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THE LAW OF DIMINISHING RETURNS[1]

By Dr. JOEL STEBBINS

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IN the Encyclopaedia Britannica under the heading, "Law of Diminishing Returns," we find that this law was first stated in relation to agriculture:

> An increase in the capital and labor applied to the cultivation of land causes in general a less than proportionate increase in the amount of produce raised unless it happens to coincide with an improvement in the arts of agriculture.
>
> In economics, then, the law of diminishing returns is merely a precise statement of what is ordinarily recognized in the affairs of the working world. Everybody knows that, after a certain point, work in given conditions yields a diminishing return unless a better method is invented applicable to those conditions.

We in this society naturally include astronomy in the affairs of the working world, and it may be instructive to trace some of the applications of the law of diminishing returns in our own field. To begin with, this law took hold of the increasing size of refracting telescopes and brought further development to a close with the completion of the 40-inch refractor some fifty years ago. True, it was the rediscovery of the possibilities of the reflecting telescope that turned the construction of new instruments into the other form. But even if there had been no reflectors it was obvious from geometrical and optical principles, not to mention atmospheric limitations, that each increase in size of the objective of a refractor was accompanied by less than a proportionate increase of power.

The same law is now holding for reflectors even if the 200-inch, as we hope, should turn out to be a complete success. I understand that at Mount Wilson the 100-inch reflector cost about four times as much as the 60-inch, while the 200-inch will cost ten times as much as the 100-inch. No one thinks for a moment that the resulting gain in power will be proportional to the outlay. These facts are elementary to astronomers but to the laymen we might quote the simple

[1] Address of the retiring president of the American Astronomical Society, November 6, 1943.

rule that the cost of a good telescope can be roughly proportional to the cube of its linear aperture.

Despite these facts how many of us, if we were offered the money for a well-endowed 100-inch telescope, would have the moral courage to say that we would prefer to build an 80-inch and use the balance of the funds for additional improved attachments and for even better operation? The difference between 80 and 100 inches is practically not as important as the difference between poor and good seeing. Ask any observer on Mount Wilson which he would rather have, a fine night with the 60-inch or a poor, or even a fair, night with the 100-inch. The answer does not mean that the larger instrument has not been a success. Certain things have been done with it for the first time which had not been done with smaller reflectors. While experience has shown that some discoveries could have been made with smaller telescopes, the fact remains that they were not made. Near the limit of observational detection the extra power of the largest telescope available is an advantage.

My own experience with the law of diminishing returns began in another fashion years ago when I was a night assistant at the Crossley reflector of the Lick Observatory. My chief at the time was Charles D. Perrine who, to say the least, was an indefatigable observer. Those were the days, or rather the nights, of long exposures, perhaps only two plates per night, and toward dawn it used to seem to me that the last fifteen minutes of a four-hour exposure were time purely wasted. Though I did not venture to say so, I was sure that on the resulting plate no one could tell the difference between an exposure of four hours and one 15 minutes shorter, so that we might as well close up and go to bed. The answer to this argument is obvious to any one, but I could maintain that by shaving off a little time from each of several exposures we might get in an additional plate at the end.

However, Mr. Perrine kept on in his industrious way and proposed to expose for 10 hours over two nights on the region of Nova Persei. I believe that the time was cut to about seven hours by an oncoming storm, but on the resulting plate Perrine found the rapid expansion of the nebulosity about the nova, the first such motion to be discovered and one which holds the record for speed which will not be surpassed, inasmuch as it presumably represents the velocity of light.

Although this episode might justly be considered to indicate the failure of the law of diminishing returns, in another sense the instinct of a sleepy assistant was sound. Perhaps he had a feeling for the failure of the reciprocity law in photography, governing the relation between intensity and exposure time. To secure constant density of the photographic image, the time must be increased in greater proportion as the intensity of the incident light is decreased.

$$It^p = \text{constant}$$

In this equation giving the relation between intensity I and exposure t, where p is less than unity, we have the exact formulation of one law of diminishing returns.

But quite apart from the photographic action for threshold images the limit for faint stars that can be photographed with any telescope is set by the brightness of the sky background. The exposures of seven or eight hours with the Crossley reflector of focal ratio f/6, made forty years ago, are not practical now with faster modern plates. It takes an unusually dark sky to make it worth while to expose more than two hours with a reflector of focal ratio f/5. What we want is some region along the spectrum to act as a sort of window and allow us to photograph or see through to the stars and nebulae with a relatively reduced sky brightness. Such a window has been found and utilized by W. Baade with the 100-inch reflector. With red-sensitive plates and a filter transmitting the region 6,000 Å–6,700 Å he has crashed through both the atmosphere and the interstellar dust clouds to record stars that can not be reached with ordinary blue-sensitive plates. In fact, the prospect is that the beautiful and valuable photographs of the Ross "Atlas of the Milky Way" will have to be made all over again with a Schmidt telescope and red-sensitive plates.

