

zoological exhibit collections of the New York State Museum, Accession No. 6342.

DAYTON STONER
LOUIS J. KOSTER

NEW YORK STATE MUSEUM,
ALBANY

THE TROPICAL CHIGOE IN CALIFORNIA

Tunga penetrans (Linnaeus), a tropical and sub-tropical siphonapterous pest, commonly known as chigger, jigger, chigoe or sand flea, has heretofore remained unreported as adult from the continental United States,¹ except for one case from New Orleans.² Thirteen gravid females³ were recently (April 7, 1942) recovered from the eyelids of a Pacific horned owl (*Bubo virginianus pacificus* Cassin), at Oceanside, San Diego County, California, by Kenneth Stager.

The life history and etiology of this flea³ are of special interest in the present emergency. Its habitat is essentially warm, dry, sandy places. Although considered free living as larvae (with the one reported exception²), adults attack not only birds, but also other warm-blooded animals, including man. Though not known to be a vector of pathogenic organisms, its entry beneath the epidermis and invasion of the stratum lucidum produces irritating skin ulcers which are frequently complicated by secondary invaders.

Southern California is known to have many outdoor camping grounds. Camp directors should therefore be on the alert for its possible appearance in infested areas. Also, with the erection of many open-air military camps in the southwest, it seems particularly desirable that special studies as to the distribution in this country be made, and precautions taken to prevent its spread.

G. F. AUGUSTSON

ALLAN HANCOCK FOUNDATION,
UNIVERSITY OF SOUTHERN CALIFORNIA

ON NUMBERING BOOK ILLUSTRATIONS

I AM reading a book on meteorology, and I come upon this sentence, "Fig. 50b shows the typical features of a towering cumulus (see also Fig. 25)." Now Fig. 50b is right under the eye; but Fig. 25? Evi-

dently it is somewhere in the fore part of the book. I am at page 81, and I make a chance dive into the earlier pages and come upon page 47. It happens to carry Fig. 32. So I thumb my way back page by page until I come to Fig. 25 on page 32. This happens to be a small book; in one of 600 pages it would be a longer chase. Now this all takes time, interrupts the attention and, with me, gives rise to an emotional turbulence which may eventuate in profanity. I am sure that many others have had the same experience, barring perhaps the emotional turbulence. "A law ought to be passed," not against the use of profanity under such circumstances, but against the use of a separate series of numbers for illustrations.

For there is no logical reason for a separate numbering of the illustrations. They are not regularly spaced as are the pages. One can not at once turn to a numbered figure in a distant part of the book, as he can to a numbered page. Their use is time-consuming and irritating.

Besides, there is a better way of handling the matter. Figures in the text should be referred to by their page number. Fig. 25, above, would then be Fig. p. 32; or even Fig. 32. One could then turn to it at once. If there were more than one figure on a page they could be distinguished as A, B, C, etc.

This suggestion concerns especially text-books in physical science and technology. It is addressed to the writers and publishers of such books. It is the duty of author and publisher to reduce the effort of the reader in every possible way; and here is one way. Any unnecessary taking of the reader's time and energy is larceny, stealing; is immoral.

Some of the best texts are already dropping the serial numbering of figures. Smith and Phillips's splendid "North America" (Harcourt, Brace and Company) is one: it omits the numbers, and when there is more than one illustration on the page it distinguishes them by letters. The use of the old system of consecutively numbered figures hangs on because of inertia and lack of imagination. Writers of text-books on science ought to be able to climb out of this rut.

LEWIS G. WESTGATE

SCIENTIFIC BOOKS

TOPOLOGY

Algebraic Topology. By SOLOMON LEFSCHETZ. vi + 389 pp. Vol. 27. Colloquium Publications of the American Mathematical Society. 1942. \$6.00.

¹ I. Fox, "Fleas of Eastern United States," p. 12. Iowa State Coll. Press, Ames, Iowa, 1940.

² E. C. Faust and T. A. Maxwell, Report of a case, *Arch. Dermat. Syph.*, pp. 94-97, 1930.

³ P. H. Manson-Bahr, "Manson's Tropical Diseases. A Manual of the Diseases of Warm Climates." Eleventh

Analytic Topology. By G. T. WHYBURN. x + 278 pp. Vol. 28. Colloquium Publications of the American Mathematical Society. 1942. \$4.75.

