SCIENCE

VOL. 100 FRIDAY, NOVEMBER 3, 1944 No. 2601

SCIENCE: A Weekly Journal devoted to the Advancement of Science. Editorial communications should be sent to the editors of SCIENCE, Lancaster, Pa. Published every Friday by

THE SCIENCE PRESS
Lancaster, Pennsylvania

Annual Subscription, $6.00 Single Copies, 15 Cts.

SCIENCE is the official organ of the American Association for the Advancement of Science. Information regarding membership in the Association may be secured from the office of the permanent secretary in the Smithsonian Institution Building, Washington 25, D. C.

EDGELESS STARS

By Dr. SERGEI GAPOSCHKIN

HARVARD COLLEGE OBSERVATORY

(1) *Introduction.* In the last three or four months there have appeared some new results in the study of the eclipsing variable stars, which seem to be gaining more and more importance. In order for the reader to understand this I should like to describe first the importance of the eclipsing variables for the study of stars in general and then proceed to the edgeless stars.

(2) *The Fundamental Parameters.* There are a few characteristics of a star which are fundamentally important, or at least considered fundamentally important; they are mass, radius and effective temperature. The eclipsing variables provide us with these characteristics. Through the eclipsing variables we think often about the stars in general. That is why the research on eclipsing variables has never ceased to be interesting, and valuable contributions are in progress in many countries all the time. It is with regret that the author can not discuss this general research on eclipsing variables. But space is limited and the new results given here seem to have little connection with the general problems.

Since the time when the first spectroscopic binaries were found, it has been realized that the application of the law of gravity to the observed motions of the components in a binary would lead to the determination of their masses. Since, however, the tilt of the orbit of the spectroscopic binaries is undetermined, the spectroscopic observations alone give only the so-called minimum masses. If it happened that a spectroscopic binary is at the same time a photometric binary, we can do some additional computations. In such a case we see how one component covers the other, producing an eclipse. From the amount of the drop in brightness of the star we can estimate the tilt of the orbit. In other words, we can remove the indeterminacy inherent in the purely spectroscopic observations of a binary. That is why the eclipsing variables are very important. Through them we determine the masses of the stars. Indeed out of about 120 well-determined masses known to-day, the eclipsing variables have provided more than 80. The rest are found from the observations of visual binaries. If one remembers that the visual binaries embrace a less varied group of stars than the eclipsing variables, the usefulness of the latter is evident.

In addition to supplying us with masses, the eclipsing variables give us the means of finding the radii of the stars. Indeed they have no rival in this field. The sun is the only star whose radius we see directly and can measure. There are a few (about a dozen) large red stars which were measured with the interferometer, and their angular radii are accordingly known. But for an idea as to dimensions for the vast majority of stars we are indebted to research on the eclipsing variables. The duration of the eclipse gives us a clue about the sum of radii in terms of the distance between the centers. For example, if the components were in contact one would have a "permanent" eclipse, and if the components were far separated the duration of the eclipse would be very small in respect to the period of revolution. Each case is naturally different in details, but altogether we now have several hundreds of radii of stars determined in this fashion.

The effective temperature of stars has its determination in the study of radiation. But, in practice, astronomers here again are indebted to the study of the eclipsing variables. For if we define the effective temperature by saying that it is a temperature (T) which makes a star so bright (L) and with such a radius (R) that

$$L \sim R^2T^4,$$

then, having found the radius in the manner described above and the intrinsic brightness from the trigonometric parallax and the apparent brightness, we have a definite value of T. From these considerations it follows that the radius is more fundamental than the effective temperature, in practical determination of their values. For the effective temperature, as far as the research on eclipsing variables is concerned, is determined after the radius has been found. It must be mentioned that the determination of radii is to a small extent dependent on the mass of the system. But the error in the determination of the mass is not great, therefore the radius depends chiefly on the type of the eclipse.

From the steepness of the eclipse we can deduce the ratio of radii of the components. For if the components are equal in size the drop in brightness during the eclipse will not be the same as when the components are different. In the former case we should never have constant brightness at the bottom of the minimum, even if the plane of the orbit were exactly in the line of sight. If the stars differ in size the eclipse may have a flat bottom. This consideration is based on the assumption that the discs of the stars are uniform in distribution of brightness. From the exposition below it will follow that one can have a flat bottom even in the case of equal radii but with non-uniform brightness over the disc.

The present methods for evaluating the essential elements from eclipsing variables are naturally not new. They have been realized many years ago, in the beginning of this century, in the works of Meyer, Dugan, Ceraski, Blazko and especially in the later work of Russell and Shapley. The technique of computations has been modified and extended notably in Holland by Fetlaar, in Russia by Krat, Nekrasova, Martinoff and Odinzov and in the United States by Wyse and especially by Kopal. The observational and computational material has been greatly accelerated in the works of Martha Shapley, Zessewitch, Henrietta Swope and through the computing bureau financed by a grant from the Milton bureau of Harvard University. Some of the recent results are incorporated in this paper.

