DISCUSSION

EXTENDED THEOREMS IN DYNAMICS

THE Faraday lines of force in any field of force constitute a familiar geometric pattern. Less well known because of its intrinsic complexity is the geometry of the trajectorial curves[1] (the paths of mass particles moving in the force field). Of course these two geometries should be related, but precisely how may we compare the two families of curves? This is what we propose to indicate.

Let a particle start from rest. The path obtained in this manner we shall call a "rest trajectory." Obviously this curve has the *same direction* as the Faraday line through the given point, but in general, since the mass particle has inertia, the two curves will *not coincide.* In fact, the only instance in which coincidence can occur is the trivial case in which both curves are straight lines. Omitting this possibility, there are always two distinct curves. The rest trajectory and the line of force will agree as to initial direction, but differ in initial curvature; the trajectory will be between the line of force and the common tangent line. The quantitative relation connecting the two curves is given in the following theorem discovered by Kasner in 1904 and since proved in many ways:

Theorem I:[1] *If a particle starts from rest in any positional field of force, the initial curvature of the trajectory is one third of the curvature of the line of force through the initial position.*

There are two important manners in which this proposition may be extended. The theorem is universally valid, but its significance is slight if the curvature of the line of force is zero at the point. If the curvature is zero, as for example at a point of inflection, the same is true of the curvature of the trajectory, and a separate discussion, involving a closer scrutiny of the situation, is necessary. In this case we study the ratio of the infinitesimal departure of the path and the line of force from the common tangent line. In the general case this ratio would be $1:3$ as stated above, because in the usual case the ratio of departures is the same as the ratio of curvatures. But in the special case of zero curvature, the ratio (which now depends on the rates of change of curvature) is found to be $1:5$, $1:7$, $1:9$, etc., depending on the order of contact of the line of force with the tangent line. The precise dependency is stated in Theorem II.

Theorem II:[2] *If the Faraday line of force has contact of order n with the tangent line, the rest trajectory will also have contact of n^{th} order; and the limiting ratio of the departure of the trajectory to the departure of the line of force from the common tangent line is $1:2n+1$.*

The second manner in which Theorem I may be generalized is to acceleration fields of higher order. A particle of unit mass is said to be moving in an acceleration field of order r if

$$\frac{d^r x}{dt^r} = \varphi(x, y) \qquad \frac{d^r y}{dt^r} = \psi(x, y)$$

where $\varphi(x, y)$ and $\psi(x, y)$ are the rectangular components of the higher acceleration field acting at the point (x, y) and t is the time. By definition, a particle will start from "maximum rest" in such a field if the initial velocity and the initial accelerations of order up to and including $r-2$ are zero (that is, all derivatives up to order $r-1$ vanish). What is the analogue of Theorem I for such higher fields?

Theorem III:[3] *If a particle starts from maximum rest in an acceleration field of order r, the initial curvature of the trajectory is $\frac{r!\,(r-1)!}{(2r-1)!}$ times the curvature of the line of force through the initial position.*

Now allow the particle to start from maximum rest in an acceleration field of order r, at a point at which the curvature of the Faraday line of force may be zero (the case in which Theorem III would no longer be significant). What is the appropriate ratio of departures?

Theorem IV: *In an acceleration field of order r, at a point at which the line of force has n^{th} order contact with its tangent line, the trajectory produced by starting a particle from maximum rest also has contact of order n with the tangent line; and the limiting ratio of the departure of the trajectory to the departure of the line of force from the common tangent line is* $\frac{(n+1)\,(r)\,!\,(nr)!}{(nr+r)!}$.

We see that all three situations previously discussed in Theorems I, II and III are now included in the single final Theorem IV. Indeed, the first three theorems become corollaries of this new result. For $r=2$, the newtonian case, the ratio is $1:2n+1$, as stated in Theorem II. For r arbitrary and $n=1$, the case in which the curvature does not vanish, we obtain the ratio of Theorem III, namely $\frac{r!\,(r-1)!}{(2r-1)!}$. And for $r=2$ and $n=1$, we have the standard dynamical Theorem I, with the usual ratio $1:3$.

All our theorems, although originally stated for two dimensions, remain valid for any number of dimensions, and in any riemannian space.

[1] E. Kasner, "Differential Geometric Aspects of Dynamics," Princeton Colloquium, 9, 1909, new edition 1934.

[2] *Idem, Proceedings of the National Academy of Sciences,* 20: 130–136, 1934.