THESE two mathematical volumes, written by leading mathematicians, published by Williams and Wilkins Company, pp. 700-703, 1940.

* After proof was received, a communication (in litt.) from the U. S. Health Service in Montana suggests this might be *Hectopsylla psittaci*, a nearly related flea from South America. Without males certain identity is difficult.

ers in their respective fields of algebraic and analytic topology, constitute another notable addition to the series of Colloquium Publications of the American Mathematical Society. There has been an extensive forward movement in the field of topology (formerly called "analysis situs") during the present century, in which Russian, Polish and American mathematicians have played a conspicuous part. In all probability these two volumes represent a kind of culmination of the abstract phase in this development.

The scientific public has for some time been aware of the abstract character of much of contemporary mathematics; this has tended to make of their mathematical colleagues a class somewhat apart. The exceeding generality of the ideas involved and the extremely technical and abbreviated terminology employed have been annoying at times, especially to those who believe that the specific situations, which arise naturally, are supremely important and that all really important ideas are basically simple. The answer to such individuals must be that the concrete intellectual object is merely the final definitive form of an abstraction, as illustrated for example by the integer; and that a "simple" concept is only one with which we have become familiar through long use, as, for instance, that of energy in physics.

It appears then that the really significant questions are whether or not topology has attained its approximately definitive abstract form and whether topological concepts are going to prove widely useful. I believe that most mathematicians who know something of the recent work would answer the first question in the affirmative. Moreover, the broad notions of "topological space" and of "metric space" which are given first consideration in both of the volumes deserve to become as well known to the scientific world as that of "linear vector space" of which ordinary Euclidean space serves as the familiar prototype. The "additive group," illustrated by ordinary and angular numbers, is likewise of prime importance. This basic concept of an additive group (in a topological space) is central in algebraic topology and has been much illuminated by the remarkable work of the Russian mathematician Pontrjagin, a good deal of which appears in Lefschetz's book in convenient form.

Broadly speaking, algebraic topology develops the algebraic machinery involved in the dissection into "complexes" and the characterization of the connectivities and other intrinsic properties, of geometric entities—lines, surfaces, solids, etc.—such as are found in ordinary Euclidean space of two or more dimensions. These are regarded only qualitatively, so that a sphere and ellipsoid are not distinguished, while a ring would be regarded as fundamentally dif-

ferent from a sphere. In algebraic topology the basic ideas of "homology" and "homotopy" are derived from that of continuous deformation: thus a small circle on a sphere is said to be homologous to 0, since it can be deformed continuously to a point.

The technical algebraic apparatus involved in homology and homotopy theory was in large part envisaged by Henri Poincaré in his classical five articles on analysis situs (1895–1904). Oswald Veblen through his excellent Colloquium Lectures on "Analysis Situs," published in 1922 and reprinted in 1931, presented the ideas of Poincaré in accurate, improved and suggestive form, and so performed a valuable service for the mathematical world; one recalls also the very useful earlier article by Dehn and Heegard on the same subject in the German Mathematical Encyclopedia. In this way interest was aroused here and abroad. Veblen's work and inspirational influence may properly be regarded as forming the starting point of the many important American contributions to algebraic topology. Lefschetz, with earlier topological investigations to his credit in the field of algebraic geometry, has done much to advance the purely algebraic side of topology.

But while all the abstract focal points involved in algebraic topology have been most successfully highlighted by means of the abstract method in the newer developments presented in Lefschetz's book, one fact needs to be emphasized: the classical open questions noted by Poincaré and others have been left largely untouched. Lefschetz, in referring to the "Poincaré group,"¹ conjectures (p. 310) that "ignorance concerning this group seems to account for the fact that many of the major problems of topology have so far eluded all attempts at solution." It probably has been J. W. Alexander and, more recently, Hassler Whitney who in this country have most advanced toward the solution of such unsolved specific questions. The appearance of Hassler Whitney's forthcoming book on "sphere spaces" will for that reason be awaited with especial interest.

On the other hand, the pure abstractionists have performed beautifully the essential task of giving topological ideas their appropriate abstract setting, and this has been work of the first order of importance.

The volume of Lefschetz owes much to the effective collaboration of various workers in the field and shows the happy effect throughout. Lefschetz mentions especially Samuel Eilenberg, W. W. Flexner,

¹ Typified in the simple case of the circle by the k -fold circuits. Thus a minute-hand makes in one hour one circuit ($k=1$), and makes 24 circuits in a day ($k=24$). Here enters characteristically the "additive group" of the integers in an elementary question of algebraic topology.