The fact that we are close to the sun has much to do with many of our conceptions, and one of them is the conception of the sharp disc: that is, that the radius has a quite definite value, which is the better known the more precise the instrument used in determining it.

In the computation of the radii of the eclipsing variables this conception of one definite radius for the star has always been axiomatic. To be sure, we have made modified computations for the radius of eclipsing variables to take into account the so-called limb-darkening, the fact that the brightness on the disc of the star may not be uniform from center to the edge. In cases where one solution (uniform or non-uniform brightness) was more representative of the photometric observations than the other, we have adopted it as giving the definite value of the radius of the star. Sometimes neither the uniform nor the "darkened" solution gave a perfect representation of the observations and the deviations were not such as could be ascribed to ordinary observational errors, whether we observed in the visual or photographic light. The radius was generally accepted as uniquely determined. In confirmation of this procedure we note that the formulae given in all text-books for the derivation of the color index of a star are based on the assumption of one radius for all wave-lengths. My results seem to lead to an opposite conclusion.

(3) *Light Curves in Several Colors.* If one compares the duration of the eclipse for the same system in photographic light and in visual, one finds for some eclipsing systems that the photographic duration is shorter than the visual; it takes less time. Since the inclination of the orbit must be the same for both sets

of observations it follows that the radii (or at least one of them) are smaller, when measured in photographic light than in visual light. This might be called a "shrinking" effect for a star; in going from the visual towards the photographic wave-length the calculated size decreases. Generally, it may be called the "edgeless" effect.

In addition, the steepness of the drop of brightness is greater in photographic light than in visual. It seems thus as if the ratio of radii expressed in the usual terms of the eclipsed to the eclipsing component is smaller in the former than in the latter case. The consequence would be that one of the components "shrinks" more than the other, or, in other words, the one component is more "edgeless" than the other. This is probably correct for the systems with widely different components. If, however, the components are nearly equal in size and in spectral characteristics the ratio of radii will probably be the same in visual and photographic light.

A third part of the same edgeless effect is that the elliptical shape of the close components differs with wave-length. When they are close to each other the usually spherical shapes are distorted into ellipsoidal shapes with the major axes pointed toward each other. Then the light of such a system will no longer be constant outside the eclipses and it is obvious that from the amount of variation of the brightness outside the eclipse we can obtain an idea about the degree of ellipticity of the components. We usually assume that both components have the same ellipticity and are strictly in alignment. The deviations from these assumptions have been discussed only in recent times (Kopal, Krat and Odinzev) and are difficult to follow observationally (Martinoff). The edgeless effect expresses itself in the fact that the ellipticity in photographic light is observed to be smaller than in the visual. Summarizing, the effect consists of three parts: The radii are smaller; the ellipticity is smaller; the ratio of radii is smaller. These results were first found on the two systems RX and SX Cassiopeiae about four months ago. The number of the known eclipsing systems with the shrinking or edgeless effect is increasing rapidly. In the table below I summarize the observational material.

I should like to accentuate one detail of this result. The discovery (or whatever it may be called) of the effect was made possible by comparison of the results obtained from visual observations with those from photographic ones. But such intercomparison of the light curves of eclipsing variables is at least twenty-five years old. And since the difference in the duration of eclipses obtained from two light curves is rather large it is possible that this difference, or this edgeless effect, has been noticed by many workers on the eclipsing variables. But if it was, the discrepancy apparently has been authoritatively disregarded and one set of observations was considered unreliable. Or attempts were made to bring the duration of the eclipses from both light curves into coincidence.

Table 1 contains nine systems, known to me, on which I have been working photographically and for which results from visual observations are published already. I have arranged the stars in respect to the length of the period, for it seems that the effect depends much on it.

TABLE 1

ECLIPSING SYSTEMS WITH "EDGELESS" EFFECT

Star	Period	Spectra	Change in radius Per cent.	Change in ellipticity Per cent.
SX Cassiopeiae	36d.6	A6, G6	140	13
RX Cassiopeiae	32.3	A5, G5	60	8
AR Monocerotis	21.2	?, K	44	5
RZ Scuti	15.2	B2, ?	13?	?
RY Persei	6.9	B6, F8	25	?
RS Can. Venat.	4.8	F4, G8	18	2
U Cephei	2.5	A0, G8	11	3
RZ Cassiopeiae	1.2	A2, ?	9	0?
ER Orionis	0.4	G1, G5?	3	4

The change in radius and in ellipticity is expressed in terms of the photographic values for the radius and for the ellipticity.