Whereas, even without the failure of the reciprocity law, the fogging by the sky light would place a practical limit to the time of exposure of direct photographs, the same limitation does not apply in spectrum plates of moderately faint stars, where there is still plenty of contrast between a star image and the sky background. It has been said that until recently the best method of getting the red or infrared spectrum of a star was simply "to wait." By postponing the work for a year or so, one could count on the development of a new plate which would be so much more sensitive in the long wave-lengths that it seemed scarcely worth while to make the effort of long exposures with faster plates in prospect. Perhaps here is an instance of the law of increasing returns. Take things easy and some one else will make your work still easier. Seriously, he is wise indeed who can strike the proper balance between using the means at hand for a given problem and waiting for or developing new means for doing the same thing better or more easily.

At one time, when our science was being divided into the old astronomy and the new astrophysics, there were more than one of the old guard who used to say that in the astronomy of precision, as they called it, one could count upon an hour's results from an hour's work, whereas in astrophysics a large part of the experimenter's time was likely to be wasted. But even in well-established routine researches the law of diminishing returns came into force. Consider, for example, the weights assigned to star positions from different catalogues of meridian observations. In the Boss "General Catalogue" the probable error of a position was considered to consist of two parts, the first being independent of the number of observations and the second diminishing with the number of observations according to the formula,

$$r^2 = r^2_0 + r^2_1/n$$

A glance at the tables in the "General Catalogue" shows that the weights almost never increase in proportion to the number of observations, and after a certain limit no greater weight is given no matter what the number of observations. We all probably treat our own measures of any kind in the same fashion. We repeat settings only to the point where additional ones are of little value because of the presence of errors which are not eliminated. This principle is applicable throughout physical science.

But it is not so much the fact that repeating the same work over and over leads to the law of diminishing returns as that some new method will revolutionize a whole field. When the late Dr. Frank Schlesinger took up the determination of parallaxes by photography with the long-focus Yerkes refractor, all previous parallaxes with the heliometer or meridian circle were soon superseded. Yet only last spring during my final visit with Dr. Schlesinger he remarked that the observational program for trigonometric parallaxes is about worked out. In forty years the law of diminishing returns has taken hold again.

It was Simon Newcomb, the first president of the American Astronomical Society, who said: "To be revised, pulled to pieces, or superseded as science advances is the common fate of most astronomical work, even the best. It does not follow it has been done in vain; if good it forms a foundation on which others will build. But not every investigator can look on with philosophic calm when he sees his work thus treated." Another president, Edward C. Pickering, while presiding over a session, once remarked, in a discussion of some of the new methods of stellar photometry, that he had made a good many visual photometric observations himself (the number was more than a million) but he expected them all to be superseded. In his later years Pickering decided that the magnitudes of the fainter stars could best be determined by photography and he diligently devised and carried on photographic programs. However, the visual work of Pickering still stands and will stand for a long time.

My own field of photometry may be considered to come under the law of diminishing returns, since the light from a star diminishes with the square of the distance and in proportion to the absorption in space. Over the years I have read a number of papers before the society, probably of diminishing value, and the present may be an appropriate occasion to summarize some of them. I believe that at one time I reported that the sensitivity of the selenium photometer had been increased 100-fold; at another time, two magnitudes more. Then the photoelectric cell was reported as being two magnitudes better than the selenium cell. In a few years more perhaps another 1.5 magnitudes was picked up with the photocell, and then the application of the thermionic amplifier was developed by A. E. Whitford, giving a four-fold increase or still another 1.5 magnitudes. Adding these reported improvements together we have $5+2+2+1.5+1.5=12$ magnitudes, or a factor of 63,000.

Strangely enough, something has been left out. When F. C. Brown and I first mounted a selenium cell at the focus of a 12-inch refractor and pointed the telescope at Jupiter there was no detectable response whatever. Since then the faintest object which Whitford and I have measured with a photocell is a star of magnitude 16.1 with the 100-inch reflector. As the probable error of measurement was about 10 per cent., the limit of detection may fairly be called magnitude 18. From Jupiter at magnitude -2 to a star at $+18$ the change is 20 magnitudes. This advance is perhaps not so much a measure of the excellence of the latest developments as of the crudeness of the first attempts. Moreover, we must allow say 5 magnitudes for the difference between a 12-inch and a 100-inch telescope, leaving 15 magnitudes or a million-fold improvement in the apparatus itself. The limit of magnitude 16 was reached six or seven years ago, and the law of diminishing returns is working now. We can predict with confidence that the next 20 magnitudes will be harder to get.

