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MATHEMATICS AND THE SCIENCES[1]

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A CLOSE inspection of the history of mathematics and that of physical science reveals the mutual dependence of the two fields of thought. At times mathematical development has been definitely stimulated by the needs of science; at other times scientific progress has been extremely rapid because of the availability of the necessary mathematical devices. It is interesting to observe, however, that serious reflection upon the actual relation of mathematics to the sciences has awaited the twentieth century. Such consideration, stimulated by a better understanding of the nature of mathematics, needs greater publicity, for it is the immediate cause of the mathematizing of parts of science previously untouched by mathematical treatment. This paper, then, will briefly review some of the factors which are of importance in any attempt to understand the relation of mathematics to the sciences. Implicit in the discussion is a broad definition of mathematics; my only apology for such a point of view is that it is the modern one.

Certainly it is true that a natural science originates with inductive procedures. The inspection of many similar situations in an effort to perceive those constant principles to be designated as laws must always remain fundamental. However, a time comes in the life history of a science when such methods are no longer adequate. Lapicque[1] has expressed the thought in the following words:

[1] Address of the retiring president of the Southwestern Division of the American Association for the Advancement of Science, Lubbock, Texas, April 30, 1941.

[1] L. Lapicque, "L'orientation actuelle de la Physiologie," in *L'orientation actuelle des sciences* (Paris, 1930). The translation employed here was given by C. N. Moore in SCIENCE, v. 81: p. 31, 1935.

Formerly, not very far back in the history of humanity, let us say a century ago, almost everything was unknown concerning the physiology in the labyrinth of the living body. Magendie said: "I wander around there like a rag picker, and at each step I find something interesting to put in my basket." This maxim horrified my teacher, Dastre, who was wont to say: "When one doesn't know what he is looking for, he doesn't know what he finds." For him the ideal of physiological research would have been to conceive in the quiet of one's study a theory explaining such and such a phenomenon, known but not understood, then to find, still by meditation, the experiment capable by a yes or a no, of proving or disproving the theory. One would come then some morning to the laboratory, and that very evening the matter would be decided. These two tendencies, each in its amusingly exaggerated form, seem to me to serve the purpose of characterizing the temperament of naturalists and that of physicists. In proportion as physiology develops, the discoveries for rag pickers become more rare, and the possibility of working as Dastre dreamed is approaching.

In the preface of Woodger's epoch-making book entitled "The Axiomatic Method in Biology,"[2] he explains his attitude similarly as follows:

> In every growing science there is always a comparatively stable, tidy, clear part, and a growing, untidy, confused part. I conceive the business of theoretical science to be to extend the realm of the tidy and systematic by the application of the methods of the exact or formal sciences, *i.e.*, pure mathematics and logistic.

What, then, is the method of mathematics? Essentially, it is typified by an organization of the propositions of a science into those which are to be accepted as primary or basic and those which may be logically deduced from them. The former propositions are known as the axioms of the science, the term axiom signifying only that the statement thus designated is not proved within the system, whereas the latter propositions are called the theorems or secondary propositions.

To a great extent the original choice between axioms and secondary propositions is arbitrary. The axioms should constitute a consistent set of statements; moreover, they should be entirely ample for the deduction of the remaining propositions of the system when the rules of inference accepted as an adjunct to the system are applied. If a proposition is found among the set of axioms which is a logical consequence of other axioms, its status, of course, should be changed to that of a secondary proposition. Also, it is frequently possible to keep the mathematical organization of a science intact by replacing a collection of the axioms by a smaller number of more primitive statements; sometimes such new axioms may not have been accepted previously as propositions within the science. As a result of this latter process, it is often true that some axioms will be of such a nature that their truth-property can not be studied directly through the medium of empirical procedures.

The subject-matter symbols of a science organized in the manner just described may not be part of the usual language of the science. In fact, the language of most sciences was not introduced for the purpose of facilitating the construction of a logical structure, and progress toward that end virtually demands some use of the symbolism of mathematics and logistic. The success of Woodger in accomplishing a rather elegant mathematical organization of some portions of biology is due partly to his use of a special set of symbols augmented by the symbolism of the "Principia Mathematica" of Whitehead and Russell.