From the nine variables, seven light curves were compared after the two photometric systems had been reduced to the international scale.

The first spectra in Table 1 are those of the star which is eclipsed at the primary minimum; it is always of "earlier" type and the observed edgeless effect refers chiefly to it. In most systems the second minimum is too shallow to discuss the reversed effect on the later type component by the earlier. Since the ellipticity is always a combined effect and since the later component is in all our systems the larger one of the two, the change of the ellipticity as given in the table is probably chiefly due to the change of the component of the later spectral type. It should be pointed out that the uncertainty in the ellipticity is smaller than in the determination of radii, so that the changes in ellipticity as given in the table are of the same degree of reliability as they are for the changes in radii.

The system AR Monocerotis is very interesting because the observed spectrum is not that of the component which is most conspicuous in the photometric observations.

Considering the table for a moment, we see that the largest effect is for the first system: the radius of its blue component is in photographic light four tenths as large as in the visual. Probably this system is an extreme case in physical and observational conditions, but we may expect that eclipsing systems with much longer periods than of SX Cassiopeiae will show an even larger difference. Looking down the table we see that there is a strong correlation between the period and the size of the effect. The small system ER Orionis, the components of which are similar to our sun (the radii are about 0.6 and the masses 0.5 that of the sun), does show the difference, but the smallest in the table. Both the sign of the difference and the photographic radii are smaller, and the degree of the difference for this system seems to be of interest in connection with our general approach to the problem of edgeless stars. The system was observed in both wave-lengths by Taylor and Alexander four years ago, whose data are given in this table. The spectrum has been determined by Struve quite recently.

Except for four systems, the rest of the table contains systems of which the chief component is of spectral class A. Probably that is why the correlation between the period and the edgeless effect is so strong. The tabulation begins with a supergiant and ends with the normal A component of RZ Cassiopeiae. Since the edgeless effect was pointed out only in recent months the number of recognized systems is, as yet, small in comparison with the two or three hundred eclipsing systems for which the visual and photographic light curves may be obtained; therefore no affirmative general statements should be made.

Some of the changes given in the table are within the limits of the errors of the observations; thus real values of the effect remain uncertain for such systems. There is, however, for all a definite indication of the edgelessness of the stars.

If we consider the physical structure of a star's atmosphere in a more general theoretical way than has usually been done in connection with the study of the sun, we can be certain that we should not expect in the small systems like ER Orionis and RZ Cassiopeiae as large an effect as is found for the large systems like SX and RX Cassiopeiae.

It is interesting that this new effect is very large in comparison with already well-known effects in eclipsing systems—the reflection, periastron, gravitation, ellipticity and alignment effects. In fact, for some systems it is larger than all these others put together.

In concluding the observational survey of our problem from a purely photometric point of view, it is only proper that some spectroscopic results should be mentioned. Struve at Yerkes and McDonald observatories was able to establish the motion of the enveloping atoms around the components of the two first systems of Table 1. If we consider his observations along with the photometric ones, we can see that the interplay of the components and their atmospheres during the two eclipses (visual and photographic) is correlated with the velocities of hydrogen and metallic atoms. The work at McDonald and at Harvard was entirely independent, yet each leads to the same results, being obtained in supplementary fields. And the spectroscopic work leads to the conclusion that the edgeless effects just described are associated with the possession of thick atmospheres.

Photometrically we observed an integrated effect, while spectroscopically the velocities of individual atoms in the thick atmosphere are observed. For some of the systems of the table large spurious eccentricities of the orbit are obtained for the blue component, while photometric results give a circular orbit. A small difference in the eccentricities has been known for a long time and it has been recently ascribed by Sterne and by Kopal to the effect of gravitation. In the light of this new effect of the thick atmosphere, the phenomenon may receive another explanation.

(4) *Theory of Thick Atmosphere.* It must be said at the outset that theoretically the idea of the elusiveness of the radius has been developed as early as 1934 by Kosirey with a specific application to the peculiar Wolf-Rayet and P-Cygni stars. Practically at the same time Chandrasekhar has developed the general theory of thick atmospheres. In other words, it seems that theoretically everything is ready for the physical approach to our problem. Following this theory, Whipple and Payne-Gaposchkin made a theoretical application to the expanding envelopes of Novae. It is needless to say that the stars which Kosirev and the above-mentioned authors considered are in many ways abnormal, although it seems that their abnormality is not so gross as has been thought twenty years ago. But if a star is conceived, not with a sharp edge but with rather diffuse surroundings, the theory will be applicable to many stars.

