremely small margin of profit between expenses and revenue; to realize how carefully such concerns must be operated, and how disastrous only a slight change in policy or methods or circumstances can be. This leads me to a discussion of a field which has *long* existed, in some form or another, but which has been only too little recognized; namely, the field of *operational* analysis. This form of analysis should be employed properly by every large concern which wishes to improve its product, and to maintain and increase a spirit of service to the public. In any group charged with operational analysis there is need

[2] Richard S. Burington, *Jour. Eng. Educ.*, 32: 4, December, 1941.

for people of many types and professions; such a group should include mathematicians with a broad understanding of the physical and engineering sciences as well as of statistics and business.

To illustrate what I have in mind, consider a transportation system—a common carrier operating a business of handling freight and passengers by air, water, rail or highway. Such a business is a tremendous one which ranges widely over the fields of finance, law, regulation, engineering, science, public relations, and so on. We can all think of isolated examples of the use of mathematical methods in certain phases of this business, such as in the routine compilation of statistical reports, efficiency studies, the design of bridges, the search for better materials, the study of the dynamics of rolling stock and the improvement of roadbeds. Most of this work is done by clerks, accountants, draftsmen or engineers. Some of it might be simplified and improved with the help of competent mathematicians. What I have in mind for mathematicians is beyond these routine activities. Such an organization should and often does have an operational group staffed by technical men from many different fields, men of broad understanding, sound intelligence and good sense. This group should include experienced and expert mathematicians and should report to a vice-president or other high-ranking officer who is primarily responsible for operations. The duties of such a group should include broad analytical studies of current modes of operations, and penetrating studies leading to improved methods and modes of operations which would in turn lead to decreased costs, more efficient service, better utilization of facilities and improved working conditions for the employees. The resulting healthy financial status would permit further desirable changes, improved facilities and equipment, and ultimately would lead to improved safety and service to the public. Such a group should be far more than a group of "efficiency experts." It should be so organized that it is subjected to a minimum amount of interference from individuals and groups who are swayed by tradition or a passion for the *status quo* or who fear a loss or change of their jobs. Similar groups should be, and usually are, set up in connection with the planning, development and research activities in the system, but I believe that the inclusion of competent mathematicians in such groups would be found to be both wise and profitable to the parent organization.

To illustrate something of what I mean, I shall give two examples:

(a) Recently a new engineer was assigned to a supervisory position entailing the design of certain types of structures which weigh many tons, and which have to be supported in water by floats. For many years the routine improvements and variations in these structures were made by engineers according to a sort of rule of thumb process which had grown up over a period of many years. None of the former engineers in charge of this work had ever even thought that the design problem could be handled successfully by mathematical methods. The new engineer thought differently but could not produce the solution. Instead of giving up he sought the advice of a competent mathematical consultant. The result was a completely new analytical method which solves the design problem by a procedure which is capable of yielding results to any desired degree of accuracy and can be carried through simply and quickly by any draftsman. The type of mathematics needed in the end was elementary, but the thinking required to produce this result involved a high order of mathematical intelligence.

(b) A few years ago an association of electric traction concerns organized and financed a group of scientists and engineers to study possible improvements in electric traction facilities and service. One of the most fruitful results of the work of this group has been the development of the light-weight, high-speed, rapidly accelerating and decelerating street-car which has so greatly improved the comfort and speed of service in some of our cities. In connection with this project, serious mathematical studies were made of the performance and dynamical characteristics of street-car trucks and of the car as a whole. These studies involved very difficult problems in analytical dynamics and function theory. They included an attempt to determine the best type of wheel, the best shape of wheel flange, the best shape of rail head, optimum spring suspension, the best way of mounting motors in trucks and the most efficient form of brake. Some of the most difficult work was done with the expert help of a man primarily trained in mathematics who in later years had become, both in theory and practice, a development and research engineer of the first magnitude.

A similar situation exists in the railroad field. Tremendous activities are afoot which point to radical changes and improvements in rolling stock.