When a logico-deductive system of the type under consideration includes no interpretation of the subject-matter symbols, it becomes a structure in pure mathematics. Of course the rules of inference are valid, and actually are more readily applied, if the basic set of axioms is uninterpreted. It is important to note, however, that the propositions within such a system assert nothing about any part of science, for they convey no meaning. In this connection we recall the familiar statement of Russell that "Mathematics is the subject in which one never knows what he is talking about nor if what he says is true." It is even doubtful that a typical non-assertive statement in mathematics should be characterized as a proposition; it merely has the form of a proposition. Also, any notion of truth-property vanishes from the system, and the concept of consistency becomes the important factor.

So, from some points of view, a mathematical structure may not possess meaning, but it certainly has form. In fact, a structure in pure mathematics may be likened to a pattern or a model or, perhaps better, to a skeleton. It has been constructed by an expert who knows how to link propositions through the use of the rules of inference, the chain starting with a few propositions which are taken as primitives. Charles Sanders Peirce, the Harvard logician, recognized this years ago when he said, "I consider that the business of drawing demonstrative conclusions from assumed premises, in cases so difficult as to call for the services of a specialist, is the sole business of the mathematician." Again he stated, "The business of the mathematician is to frame an arbitrary hypothesis, which must be perfectly distinct at the outset, so far, at least, as concerns those features of it upon which mathematical reasoning can turn, and then to deduce from this hypothesis such necessary consequences as can be drawn by diagrammatical reasoning."

[2] J. M. Woodger, "The Axiomatic Method in Biology," p. vii. London: Cambridge University Press, 1937.

Through the ages, mathematicians have constructed many of the symbolic skeletons which constitute the field of pure mathematics. Some of them are only superficially different, but that fact is irrelevant to this discussion. Suffice it to say here that there has been a frenzy in mathematical circles in recent years; the pace in mathematical research has become faster and faster as new mathematical structures are created and old ones perfected or extended. Such matters are of interest to the scholar, but the writer of this paper must insist that mathematics would become a dead subject and mathematicians an economic liability if the structures of pure mathematics should cease to be of great importance in the sciences.

The task of covering a mathematical skeleton with the flesh which is the substance of a science is not always simple. It requires, first of all, the discovery of a mathematical structure which possesses an axiomatic basis capable of becoming the foundation of the science under consideration when the subject-matter symbols are properly interpreted. In other words, a mathematical structure becomes a system in theoretical science when the subject-matter symbols are properly particularized in meaning. When such precise correspondence, as is implied here, is attained between the fundamentals of a mathematical structure and the primitives of a science, the same definite correspondence is maintained throughout the two systems; that is, the system in pure mathematics and the science organized through its use are identical in form or are isomorphic. In view of the extensiveness of most mathematical structures which are available, success in fitting a mathematical structure to the data of a science may immediately increase knowledge relative to that science many times over. Scientific discoveries which have attended the use of the method have been little short of astounding.

At this point a brief consideration of a very simple mathematical system might be of interest. It should be recalled that meaning is not a necessary ingredient, so the uninitiated may regard a mathematical system as mere jargon. The symbolic system which characterizes "simple order" is of frequent use to mathematicians, and is concerned with a set of elements, A, B, C, etc., and a relation designated by the symbol R. There are three axioms; namely,

1. If A is different from B, then either A R B or B R A.
2. If A R B, then A is different from B.
3. If A R B, and B R C, then A R C.

Not many propositions can be logically deduced from these axioms, but a typical consequence is the proposition,

4. A R B and B R A is false.

An application of the mathematics of simple order may be found in biology when studying the procreation of yeast cells. A new yeast cell first appears as a bud upon the parent cell. The young cell ultimately separates from its parent, becomes mature, and then begets new cells, one at a time. Every cell has essentially the same kind of a life history. If, now, some one cell is designated by a letter of the alphabet exclusive of R, its first offspring by another letter, the first progeny of the second lettered cell by another letter, and so on, the axioms just given will be satisfied if R is assigned the interpretation, "is an ancestor of." In fact, the axioms become

1. If yeast cell A is different from yeast cell B, then either A is an ancestor of B or B is an ancestor of A.
2. If A is an ancestor of B, then A is different from B.
3. If A is an ancestor of B, and B is an ancestor of C, then A is an ancestor of C.

Now by referring to the mathematical proposition 4 which was deduced as a logical consequence of the original axioms, the valid assertion may be made that

4. A is an ancestor of B and B is an ancestor of A is false.

Such a conclusion is obvious, for the situation studied is a simple one, and the mathematical system employed is elementary. Perhaps, however, persons unfamiliar with mathematical studies can now partially appreciate how a similar technique can be of value in the study of complicated situations when involved mathematical systems are necessary.

