DISCUSSION

NATURE OF GROUP THEORY

FOR thousands of years the operations of addition and multiplication of ordinary numbers have commonly been regarded as two distinct operations but in group theory, with the exception of zero in multiplication, they are regarded as belonging to the same more general operation. This is due to the fact that in group theory we restrict our attention to what is common to these two operations. In the first place, both of them relate to the combination of elements so as to obtain a single element of the same kind, and the resulting element is independent of how these elements are associated before they are combined. That is, in both of these operations the associative law of combination is satisfied. In fact, the commutative law of combination is also satisfied in both of them but for the sake of greater generality and a comparatively small loss in simplicity this law is not assumed as fundamental in group theory and an essential part thereof.

In 1870 L. Kronecker (1823–1891) read a paper before the Berlin Academy of Sciences in which he remarked that the extremely simple principles upon which Gauss's method is based occur not only at the place to which he had referred but also at many other places and they are used already in the most elementary parts of number theory. This circumstance points to the fact, which can readily be verified, that these principles belong to a sphere of more general and more abstract ideas. Hence it seems desirable to develop them independently of all unrelated matters so that a repetition of the same arguments in their use on different occasions may be avoided. The resulting advantages appear already in the developments themselves, and the presentation thus gains in simplicity and perspicuity by the restriction to what is essential.

These observations were followed by a system of postulates for what is now commonly called an abstract abelian group, but the term group was not then used by him. They illustrate the fact that group theory is not only a subject of mathematics but also an isolation of ideas which are fundamental in various mathematical subjects and a development of these ideas with a view to avoiding repetitions when they present themselves in different subjects. It therefore contributes to the economy of thought and as it relates also to the most elementary subjects of mathematics it may reasonably be expected to be permanently useful. This may account for the recent group theory week at what was then a leading mathematics center of Europe, Göttingen, Germany, and was reported in *Crelle's Journal*, volume 182, pages 129–248 (1940).

The fact which we aim to emphasize here is that abstract group theory was developed before the postulates on which it is based were explicitly formulated so that these postulates resulted from observing basic facts underlying these developments. It is true that after this formulation these developments were greatly extended and gave rise to an autonomous subject of considerable extent which differs widely from the subjects which gave rise to it. Just as the operations of addition and multiplication will probably always be regarded as distinct operations, notwithstanding their union in group theory, so in other fields where the group theory point of view led to interesting broader points of view the original methods have not always become obsolete as a result of these broader views.

Since the term "group" was used in the non-mathematical literature long before it was adopted by the mathematicians as a technical term and its use as such a term grew unusually rapidly in recent years on account of its wide applications, it may not be surprising to find that it is still often misused, especially by popular writers. Some of our best and most recent dictionaries of the English language still fail to give a definition of this term which is in accord with those commonly employed in the modern treatises on the subject. In particular, the associative law is often omitted in the definitions found in these dictionaries. This makes it more desirable to emphasize the nature of group theory as a scientific subject of growing importance, especially in view of the numerous recent misstatements relating thereto by men in positions of great influence.

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THE STATUS OF EXPERIMENTAL PSYCHOLOGY AMONG THE LABORATORY SCIENCES

IN order to determine to what extent experimental psychology is being recognized as a laboratory science that will satisfy laboratory requirements toward the A.B. degree, the following questionnaire was sent to 75 of our leading universities and colleges:

1. Does your College of Arts and Science require a minimum number of hours of laboratory science as a prerequisite to the A.B. degree?

2. Is the course in Experimental Psychology recognized as a laboratory science that will satisfy the above prerequisite for a degree?

The replies to this questionnaire indicate three distinct trends in the status of experimental psychology.[1]

The first trend is represented by those institutions that require a laboratory science for the A.B. degree, and that recognize experimental psychology among

[1] Data obtained in 1937.