$$\Delta\bar{x} = \frac{4\,v_0^3}{3\,g_1^2}\omega\cos\phi_1\sin 3\beta \begin{cases} > 0 \text{ for } 0° < \beta < 60°, \\ = 0 \text{ “ } \beta = 60°, \\ < 0 \text{ “ } 60° < \beta < 90°, \end{cases}$$

with the positive sense of x to the east.

If the projectile be fired to the west, for which $\alpha = 90°$ or $\sin\alpha = 1$, we have, in particular,

$$\Delta\bar{x} = -\frac{4\,v_0^3}{3\,g_1^2}\omega\cos\phi_1\sin 3\beta \begin{cases} < 0 \text{ for } 0° < \beta < 60°, \\ = 0 \text{ “ } \beta = 60°, \\ > 0 \text{ “ } 60° < \beta < 90°, \end{cases}$$

with the positive sense of x to the west.

In both of these cases the *deviation in range* (*i.e.*, $\Delta\bar{x}$) extends to the east if $0° < \beta < 60°$, and to the west if $60° < \beta < 90°$, and this deviation is zero if $\beta = 60°$. The first term in formula (1), namely: $\sin 2\beta \cdot v^2{}_0/g_1$, represents distance from the gun to the east if $\alpha = 270°$, and to the west if $\alpha = 90°$. It is from the terminal points of both of these distance-vectors that $\Delta\bar{x}$ extends to the east if $0° < \beta < 60°$ and to the west if $60° < \beta < 90°$, and produces no augmentation if $\beta = 60°$. We have thus proved the statements made in the second paragraph.

2nd Proof: Let us refer the motion of the projectile to a set of cardinal axes $0 - \xi, \eta, \zeta$ of which the origin 0 is at the muzzle of the gun, and the positive senses of the axes of ξ, η, ζ are to the south, east and zenith, respectively. Denoting the time derivatives of the coordinates ξ, η, ζ by ξ', η', ζ', respectively, the components of the muzzle velocity-vector (v_0) are $\xi'_0, \eta'_0, \zeta'_0$. Again referring to the above mentioned monograph, we find that equations (115), on page 63, express the coordinates ξ, η, ζ of the moving projectile in terms of the time t. If we equate to zero the expression for the altitude ζ and solve the resulting equation for t, we obtain the expression (123), on page 66, for the time of flight of the projectile. Substituting this value of t in the first two equations (115), we obtain for the coordinates of the *point of fall,* the expressions

$$\bar{\eta} = \frac{2\,\eta'_0\zeta'_0}{g_1} + \frac{4\,\omega}{g_1^2}\left[\zeta'_0\{\eta'^2_0 - \tfrac{1}{3}\zeta'^2_0\}\cos\phi_1 - \xi'_0\zeta'^2_0\sin\phi_1\right],$$

$$\bar{\xi} = \frac{2\,\xi'_0\zeta'_0}{g_1} + \frac{4\,\omega}{g_1^2}\eta'_0\zeta'_0\left[\xi'_0\cos\phi_1 + \zeta'_0\sin\phi_1\right].$$

If, in particular, the line of fire is along the east-and-west line (*i.e.*, along the axis of η), we have $\xi'_0 = 0$, and then the preceding formulae become

$$\bar{\eta} = \frac{2\,\eta'_0\zeta'_0}{g_1} + \frac{4\,\omega}{g_1^2}\{\eta'^2_0 - \tfrac{1}{3}\zeta'^2_0\}\zeta'_0\cos\phi_1,$$

(2)

$$\bar{\xi} = \frac{4\,\omega}{g_1^2}\eta'_0\zeta'^2_0\sin\phi_1.$$

Since now (*i.e.*, for $\xi'_0 = 0$) we have $\eta'_0 = v_0\cos\beta$, $\zeta'_0 = v_0\sin\beta$, (where β may now be regarded as measured from the positive η-axis and capable of ranging in value from 0° to 180°, so as to include the case in which $\alpha = 90°$ as well as that in which $\alpha = 270°$), the first of formulae (2) becomes the same as formula (1) when $\alpha = 270°$, and the second takes the form

$$\bar{\xi} = \frac{4\,v_0^3}{g_1^2}\omega\sin\phi_1\cos\beta\sin^2\beta,$$

which is the same as the expression for the drift (distance of the point of fall from the line of fire) given by the third of formulae (129), page 68, of the monograph, for the special case in which $\alpha = 270°$ or 90°.

The first of formulae (2) gives us the information we desire. For since $\zeta'_0 > 0$, it follows that the first term changes sign with η'_0 (since it enters to the first degree), whereas the second term *does not* change sign with η'_0 (since it enters to the second degree). Furthermore, the second term is positive, negative or zero according as

$$\eta'^2_0 - \tfrac{1}{3}\zeta'^2_0 = v^2_0\,(\cos^2\beta - \tfrac{1}{3}\sin^2\beta) = v^2_0\,(1 - \tfrac{4}{3}\sin^2\beta)$$

is positive, negative or zero, *i.e.*, according as the angle of elevation β is less than, greater than or equal to 60°.