Among the numerous other applications of the mathematics of simple order is the specific ordering of a set of temperature readings. This may be accomplished by employing the letters, A, B, C, etc., to denote various temperatures, and by giving to R the interpretation, "is higher than."

The studied use of mathematical methods in science is not new. Archimedes organized a treatise upon some aspects of mechanics before the second century, B.C., in which the deductive procedures of mathematics are brilliantly displayed. Archimedes had been schooled in Euclidean methods while at Alexandria, and his contributions to geometry and mechanics are a manifestation of his rigorous training. The first book of his treatise on mechanics entitled "On Plane Equilibria or Centres of Gravity of Planes" contains fifteen propositions deduced from seven axioms, and demonstrations are given for the determination of various centers of mass which are virtually identical with those still employed in elementary books upon mechanics. His second book of ten propositions extends the work of the first book to more difficult consideration.

It appears that Sir Isaac Newton believed in the possibility of inventing a theoretical science which would be of universal application to the study of the physical universe. In attempting to organize his science, he assumed mass points of invariable mass to be the basic entities. He then proceeded to the consideration of the necessary fundamental propositions involving such mass points. The foundation which he conceived is familiar to every student of physical science; however, it is incomplete from a mathematical point of view.

In 1788, Lagrange published his analytic mechanics. For the first time, a science of mechanics was systematized by the use of mathematical methods. In the preface to his masterpiece, Lagrange wrote, "No diagrams will be found in this work. The methods which I expound in it demand neither constructions nor geometrical or mechanical reasonings, but solely algebraic operations subjected to a uniform and regular procedure." Within his organization he explicitly stated a hypothesis, for example, upon which the well-known principle of the composition of forces is founded. Throughout the treatment, Lagrange insisted that the principles of mechanics are developed from assumptions, and, apparently, he did not believe that such principles form a system of absolute truths discovered by some group of scientists working in partnership with the Deity.

In modern times, the use of the mathematical method in science is becoming common. Some parts of the axiomatic basis for the theory of relativity are probably better known than are other aspects of the theory. The beginning student in mechanics should be given the opportunity to read Huntington's modern work entitled "The Logical Skeleton of Elementary Dynamics,"[3] for the mathematical approach in Huntington's development is quite satisfying. The economist with ample background is usually impressed with the possibilities of which he has a glimpse in some modern mathematical studies upon economic problems.[4] The work of Woodger in biology has already been mentioned. The number of such studies is rapidly increasing, and a definite impetus has recently been given to the careful consideration of the organization of a science by the early publications of the committee sponsoring the "International Encyclopedia of Unified Science."[5]

It seems foolish to the mathematician for any one to advocate that the use of the mathematical method is the certain cure for all the ailments of science. Yet achievements resulting from its use have been so notable that some men have made the doubtful declaration that what Descartes dreamed is true: that it is possible to arrive at a complete mechanical interpretation of the world in the exact terminology of mathematics. This expresses the attitude of the extreme mechanist. Irrespective of one's point of view upon this controversial question, all will admit the potency of the mathematical method when circumstances are such as to justify its use. In fact, many persons, even scientists, have developed a certain awe of mathematics. For them it may be surprising to read Bridgman's statement, "It is the merest truism, evident at once to unsophisticated observation, that mathematics is a human invention."[6] In other words, one of man's best-known devices for interpreting nature possesses the same elements of strength and weakness that belong to man himself. The significance of this fact is closely related to the underlying philosophy of all science.

The subject-matter of any science is a collection of sense-experiences which originally appear as a chaotic variety. In attempting to interpret such a collection of experiences, science seeks some pattern to which they appear to conform. Thus the recognized object of science is the development of mechanisms, a mechanism being simply a man-made schema or model which purports to relate a set of natural phenomena in a rational manner. A mechanism may be pictorial, as is the conventional atomic model portrayed to elementary students of physical science, or it may be diagrammatic like the device employed by the organic chemist to display the manner in which a large number of atoms may cling together to form a complex molecule. So, just as the architect's blue-print possesses a correspondence to the finished house, the mechanism of the scientist is made to correspond to some part of nature.

A mathematical structure when applied as a correlating agent to the data of a science merely becomes a mechanistic device, and must be regarded as such by the scientist. It is the belief of many, however, that the mathematical mechanism has merits which others do not possess. For example, deductive reasoning as rigidly employed in mathematics is the only means yet developed for isolating hidden assumptions and for following the subtle implications of the various hypotheses. Moreover, the basic entities of a science are conveniently recognized as those which are represented by subject-matter symbols that are not explicitly defined within the mathematical system employed; in fact, such symbols are given an implicit definition by the set of primitive statements in which they occur.