An example of another type of problem met by a railroad is the freight tariff. Any one who could make a freight tariff which is simple to read and to understand and which still meets the many rulings which have accumulated over the years would do a wonderful service. I have often thought that a good mathematician, once he learned enough about tariffs, might make striking progress in this direction, provided the multitudinous pressure groups, regulatory bodies and dyed-in-the-wool tariff men would let him.

One of the difficulties and challenges which an operational group faces arises from the fact that the actions of any transportation system (and of many other businesses also) are subject to serious boundary conditions—conditions which often are incompatible and tend to be destructive. These boundary conditions arise in many ways; the dominant sources may easily be traced to two essentially independent sets of statutes one of which is administered by a body which regulates revenue by the fixing of rates, and the other by a board or other regulatory system which fixes the cost of labor. In most cases the actions of these regulatory bodies are entirely independent. Some of the effects of this system of regulation are evidenced in the set accounting procedures, safety regulations and rules governing the utilization of employees. Many an obvious improvement is never put into effect because of the seemingly unsurmountable difficulties resulting from a multitude of laws, regulations and traditions, and because of the difficulties involved in getting these hindering laws, regulations and traditions revised to permit the proposed improvements. These complicated boundary conditions are a challenge to the management of any transportation system and, in a similar way, to any operational group. The mathematician should be of value in solving such problems because of his special ability to organize a mass of details into a few simple formulas or principles.

What I have just said concerning a transportation system applies in a similar manner to almost every industry and business. There are many instances where mathematicians would prove to be of service if they were properly trained.

(8) *Cooperation with Engineers and Scientists on the Part of Mathematicians is Essential, but the Autonomy of Mathematicians is Desirable.* For a business to get the most out of a mathematician, the business administration must have an appreciation of the scope and limitations of mathematicians. Experience shows that, in order for a mathematician or other consultant to be of continuing service to industry, he must have his duties so arranged that he can keep up with the scientific literature and the developments in his own and related fields. Some universities have appreciated this need and have been reasonably careful in assigning loads to those members of their faculties who were interested in or engaged in research work. Industries, research laboratories and other agencies who expect to make the best use of scientific help should fully realize this. There is no such thing as mass production of creative thinking.

It is well to observe that mathematics is just as much a profession as physics or chemical engineering. It is necessary to maintain the autonomy of mathematics just as completely as that of physics or mechanical engineering. Domination may drive, but it can not inspire. If mathematics is to continue to develop and furnish improved tools for analysis and computation, human experience shows that mathematics must retain its entity—subject, of course, to the sort of cooperation which is the theme of this paper. This policy is essential if the storehouse of mathematical knowledge is to be maintained and increased for the benefit of the other sciences and for mankind as a whole. These are some of the principal reasons why mathematics has continued in most universities as a department on its own, as it should, and has not been made a part of a physics department or of some engineering department, wherein it would lose its identity.

(9) *Frontiers for Investigation in Physical Mathematics.* In recent years the style of research in physics has been toward quantum mechanics, nuclear physics, and the like. It has been commonly thought that there was no need for further investigations into the broader so-called classical fields of physics, such as electricity, magnetism, sound, and hydrodynamics. Our experience in this war has only too clearly shown how shortsighted this attitude has been. Actually we know appallingly little about most of these broad fields; here there are many matters for fundamental study by both physicists and mathematicians.

There is no field of physics, either ultra-modern or classical, that is not a veritable mine of inspiration and guidance for further mathematical inquiry. I can heartily recommend to any mathematician, pure or applied, or of the type that I have described above, that he examine one or more of these fields. If he is any kind of investigator at all, he will come away with inspiration for thought and search. If he is a pure mathematician, while he can not become an expert in an applied field over night—as some have thought during this war—he will at least have some vivid and stimulating impressions. He may sense the need for improved mathematical descriptions of physical phenomena—particularly for descriptions which can be derived from observation and experiment. If he looks a little further, he will find that his natural tendency as a pure mathematician to develop his theories and to obtain the greatest possible generalization is not always the most fruitful pursuit. He will find that, more often than not, some physical restraint usually limits generalization in a way which he, as a pure mathematician, may think to be wrong; but unrestricted generalization will commonly lead to results which are completely worthless in the physical sense.