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SUGGESTED CASTE TAXONOMY FOR THE COMMON TERMITE

FOURTEEN years have passed since the beginning of the work of the Termite Investigation Committee under Drs. C. A. Kofoid and S. F. Light, of the University of California. Early in the work, good fortune permitted my selecting as a subject of study the genus *Reticulitermes* with a caste system, more complex than that of any other wide-spread Nearctic genus of termites. I used, of course, the regular caste taxonomy of the time. Primary, secondary and tertiary reproductives were each studied as a caste. But long-time collecting yielded a series of specimens bridging the gap between the primary reproductive and the worker; the supplementary reproductives, far more prolific than the primary, gradually assumed, in a new line of thinking, the status of an intercaste. Nanoids, both soldiers and workers, were found, and at first were recorded as castes; then rare intermediate soldier-workers and still rarer soldier-reproductives had to be classified. At last I came to feel that we who are interested in the complicated society of the termitarium might be "picking castes from the air." Independent work since has extended my laboratory observations to include the chief species of *Reticulitermes* in each of the termite-yielding sub-regions of the Nearctic. I find that given sufficient time, we can produce or at least predict the appearance of most of the outstanding forms found in the labyrinths. This brought not a "break with the old system" but a gradual drifting away from it. What I regarded as a caste, fourteen years ago, may now seem no more than subcaste or intercaste; instead of listing newly discovered sizes and forms, each in a separate caste, it seems better to seek to relate each to one of the three more common castes: Reproductives, workers

and soldiers. Interblendings of these and maturity attained in different instars by different individuals would seem to account for all other forms and sizes, remembering, of course, the absence of nasutes in *Reticulitermes*. In *South Carolina Natural History* (No. 29, 1937) the existence of intermediates and subcastes is indicated, but I did not feel free to break completely with the idea of supplementaries and nanoids as castes. In *Neighborhood Research* (2: 3, 1938) I am inclined to treat secondary, tertiary and soldier-like reproductives as well as soldier-workers as intermediates of the three regular castes, and in the same publication (4: 1, 1940) I have outlined the scheme of classification of castes I am still using in my study of *Reticulitermes,* save for the substitution of what seem better names for some of the forms observed. Thus all forms have been found capable of relation to this classificatory scheme:

CASTE 1. Primary, primitive or archaic reproductives; the "king and queen" or "royalty" of older writers. Fully winged previous to mating and thoroughly pigmented.

Intercaste 1. Supplementary reproductives; "vice-royalty"; "secondary" and "tertiary" reproductives; brachypterous and apterous reproductives; neoteinics. Most of the forms here will be found to be the white "brachypterous" type. The older lines are too hard and fast; a gradual transition in form may be traced from the primary reproductive to what is apparently the worker form. We should here include all below the perfectly winged type and all above the infertile worker, in a broad reproductive-worker intercaste.

CASTE 2. Workers; ergatoids. Wingless individuals who do the labor. They are found in four sizes, which have been termed sub-castes:

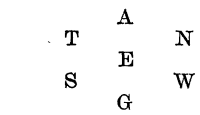
i. Midgets, or Nanoids, apparently adult at fifth instar
ii. Dwarfs, or Parvuloids, apparently adult at sixth instar
iii. Regulars, or Megaloids, apparently adult at seventh instar
iv. Giants, or Gigantoids, apparently adult at eighth instar

Intercaste 2. Worker-soldier or soldier-worker form. Known, so far only in the midget, and perhaps the dwarf form, but only one size positively known. Thus division into sub-intercastes is not justified. For sake of a different initial in graphs I have called this intercaste form the *guerrilla*, which word bears a suggestion of the diminutive, or at least the "small-time" warrior.

CASTE 3. Soldiers; guards. Fighters, who are unable either to reproduce or to work. They occur in the four sizes, or subcastes, given above for workers.

Intercaste 3. Reproductive-soldier or soldier-reproductive. Known so far only in a larger size corresponding to the regular-sized or perhaps the giant-sized soldier or worker. I have termed this a *tiro,* following the Latin and Austin Dobson's spelling of the word, which originally signified a newly levied soldier, or beginner. It really seems to be the last of the known forms to make its appearance, and its tyro—using the common spelling—nature shows in its being both soldier and reproductive but doing neither well.

Let us take now the initial letter of each of the names given to castes and intercastes, *viz.,* A from the alate or better the archireproductive caste, N from the "neoteinic" or better the neoreproductive intercaste, and so: W, worker; G, guerrilla; S, soldier, and T, tiro, with E for the egg-mass deposited during the entire history of the community. Arrange them thus:



Some individuals hatching from E the egg-mass will develop through about seven stages to become alates, the youthful archireproductives. Many more will develop through five to eight stages or instars toward W to become workers. A smaller number, through a similar number of instars, grow toward S to become soldiers. Now can any one believing in the modern scientific ideas suppose that the eggs of the ancient roach-like termite following a normal course and producing individuals that developed along a straight course, which we may readily visualize as an unserifed, or Gothic, capital I, thus normally growing into reproductives like their parents, at some time suddenly began following some strange V-shaped path into reproductives and soldiers, and in later and higher termites an equiradiate Y into reproductives, soldiers and workers? Such a theory smacks too much of some force leading the insects along the mystic Pythagorean monad, duad and triad, shaped respectively like Gothic I, V and Y. Whether they arose by saltations or more gradual development true castes must have reached their more distinct forms through a line of intermediates more nearly like the primitive ancestors than appears at a later time. One group of these intermediates has proved so successful a deviation that most of the work of reproduction is carried on by it, and the primitive form has assumed a sort of secondary role in the matter of egg-laying.