N. E. Steenrod, John Tukey and Claude Chevalley. At the end appear two valuable Appendices, one on "homology groups" by Samuel Eilenberg and Saunders MacLane, and the other on "periodic transformations" by P. A. Smith. Both this book and Whyburn's are practically flawless in typography. The earlier volume written by Lefschetz in the Colloquium series ("Topology," 1930) may be regarded as more or less superseded by the new work under review.

The general plan of the book by Lefschetz is roughly the following: general spaces and additive groups (Chapters 1, 2) complexes and nets of complexes (Chapters 3-6), general homology theory (Chapter 7), topology of polyhedra (Chapter 8). Both of the excellent introductory chapters will be carefully read by a very wide circle of mathematicians. In Chapter 3, the approach to the basic theory of complexes is that of A. W. Tucker. The Czech "homology theory" of topological spaces is made fundamental, of which the Alexander-Kolmogoroff, Alexandroff, Kurosch, Lefschetz and Victoris theories appear as specializations. The fundamental work of Alexander and Pontrjagin, extending the Poincaré principle of duality, and that of Lefschetz on intersections, coincidences and fixed points, is developed in the later chapters.

The reviewer has seen at least one attribution in the volume of Lefschetz which appears to him to be unsatisfactory, even if it has been widely accepted, namely calling "theorem of Zorn" (p. 5) a result which, as Lefschetz states, had been essentially given by R. L. Moore earlier.² It seems to the reviewer that the theorem would be more appropriately designated the "theorem of Moore-Zorn."

The interest in general types of spaces, important alike for algebraic and for analytic topology, began with the thesis of Fréchet (1905) in which "metric spaces" were defined and studied. Stimulated by this work and that of Hilbert and of Erhard Schmidt, E. H. Moore, the outstanding American mathematician of his day, conceived of absolutely general spaces in which the elements (points) might be of wholly arbitrary type. A little earlier he had been interested in the foundations of geometry and other logical questions. At that time there were working at Chicago with E. H. Moore a group of extremely able young men, among them Veblen, R. L. Moore and N. J. Lennes. The subsequent role of Veblen in the development of algebraic topology at Princeton has already been mentioned. R. L. Moore was destined to become the creator of the important American school in analytic topology at Austin, with G. T.

² See, for instance, his notable Colloquium volume on analytic topology, "Foundations of Point Set Theory," 1932, p. 84.

Whyburn and R. L. Wilder as outstanding students and co-workers among an important group. Veblen, R. L. Moore and Lennes had nascent ideas in the field now called analytic topology, which treats largely of connectedness and continua in topological space, where "point" and "neighborhood of a point" constitute the only primitive ideas. For example, Lennes proposed about that time to define a simple arc AB as a closed³ connected set of points containing A and B which contains no closed connected subset likewise containing A and B.

A very important later idea of R. L. Moore which finds its proper place in Whyburn's book is that of "upper semi-continuous collection." Imagine the ordinary Euclidean plane to be constituted of closed continua, U_P , one and only one containing an arbitrary point P. Suppose further that if the point P tends to a point Q, then always U_P tends toward all or a part of U_Q . The collection of sets U_P then constitutes an upper semi-continuous collection in the sense of R. L. Moore. This idea has dynamical applications as I have found, although, interestingly enough, it was invented by Moore simply as a mathematical *jeu d'esprit*, by the esthetic combination of ideas.

The general plan of the book by Whyburn is roughly the following: Introductory topology (Chapter 1), mapping theorems (Chapters 2, 8-11), theorems on connectedness (Chapters 3-6) upper semi-continuous collections (Chapter 7), periodic transformations and fixed points (Chapter 12). A considerable portion of the book is devoted to advances made by R. L. Moore and by Whyburn. Being slightly less condensed and involving a less extensive range of ideas, this book makes easier reading than that of Lefschetz.