If we write, in accordance with Kosirev and Chandrasekhar, that the density gradient in the thick atmosphere has the general form

$$\tau \sim 1/r^n$$

where τ is the so-called optical depth, r is the radius of the star at the optical depth unity, and n a constant which has to be evaluated from other considerations, then for the normal atmosphere like that of the sun, n will have a value much larger than 1.0 (probably around 3 or 4), and for Wolf-Rayet stars n is close to 1.5, as follows from the work of Kosirev. As we go on taking different values of n, especially between 1.1 and 3.0 we obtain many gradations of the elusiveness of radii. Kosirev and Chandrasekhar have shown that the problem of the transfer of radiation has a solution for such atmospheres, which is of more general form than it is for the sun.

Payne-Gaposchkin and the author have recently evaluated the distribution of brightness over the disc (or whatever it may be called) of stars with such thick atmospheres. The brightness depends on the above-mentioned *n*, the wave-length, the initial temperature, the distance from the center and the angle of sight. Our interest was chiefly confined to showing whether or not the theoretically predicted eclipses will have a dependence on the wave-length in the same sense as observed. The application was made to thirty-six cases, depending on difference in brightness and size of the components, *n* and on wave-length. It may be mentioned before publication that the theoretical results seem to be in full agreement with the observations. The photographic minimum is narrower than the visual. Of course, in a strict sense, both eclipses begin at the same time, but the distribution of the brightness over the disc in the photographic light is so different that the eclipse apparently begins much later. It would follow from this that the usual law of darkening towards the limb is only a limited case of this general distribution of the brightness over the disc with the thick atmosphere.

It is evident that the real observational results are more complicated than we can now theoretically handle. There is a serious question about the effect of ellipticity, and if we remember that the thick atmospheres of the two components may fill the whole space between the components, so that a great distortion in the isophotes and in velocities of atoms can be expected, the problem is indeed complicated. Then there is the inherent difference between the two components in some systems. It seems, however, that there is no doubt that the first application of the theory to the observed difference in the light curves has met with some success.

(5) *Conclusions.* The theory of thick atmosphere of Kosirev and Chandrasekhar, the spectroscopic observations of Struve and our photometric results seem to indicate that elusiveness of the radius is a more general phenomenon than has previously been thought.

The application of such an idea may have a wide scope and it may be that some observations on peculiar stars with envelopes, on large eclipsing variables like VV Cephei, ζ Aurigae and others or even on new stars (as with the duplicity of Nova Herculis) may receive a new explanation. Some other branches of astronomical research may also be influenced.

Since this effect has been found from the comparison of the visual with the photographic light curve, the visual observer of eclipsing variables will probably find a new stimulus. In the last decade many visual observations were considered unnecessary because of the ease and richness of the photographic work. Now the visual observations should probably not be confined to fixing an epoch of minimum, but will have another and more physical meaning. It is worth while to mention that the comparison in two wave-lengths does not need the basic photometry of the zero point, only of scale. It would seem that some revision of the sizes of large stars will be in order. Some peculiarities in their characteristics may be explained through the recognition of this edgeless effect.

It may also be that the internal constitution may receive a modified approach, for of the three fundamental parameters, two are becoming less definite, or have to be more precisely defined. If the effect is general, a great deal of work will probably be done in the future to determine the degree of thickness (the number *n*) in order to approach our established methods in computations of eclipsing variables with the inclusion of this effect.

In a recent communication Sir Arthur Eddington remarks with his usual simplicity and profundity that in our problem the conception of "occultation" radius and "bright" radius of a star should be kept apart, and though the problem is probably complicated this difference in the conceptions may contain a valuable element of bridging the new elusiveness of the radius with the old hard and fast conception of it.

OBITUARY

JAMES ALEXANDER SHOHAT

JAMES ALEXANDER SHOHAT, professor of mathematics at the University of Pennsylvania, died in the University Hospital on October 8, 1944, after an illness of three months. The cause of his death was bacterial endocarditis.

Professor Shohat was born in Brest-Litovsk, Russia, on November 18, 1886. He graduated from the University of Petrograd in 1910. He continued graduate studies there and in 1922 was awarded the degree of master of pure mathematics.

In 1922 he married Nadiaschda W. Galli, a physicist who later taught in the Physics Department of the University of Michigan, Mount Holyoke College, Rockford College and Bryn Mawr College. He and Mrs. Shohat came to the United States in 1923. He became a United States citizen in 1929.

From 1913 to 1917 he was an instructor in mathematics at the Polytechnic Institute, Petrograd. From 1917 to 1921 he was professor at the University of Ekatherineburg and from 1921 to 1923 at the Pedagogical Institute of Petrograd.

When he first came to the United States, he was an assistant in mathematics at the University of Chicago.