[3] E. Kasner and D. Mittleman, *Proceedings of the National Academy of Sciences,* Feb., 1942.

Theorem I has been applied[4] to the study of the famous deviation problem (a particle falling from rest to the earth allowing for rotation or ellipticity). Our new extended theorems will also have applications, direct and indirect, in physical situations dealing with interacting particles. The forces need not be conservative.

EDWARD KASNER
DON MITTLEMAN
COLUMBIA UNIVERSITY

A STUDY OF LAMPBRUSH CHROMOSOMES BY THE ELECTRON MICROSCOPE

LAMPBRUSH chromosomes have been photographed by the electron microscope. The nuclei were removed from the oocyte according to the method of Duryee[1] and were placed on the collodion film suspended on the wire mesh which is used in place of the slide in the electron microscope. The nuclei were torn apart so that the enclosed material spread over the film. The membrane was then removed since it was too thick for penetration by the electron beam when collapsed. The preparation was allowed to dry in air.

The photographs seem to verify early descriptions. Some chromosomes appeared to be highly branched and subbranched. They were fern-like in appearance. The threads were crystalline and single. Other chromosomes showed less numerous, thicker, more globular side branches. Many side branches had been lost between the first and second type. Finally some showed no branches. There were as many as four threads twisting about one another separating into twos at some points and rejoining at others.

No loops, as described by Duryee, were seen. However, chromosomes from full-sized eggs only have been examined. Further investigations, in which the nuclei of half-sized eggs will be used, are in progress. It may be that these will verify the loop theory as put forth by Duryee.

Blanks were run in which only cell debris, from which the nucleus had been removed previously, and the nuclear salt solutions were dried and photographed. No similarities between these preparations and those of the nucleus were observed.

The investigations are being extended in the belief that they will throw added light on the structure of such chromosomes and will clear up such problems as the time at which the chromomena thread becomes doubled.

G. L. CLARK
M. R. BARNES
E. R. BAYLOR
UNIVERSITY OF ILLINOIS

[4] W. H. Roever, *Bull. Amer. Math. Soc.*, 456, 1915.

[1] W. R. Duryee, "Cytology, Genetics and Evolution," University of Pennsylvania Press, Philadelphia, 1941.

CONSIDERATION OF THE ADEQUACY OF BIOMICROSCOPY AS A METHOD OF DETECTING MILD CASES OF VITAMIN A DEFICIENCY

RECENTLY Dr. H. D. Kruse has reported on "The ocular manifestations of avitaminosis A with especial consideration of the detection of early changes by biomicroscopy."[1] He has suggested that "xerosis conjunctivae probably precedes night blindness as an early sign of avitaminosis A," and recommends biomicroscopic examination as a "simple, convenient, objective method" for the detection of avitaminosis A in surveys.

In view of the importance of finding reliable tests for detecting mild degrees of the various avitaminoses, it is relevant to call attention to certain discrepancies between the above-mentioned observations and those reported in a study by Booher, Callison and Hewston in which impaired dark adaptation was produced in five adults by the consumption of a diet adequate in every known dietary essential except vitamin A.[2] The Hecht and Shlaer Adaptometer was used to determine the dark adaptation curves of these subjects. Dysadaption occurred in from 16 to 124 days after the vitamin A-deficient diet was begun and for four subjects was allowed to proceed until the visual threshold after 30 minutes of dark adaptation was elevated by 1 logarithmic unit; subject I of this group was continued on the experimenal diet until the 30-minute threshold was 4 logarithmic units above normal, while at this time of greatest visual impairment, the rod structures were not functioning at all below the scotopic threshold of the cones. Thus, there was no question of the existence of hemeralopia in any of the five subjects.

During the period of greatest impairment in retinal function, a slit-lamp examination was made on subject I by Dr. Alan C. Woods, of the Wilmer Ophthalmological Institute of The Johns Hopkins Hospital. There was no evidence of abnormality. The remaining four subjects were examined with the slit-lamp by Dr. William M. Rowland of the same institution both before and during impaired adaptation, as well as after that function had returned to normal following the administration of moderate amounts of vitamin A. Neither did any of these subjects show conjunctival or corneal changes at any of the examinations.

Attention should also be called to the work of Youmans *et al.*, who conclude that mild degrees of vitamin A deficiency can exist without any modifica-

[1] H. D. Kruse, *The Milbank Memorial Fund Quarterly*, 19: 207, 1941.

[2] L. E. Booher, E. C. Callison and E. M. Hewston, *Journal of Nutrition*, 17: 317, 1939.