One fact which some pure mathematicians never seem to learn is that the act of writing a mathematical paper in physical language does not make it an applied mathematical paper. Such practices can actually be bordering on fraud. On the other side, many physicists, engineers and chemists mistake papers involving mathematical processes for mathematical research, whereas such papers are not mathematical research at all, but physical studies involving mathematical language.

Another fact which a pure mathematician can not learn too quickly is that, in physical mathematics, a job is often not finished until actual numerical results are obtained. This fact suggests a consideration of computational processes.

(10). *Computational Processes.* Problems, often of a very difficult nature, are continually arising which involve extensive computation and which, therefore, require a large amount of work to carry through. Great strides have been made and are now being made in the development of machines capable of computing a vast variety of things. Even after such machines have been adequately developed by men skilled in such fields there is still a big job for the mathematician. These machines do not and can not think, and, therefore, can not evolve new processes. Mathematicians should try to discover mathematical processes which will greatly reduce, if not completely eliminate, the need of these machines in some processes. For example, I know of certain calculations being done by machine which can be done much more simply and more quickly by a graphical process with which any eighth-grader can obtain an answer as accurate as the problem may require. This process was evolved by excellent mathematical methods.

One field which is encountered frequently in working problems in electricity, elasticity and hydrodynamics involves the solution of partial differential equations with very nasty boundary conditions. True, there are various analytical methods of obtaining the complete solution of some of these problems, and numerical processes have been devised for carrying through approximate solutions in many cases. I believe, however, that there is much room for advance in these fields. A practical solution might entail new types of computing machines, such as the engineer has developed for the study of strains in structures by photo-elastic methods suitable for two-dimensional studies but with the scope of the method extended to three-dimensions. The alert mathematician should, however, contemplate an entirely *new* mathematical approach for solving such problems numerically without the need of such elaborate model studies with, perhaps, either no need for a machine or, at most, a machine which is simple and produces results with speed.

(11) *Statistics.* Much of the statistics commonly in use in business and industry has been developed by people who are not mathematically trained. This product has been useful and some of it is very good. I am sure it would be done even better and would be of more use if mathematically trained men were to assist in its development. In particular, it would have been possible to avoid numerous examples of profound nonsense which have found their way into routine statistics, had more good mathematicians been consulted in the past. In this field alone, if mathematicians had taken more interest long ago, there would have been a much greater market and demand for mathematically trained people. To-day there are several excellent schools of mathematical statistics, and they are engaged in consulting service which is at times valuable to those who make use of them.

One of the things I have noticed about mathematical statisticians is that they often produce reams of tables and graphs with the result that they completely bewilder the individuals or the concerns interested in the results. Statisticians must learn how to present their results simply and attractively and must present those results in such a way that any normal individual in the business concerned can grasp their import readily and quickly. If they are unable to come to any proper conclusions because of lack of information or for some other reason, they should bring the matter out completely in the open. In any case, they must be honest and must clearly state the nature of their premises and the degree to which their results depend thereon for reliability.

(12) *Algebra.* There has been a very decided rise in the level of algebraic processes used in engineering design and research in general. Since there is still a great lack of simple numerical means of carrying out needed calculations, I think that a fresh approach to these problems is required with the primary purpose not the discovery of new algebras, but rather the development of simple computational schemes. For example, many uses of the discriminant are encountered in dynamic problems where numerical values of the roots must be found. You may say "get a computing machine," but what is needed is something far better. Mathematical improvements in the calculations relative to the theory of sets of quadratic forms and related matrix algebra would find an immediate use with those who use such mathematics in electrical circuit theory, in dynamics and in the study of the physical properties of crystals.

Many investigators in mathematics have the opinion that as soon as they have exhibited a set of fundamental theorems they are through with the job; actually, they have just started in the practical sense. For, while the theoretical processes may have been reduced to a minimum of simplicity, the practical problem may still be of immense difficulty. The true mathematician should see his work through the ultimate stages of its practical and effective use, and he must insure that, in its final form, the process is ready for utilization by men of little training or background.