It is significant that both volumes terminate with a discussion of "fixed point theorems." A very simple example of such a theorem is furnished by the continuous mapping of a linear segment AB of a line on part of itself, as A'B'. It is clear that as a point P travels from A to B, its image P' passes from A' to B', and coincides with or intersects P in a "fixed point" an odd (or infinite!) number of times, since P passes P' in one sense $k+1$ times and k times in the other (negative) sense. Thus we have

$$(k+1) + (-k) = 1.$$

As the Dutch mathematician L. E. J. Brouwer and others noted long ago, there is a very general type of geometric situation in which the algebraic sum of the intersection numbers (Kronecker indices) is unique and determinate, regardless of all internal details. Hence it is only necessary to make the

³ A point set is said to be "closed" if it contains all its limit points.

algebraic count of intersections in one special case to get the number of fixed points, algebraically taken. Lefschetz's well-known theorems on intersections, coincidences and fixed points describe situations belonging to this general category, of which various important special cases were previously well understood. The Appendix B by P. A. Smith with which Lefschetz's book closes is devoted to a study of the "Fixed Points of Periodic Transformations," a subject still closer to the dynamical applications. Similarly, the last chapter of Whyburn's book is entitled "Periodicity. Fixed Points," and references to the dynamical origins of this type of question are there made. Here Whyburn presents interesting work due in part to Kerékjártó, to Ayres, to Montgomery and to himself.

In this way one receives a concluding tacit suggestion in both cases that the abstract phase in the development of algebraic and analytic topology is about to pass into a second phase, less abstract and closer to basic dynamical ideas.

The significance of this rich mathematical source was realized first by Poincaré, who in the third volume of his celebrated "*Méthodes nouvelles de la Mécanique Céleste*" found it necessary to analyze the connectivity of certain manifolds of states of motion, to consider transformations and fixed point theorems and to evolve the concept of dynamical "probability." In fact it seems that *all* topological questions are presented naturally in purely dynamical contexts. Certainly there are numerous fascinating and important questions of this sort as yet unanswered. For example, questions concerning measure-preserv-

ing transformation (like rotations which preserve areas or volumes) are essentially topological in character, since these are the transformations which can not take any continua or set of continua into a part of themselves. As yet such "conservative" transformations have been little studied, although recently Oxtoby and Ulam have treated them to great advantage.

It seems to be regrettable that up to the present time so little has been accomplished by the topologists that is directly serviceable for application in the dynamical field. Since I have long worked in theoretical dynamics, on the borderland of and in what is essentially pure topology, I may be allowed to testify to this fact. More than any one else, it has been Marston Morse (see his Colloquium volume, "*The Calculus of Variations in the Large*," 1932) who has shown algebraic topology at work in the applications, through his notable "critical point relations." Likewise, as stated above, the upper semi-continuous collections of Moore in analytic topology have turned out to be valuable for the understanding of certain dynamical situations.

If further development in the direction of the applications continues, topology will indeed greatly increase in scope and significance. In any case, mathematicians generally will rely upon the books of Lefschetz and Whyburn, as representing the present high-water mark of topological development, and as furnishing first-hand and notable accounts of two basic aspects of abstract topology.

GEORGE D. BIRKHOFF

HARVARD UNIVERSITY

SOCIETIES AND MEETINGS

CENTENARY OF THE AMERICAN ETHNOLOGICAL SOCIETY

THE centenary celebration of the American Ethnological Society, held on November 14, marked the founding of the oldest anthropological, and one of the oldest scientific, associations in the United States.

The society was founded in November, 1842, by Albert Gallatin, Secretary of the Treasury under Thomas Jefferson. Its headquarters have always been in New York, and it is now affiliated with the New York Academy of Sciences and the American Anthropological Association. The American Museum of Natural History in New York has consistently cooperated with the society. In 1943, the first year of the newly organized Inter-American Society for Anthropology and Geography, the society will likewise act as an affiliate and council member of that group. Membership is at present largest for the United States, but likewise includes individuals from Mexico,

Central and South America and, until December 7, 1941, from Europe, India and the South Seas. It is noteworthy that a few English members still keep their accounts active.

The celebration was originally planned to cover at least two days and to include speakers from the country as a whole, but, at the request of the Office of Defense Transportation, it was telescoped into a single meeting terminated by a dinner, and its roster of speakers was limited to the eastern seaboard from Boston to Washington.

The afternoon meeting, held at the American Museum of Natural History, consisted of three sessions on the general topic of acculturation or culture-contact, oriented toward administration. One session was devoted to each of three geographical areas: Oceania, Latin America and North America. The speakers were Ruth F. Benedict, Raymond Kennedy, Clyde Kluckhohn, Ralph Linton, Margaret Mead and