When my friend and colleague Jakob Kunz passed on some five years ago he was still optimistic about getting a more sensitive photoelectric surface of potassium hydride. In fact he had actually produced cells which gave ten times the response of some of the best cells on hand, but unfortunately they were not stable and the surfaces deteriorated in a few days or weeks. There are other blue-sensitive cells now available which give a response in micro-amperes per lumen considerably better than the best Kunz cells, but the commercial cells are usually not constructed with the extreme insulation needed for detecting small currents. It is the old story of what you want versus what you don't want, or the ratio of signal to noise, as they say in radio. Suffice it to say that there already exists the possibility of a photoelectric surface which, if deposited in the right way in the right cell or tube, will give greater effective sensitivity than anything so far available, but whether the improvement will be as much as 10-fold is more than I can tell.

But how about the relative precision of measurement as the effective sensitivity of a stellar photometer was being increased? The answer is definite; while six decimal places in sensitivity were being picked up, a single decimal place in increased precision was scarcely achieved. I used to have a goal of a thousandth of a magnitude for the probable error of the magnitude of a star, but I have never reached it. However, that precision has been reached by Gerald Kron at the Lick Observatory. He has established the light-curve of at least one variable star with normal magnitudes having a probable error of ± 0.001 magnitude (*Lick Obs. Bulletin,* No. 499, 1939). As Kron has remarked to me, the limit of precision for bright stars observed with a small telescope, say up to 36 inches, is fixed not by the photocell, not by the amplifier, not even by the astronomer; it is fixed by the quality of the seeing. It took me a long time to learn that fact, but it is true, and it has been demonstrated by Whitford at Madison and Mount Wilson. Using a photocell, a short-period amplifier and an oscillograph to reveal the rapid fluctuations of a star at the focus of a telescope, he has recorded the difference between poor and good seeing. In poor seeing a star may vary four- or five-fold in intensity within 1/20 or 1/30 second, while in good seeing the maximum deviation may be not more than 10 or 15 per cent. from the mean in the course of a whole second. With a galvanometer and circuit requiring 10 or 15 seconds to give a full deflection these irregularities are smoothed out to a great extent, but under ordinary conditions with a 15-inch telescope jumps of 0.5 to 1.0 per cent. at the end of a long-period deflection are quite common. With the 60-inch or 100-inch telescope the galvanometer is much steadier at the top of a deflection, and 0.1 per cent. or 0.001 magnitude does not look so far out of reach.

An example of the futility of taking more than a reasonable number of measures of the same thing is furnished by the light-curve of the variable, Delta Cephei, which I determined visually some 35 years ago. About 7,000 individual settings were made on 72 nights, and I decided that nothing would be gained by further observations. This visual light-curve may be compared with recent curves determined with the 60-inch reflector at Mount Wilson, using a photocell and filters which isolate six different regions of the spectrum. In about a third of the observing time which had been devoted to the visual work, it was possible to get light-curves in the six colors, each one being superior to the visual light-curve. The results furnish material for the theoretical study of this type of light variation. The new curves are uniformly smooth, showing no secondary fluctuations or humps. The amplitude of variation at 3,530 Å is about $3\frac{1}{2}$ times the amplitude at 10,300 Å. There is a retardation of phase for the longer wave-lengths in the sense that the maxima and minima of light are later in the infrared than in the ultraviolet. The colors of the star at different phases are quite close to the colors of normal giants, ranging from F4 at maximum to G2 at minimum in good accordance with the changes in the spectrum.

During the past ten years with my colleagues, Huffer and Whitford, I have spent a good deal of effort in determining color indices of stars with a photocell. These results have been criticized, principally by ourselves, because of the short base line or leverage furnished by the cell and filters used, the difference between the two spectral regions being little more than half the corresponding difference in the international system. The new six-color measures give so much more information about the radiation from a star that the old two-color measures are already out of date. Moreover, the extreme base line from 3,530 Å to 10,300 Å gives a scale some 7.5 times our former scale of color index. But here again, even while we are enjoying the power of the new method, the law of diminishing returns seems to have set in. The measures in the ultraviolet have been very useful in the application to extragalactic nebulae and in the determination of the law of interstellar absorption from reddened B stars, but experience has shown that for most stars a less extended base line will serve just as well. When a star's radiation is nearly a linear function of $1/\lambda$ there is no advantage in a very long base line, because of the dispersion in the characteristics of stars of the same spectral class. Moreover, the ultraviolet measures are especially affected by hydrogen absorption in some stars and by variations in our own atmosphere. Therefore, it may be best to reduce the number of colors from say six to four, omitting the ultraviolet and the blue or green.