In the rise and decline of a common termite community, each caste, intercaste and subcaste has apparently a place and time for its appearance approximating the exactitude with which dramatic players come

upon the stage for their parts. At swarming time the alates emerge, dealate themselves, sometimes after, sometimes without flight, and after pairing off and constructing a small cell begin a primary community. In *Reticulitermes* the diminutive nanoid worker is usually the first form to be distinguished as an offspring of one of these pairs. Soon after, however, we expect a nanoid soldier, and in those groups in which they appear the soldier-workers or "guerrillas" in nanoid size are due about the same time. In isolated groups there may be a delay in the arrival of new forms and sizes for even two years. Fortunately, however, several of these families may be neighbors, since the alates swarm often by thousands and rarely fly very far. A struggle now begins which we can liken only to a small war of imperialism. A number of parents may be killed off by invasion and their offspring annexed to a larger group headed by a surviving pair of reproductives. With more communal feeding parvuloid and megaloid workers and soldiers appear in order. The guerrillas or soldier-workers disappear, and with a larger population none of the soldiers nor workers become adults in the earlier instars, and so midget and dwarf forms yield to the larger sizes, and in very old and populous groups some appear to retain a sort of perpetual youth and to grow into an eighth instar, so becoming giants or gigantoids. But before this group arises the population has grown sufficiently to enable it to support a number of short-winged potential reproductives. I have found these within three years and six months from the time of the establishment of the original cell by the dealated primaries. Six months later, or four years after the spring in which the community began its history, the first swarm of alates issued, the cycle having worked back to the same form that established the said community. The soldier-reproductive or tiro remains unaccounted for. Apparently these arise from small groups of workers cut off from the main body and left without brachypterous potential reproductives. If we carefully hand-pick under the microscope a group of wingless workers and set them apart in a separate termitarium we may expect after a long period, as much even as ten months, a wingless or almost wingless reproductive to arise. At times, however, in such a group or perhaps in a remote part of the labyrinth a juvenile with a tendency toward the soldier form is apparently transformed into the reproductive before any worker can be so transformed. It is the rarest of the regularly recognized forms, and seems to occur only in one of the larger subcaste sizes. Note this carefully. While the soldier-worker or *guerrilla* is to be expected in the low populations of young and growing communities and in a smaller subcaste size, the soldier-reproductive or tiro is rather expected in the low populations of old and decadent communities and in a larger subcaste size. It would probably take years of experimenting, but it would be interesting to place the eggs of young primaries in these old, decadent groups to see if any of the resulting young developed into the reproductive-soldier form. Incidentally, it is quite likely that such eggs placed in a highly populous colony would not produce any of the lower subcastes, but that the young would grow into a larger size before becoming definitive workers or soldiers. Are guerrilla and tiro mere size variations or subcastes of the same form generations apart? At present, it seems better to regard these two intercastes as separate forms, both perhaps almost vestigial in *Reticulitermes,* but in a more primitive genus, *Zootermopsis.* Prof. G. B. Castle has found a number of fertile soldiers, the females capable of producing eggs, the males with large and well-developed testes (Kofoid and Light, "Termites and Termite Control," 1934). Some of the more tropical genera, still higher in development than *Reticulitermes,* have produced what I am inclined to regard as a fourth caste, the nasutes. Incidentally some nasutes have lost the mandibles, while on one hand others have long soldier-like mandibles, and on the other hand some have short mandibles that may relate them to the workers. Such genera it has not been my privilege to study closely in life, and I make the suggestion, merely as a possible explanation of the origin of the nasutes, worth at least a thought. Was the intercaste between the worker and the soldier which plays so little a part in the *Reticulitermes* group as to generally disappear early in the history thereof, retained in more advanced genera to be developed into other forms?

Anthropomorphism long troubled the zoologist; apimorphism still troubles the student of castes; too long he has been bee-minded. Apparently the worker-bees can make conditions that result in a "queen," but the termite reproductive evidently yields some secretion that inhibits the normal sex-development of its associates and keeps them in the condition of workers. Remove the reproductives, and others arise, which in turn secrete an inhibiting substance, that continues to hold the less forward individuals in the energetic but unfertile condition. As the colony grows, many succeed in wandering to parts of the labyrinth where they are free from this influence and so develop into normal reproductives. This increase of population, with more abundant intra-communal feeding, also allows the development of larger sizes of subcastes previously mentioned.

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