

attached to an algebraic curve (1826, 1857) has been ever since the object of intensive study by analysts and geometers alike (Poincaré, Picard, Castelnuovo, Enriques, Severi, etc.) who were attracted to it either by its profound analytical content or by its power as a tool for the discovery of geometric properties of algebraic varieties. The theory gives rise to many difficult questions, and the difficulties increase with the dimension of the variety. Thus, while the theory of abelian integrals is almost a finished chapter in the case of algebraic curves and is a richly developed chapter also in the case of algebraic surfaces, it is on the whole unexplored territory in the case of varieties of higher dimension. The unsolved problems in which it abounds have to do mainly with the *classification* of the abelian integrals according to their singularities and with the properties of their *periods*. By the period of a *multiple*, say *m-fold*, abelian integral is meant its value which we obtain by integrating along an *m-dimensional cycle* of the variety (cycle = a closed *m-dimensional* region; a closed curve, if $m=1$; a closed surface, if $m=2$; etc.).

A typical and important problem, which remained unsolved for a long time, was the following: do there exist integrals which have no singularities and whose periods are all zero? Hodge was the first to prove that such integrals do not exist. This proof marks the beginning of a long series of investigations by Hodge which have materially contributed to a new and substantial progress of the theory of abelian integrals. One of the most significant aspects of these investigations is the diversity of the methods used. In addition to the purely analytical and algebro-geometric methods, topology and differential geometry come effectively into play.

Topological methods, already used by Poincaré in questions of the theory of algebraic surfaces, have been developed in this country by Lefschetz. The topological point of view in algebraic geometry plays an essential role in Hodge's solution of the problem mentioned above. Topology also forms the basis of de Rham's theory of integrals on arbitrary topological manifolds, a theory which is in close connection with Hodge's theory of harmonic integrals.

However, it is the adaptation of the methods and ideas of differential geometry (Riemannian geometry) to the theory of abelian integrals that best reveals the originality of Hodge's theory. Here the central concept is that of a *harmonic integral*. This is not the place to discuss this concept in any detail. It will suffice to say that the concept of a harmonic integral is more general than that of an abelian integral, but is more concrete than the concept of an integral attached to an arbitrary topological manifold. The fruitfulness of the concept of a harmonic integral lies

perhaps just in its being a "harmonious" mixture of the right amounts of generality and concreteness.

The mathematical reader will find in Hodge's monograph a very well-written and highly stimulating account of a young and active mathematical theory. He will find ample evidence of the diversity of methods which we have mentioned above. As an ingenious blend of algebraic geometry, analysis, topology, tensor calculus and differential geometry, the monograph should prove of considerable interest to specialists in these various fields.

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ELECTRICITY AND MAGNETISM

A Textbook in Electricity and Magnetism. By H. C. KELLY. vi + 356 pages. New York: John Wiley and Sons, Inc. 1941. \$3.75.

THIS elementary text attempts to give the reader an acquaintance with a great deal of modern physics as well as to cover the conventional topics suggested by its title. The twenty-eight chapters are short and emphasize the historical and descriptive aspects of the subject. The diagrams are numerous and well drawn, and a number of simple problems are placed at the end of each chapter. In the third chapter the author goes to some pains to find the potential due to a point mass without the use of the calculus, but does not hesitate to introduce integration by the time Chapter 7 is reached, and to discuss the partial differential equations of the electromagnetic field and the wave equation in Chapter 28. In discussing induced electromotive forces the author makes the common mistake of attributing the e.m.f. to "relative motion between a wire and a magnetic field."

The book should prove very useful in a survey course intended to cover modern developments in physics as well as the elements of electromagnetism. To cite a few of the modern topics presented, we find mention of superconductivity, the Barkhausen effect, the Stern-Gerlach experiment, the cyclotron and the electron microscope, and discussions of space-charge, the photo-electric effect, the Bohr theory and the mass spectrograph.

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HYDROBIOLOGY

A Symposium on Hydrobiology. By JAMES G. NEEDHAM and 51 other contributors. ix + 405 pages. Madison: University of Wisconsin Press. 1941. Price \$3.50.

THIS book contains the 32 formal papers presented at a symposium on hydrobiology held at the University of Wisconsin on September 4-6, 1940, and the abstracts of 16 volunteer papers given at one of the