[3] E. V. Huntington, *Amer. Math. Monthly,* 24: 1–16, 1917.

[4] Note, for example, G. C. Evans, "Mathematical Introduction to Economics." New York: McGraw-Hill Company, 1930.

[5] Note Volumes I and II. "Foundations of the Unity of Science," edited by Otto Neurath. Chicago: University of Chicago Press, 1938.

[6] P. W. Bridgman, "The Logic of Modern Physics," p. 60. New York: Macmillan Company, 1927.

The systematization which mathematics gives to a science is never static, and the science thus organized takes on a directed growth. Some investigators will always be concerned with the reorganization of the axiomatic base of the system, and especially with the possibility of decreasing the number of the axioms. Other students of the science will be making additional deductions from the accepted body of propositions, and new propositions obtained thereby will furnish the suggestion for more experimentation. In fact, the mathematization of a science must never be regarded as a substitute for experiment, for experimentation is continually necessary for confirmation of the theoretical structure. One experimental result contrary to that predicted by the mathematical theory may be sufficient to cause a thorough revision of the theory, or perhaps relegate the whole thing to the grave of false hopes. Of course, many factors must be considered before a theory is actually discarded; for instance, a simple theory furnishing quite approximate results may be employed in preference to a very complex theory which is considerably more accurate in its interpretation of nature.

There is a strange fact about all these mechanistic devices which have been invented and employed by man in his effort to comprehend nature. They are first called laws of science, then, perhaps, laws of nature. After a while man is inclined to forget that they are products of his own imagination, and comes to believe that they are real and a part of creation. This fact has been responsible for many unfortunate attitudes and points of view. So some comments pertaining to the true relationship between a mathematical theory and that portion of nature which it is designed to interpret may be appropriate.

First of all, it must be emphasized that modern science recognizes the ultimate complexity of nature, and any theory which science may employ is too simple to have exact structural similarity to any part of nature. The mathematician may seek a linear formula that best represents the trend of a random set of points which are distributed, however, so as to suggest a straight line; in like manner, the scientist systematizes his study by the use of a mathematical pattern which can reflect only the general behavior of the data of his science. Moreover, it is doubtful that there is a unique theory to be sought by the scientist laboring in any field, for as Bliss[7] has said, "There are always more mathematical theories than one whose results depart from a given set of data by less than the errors of observation." The Ptolemaic and Copernican theories of the solar system furnish illustrations of two essentially different theories which, after slight modification of the former, describe equally well the behavior of the planets. The modern popularity of the Copernican theory is due chiefly to its relative simplicity.

A serious misunderstanding in regard to the mathematizing of science is apparent in the writings of some popularizers of scientific theory. In many instances, such writers read into nature a lot of fantasy which has its origin in some mathematical property of the theory under discussion rather than in the data from nature which the theory is designed to systematize. Of course, an adequate discussion of such matters must penetrate deeply into the subject of scientific methodology. An example of this type of misunderstanding is to be found in the insistence of some persons that the universe is finite, simply because the finite geometry of Riemann has been used with considerable success as a correlating agent of the data of the astronomical universe. Similarly, there is no justification for stating that continuity is a property involved in a set of data when a calculus of continuous functions has proved valuable in studying it. Many mathematical properties, as a matter of fact, are ideal, and their precise mathematical meaning could not be realized in the physical universe.

It should be evident by now that there are many interesting problems involved in any consideration of the relationship of mathematics to the sciences. In truth, as a field of study, science and philosophy have only touched the fringe. Real progress in analyzing the many difficulties involved demands more investigators with greater versatility of interest and preparation. Mathematicians need to become more familiar with the sciences, and many scientists must appreciate that a knowledge of mathematics consists of more than a mere ability to manipulate a few mathematical symbols. In the meantime, humanity awaits the many fine accomplishments which will result from a greater mutual understanding between mathematicians and the scientists.

FORTIFICATION OF FOODSTUFFS[1]

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IT is doubtful whether a single nutrition conference, out of the many that have been held in the past year, has not given some attention to the fortification of foodstuffs with vitamins and minerals. The interest of the public and of the food manufacturer in the problem is evidenced by the increasing number of

[7] G. A. Bliss, *Am. Math. Monthly*, v. 40, p. 472, 1933.

[1] Nutrition Conference: University of California, Berkeley, California, May 3, 1941.