To utilize an intermediate base line for color index we have recently tried out a cell and two filters with effective wave-lengths of 4,200 Å and 7,900 Å for a source at 10,000° K, giving a scale about 4 times the old photoelectric scale. This combination is good for the detection of small amounts of space reddening in early-type stars and is generally useful in measuring color indices of all stars. The red filter, however, transmits the radiation from what we believe to be one or more auroral lines in the neighborhood of 10,000 Å, certainly of wave-length longer than 8,500 Å. When this cell is exposed to the sky alone through an infrared filter, the relative intensity near 10,000 Å, compared with a solar type star, is 10 times the intensity of any other part of the spectrum. A further study of this line or lines awaits the spectroscopists, but we know that the radiation is atmospheric from its irregular behavior throughout the night.

A communication from Dr. V. M. Slipher confirms the presence of this strong infrared radiation in the night sky, but he has not yet determined the wave-length. A rough comparison of the observed relative galvanometer deflections along the spectrum of the sky with the deflection from a star of spectrum dF7 is as follows:

Wave-length	3,530	4,220	4,880	5,700	7,190	10,300
Deflections, dF7	8	8	10	8	8	8 mm
" Sky	8	5	5	8	13	112 mm

Also, compared with other regions of the spectrum, the infrared has varied from summer to summer somewhat as follows:

1941	60
1942	80
1943	112

One is naturally suspicious of a connection with the sunspot cycle.

Even with this handicap of the sky radiation the new color system which we call C_3 can be used on the brighter nebulae, and we have started of course with M31, the Andromeda nebula, just to see what would happen. It turns out that there is a difference in the color of the two sides of the nebula which, if interpreted as the effect of space reddening like that in the galaxy, gives at once the ratio of total to selective absorption. The new results may be summarized briefly. Let A_{pg} be the total photographic absorption, E_1 the old, E_3 the new, and E the international color excess, respectively. Then the different relations and the basis for each are as follows:

(1)	$A_{pg}/E_3 = 2.01 \pm 0.10$ (p.e.)	Andromeda nebula
(2)	$E_3/E_1 = 3.86 \pm 0.13$	Reddened B stars
(3)	$A_{pg}/E_1 = 7.8 \pm 0.5$	(1) × (2)
(4)	$E/E_1 = 1.90 \pm 0.12$	Seares, A stars
(5)	$A_{pg}/E = 4.1 \pm 0.4$	(3)/(4)

The weak step in the sequence is presumably in (4), the ratio of the international to the photoelectric scale. The result by Seares is from 52 A-type stars near the north pole, and is actually the ratio of the colors C/C_1 rather than the color excesses E/E_1. The latter ratio is probably higher than the former. Unfortunately, nature has given us few B stars in the vicinity of the pole, and the interstellar absorption there is too small to give a reliable comparison of the two scales of space reddening.

It should be emphasized again that these results depend upon the assumption that selective absorption in the Andromeda nebula is the same as in the galaxy, also that the apparent surface brightness of the nebula for the regions measured would be symmetrical about the nucleus if there were no such absorption. However, the ratios in equations (3) and (5) look reasonable and the value $A_{pg}/E = 4.1$ will probably be welcomed by those who have claimed that a higher value of this ratio does not agree with the conclusions from star counts and other evidence in the galaxy.

Incidentally, if we assume that the absorption is caused by a thin layer near the median plane, these photoelectric results are in agreement with the view that the main dark lane of the nebula is on the near side, and therefore that the direction of rotation is such that the arms of the spiral are trailing.

I have included these examples from photometry not so much to illustrate the application of the law of diminishing returns as to show some efforts to combat that law. Perhaps the difference between the law of diminishing returns and the law of increasing returns is merely the difference between looking backward and looking forward. It has been well said that just as soon as a problem becomes easy it ceases to be research; if you are doing real research you are likely to be in difficulties most of the time. We have it from Bobby Jones that there is no easy shot in golf. If it is easy to get your ball on the green you should be aiming at the pin.

If I were to draw any moral from these remarks, it would be to remember that it takes only a slight improvement over what has gone before to open up entirely new opportunities. If the law of diminishing returns seems to prevent us from doing something better, we can always try to do something different.

OBITUARY

HERMAN LEROY FAIRCHILD

1850–1943

ON the 29th of November Emeritus Professor Fairchild, of the University of Rochester, long an outstanding figure in American geology, passed on at the age of 93 years. Professor Fairchild was the last of a famous geological group belonging to an earlier generation, boasting many names that will be remembered as long as geologic science, as we know it, lasts.

For more than 70 years he devoted his life to education in science—teaching, lecturing, organizing, advising, investigating and writing—all with marked success. His more than 200 published writings covered a wide field and shed luster on the institution that he served with great devotion for more than a half century. He contributed much to organized science. No one in his time was more continuously engaged or more successful in developing scientific organizations to larger usefulness. He was a constructive person. Whatever he touched seemed to be improved. Every