(13) *Invariants.* Most mathematicians have a good

understanding of the role that invariance plays in various branches of mathematics. But the mathematician will learn that he must pay more attention to invariants when he embarks upon any physical application of mathematics, for in some form or another, invariants and their discovery are of fundamental importance in the development of any physical science. The fact that the heyday for the researcher in algebraic invariants passed with the discovery of Hilbert's basis theorem in no way reduces the importance of a conscientious search and high respect for invariant properties when working in physical mathematics. Thus, with the rapid development of plastics, glass, plywood, fabrics and wood products, a whole new study must be made to determine their invariantive properties, such as strength, brittleness, fatigue resistance, viscosity, crack-propagation and the influence of external conditions. The uncertainty in the application of the earlier theories of elasticity is a problem in this connection. Serious attention must be given to more general or new theories of elasticity, and along with this must come a search for an improved mathematics capable of describing adequately the theory and its adaptations to the use of engineers. Some of this theory may require a broader use of statistical and probability methods. Undoubtedly, lying moldering in the archives of libraries, there are general solutions to some of these problems already nearly worked out which, with some modernization and extension, may yet prove of real value. Real progress will not result until the practical problems of the engineer can be solved quickly in the form in which they arise in practice. Model studies have been useful, but there is still wide scope for true progress in this field.

(14) *Biophysics.* In recent years there have been many papers in mathematical biophysics which are concerned with quantitative theories of a wide variety of biological phenomena. Quantitative types of biological theory are many years behind their analogues in physical science. As these theories progress, it may well be that they require new mathematical theories or that they require a more expert use of known mathematical techniques than biologists and the medical profession can be expected to possess. This field may need the services of mathematicians with a training and interest in the biological sciences.

(15) *Hydrodynamics Aerodynamics.* Much of the mathematics which is being written on problems in aerodynamics and hydrodynamics can not adequately describe or predict such phenomena. Mathematicians who attempt to work in those fields should concern themselves with an understanding of the phenomena which can be treated by their theories. They should spend a considerable period in studying experimental data in order to adapt their theory more nearly to actual situations. Aside from a great multitude of aerodynamic problems which are still to be solved, the mathematician with good physical background will find large fields of research relating to the nature of the destructive action of cavitation, the behavior of blade profiles when the velocity approaches that of sound, or the theory of stress due to the collapse of vapor bubbles in a liquid. These, and other fields, are still scarcely touched.

(16) *Importance of This Program to the Nation. Summary.* It is without question most necessary for the national interest and for the preservation of peace in the years to come that we encourage and maintain a high level of research in this country, both in the pure and the applied sciences. Accordingly, I consider the program proposed herein to be of considerable importance.

I have indicated some of the directions in which mathematicians should concentrate their efforts. I consider the program suggested herein as intended to *supplement* the older philosophy which has prevailed for many years.

I have urged that pure mathematical research be given continued emphasis and encouragement, but in addition, it is most necessary that a new high level of research in physical mathematics be encouraged and maintained in the years to come to a far greater extent than ever in the past.

This program entails a new training program to produce new types of mathematicians; and this will require the whole-hearted cooperation of educational institutions and many businesses and industries.

It would then seem most natural for mathematicians to find their way into industry on a much larger scale than ever before, and for industry to take much greater advantage of existing mathematical knowledge and talent than ever before. So I say to mathematicians, "By your deeds prove your worth"; and to industry, business and research organizations I say, "Take fuller cognizance of mathematical methods and mathematicians, and by all means make a fuller and wiser use of them than you ever have before."

OBITUARY

RECENT DEATHS

DR. EDMUND BURKE DELABARRE, professor emeritus of psychology of Brown University, has died at the age of eighty-one years.

DR. JOHN C. GRILL, head of the department of pathology of the Marquette University School of Medicine at Milwaukee, died on March 17 at the age of fifty-two years.