tion $t(\theta)$ of C, will yield a general representation $p_i[t(\theta)]$ of g. This representation is formally written pt and termed a product representation. Morse and Tompkins use these product representations whenever there is more than one contour. One advantage lies in the fact that the restricted representations converge for bounded D only to restricted representations, while the Möbius transformations converge in a simple manner to Möbius transformations or degenerate transformations regardless of whether D is bounded or not. Another advantage is that D, regarded as a function of these restricted and Möbius transformations and of the ratio ρ depends continuously on the Möbius transformations and ρ.

With these new variables the definition of the integral D can be extended by a unique limiting process to the cases where $\rho=0$ or where one or more of the Möbius transformations are degenerate. The resulting function of these new variables is shown to satisfy the conditions of the general theory, which may then be applied.

We shall give a typical theorem, using conditions somewhat less general than those which will appear in our paper, in particular not introducing the "chord arc condition."

THEOREM. *Let g_1 and g_2 be two simple closed curves with continuously turning tangents. Suppose that g_1 and g_2 are separated by a plane and have convex projections on suitably chosen planes. If g_1 and g_2 bound a ring type minimal surface of minimum type they also bound a ring minimal surface of non-minimum or unstable type.*

The conditions of the second sentence of the theorem can be considerably lightened. The theorem may be illustrated by a classical example in which the boundaries are two circles with centers on a common axis and planes orthogonal to this axis. If the planes of these circles are near enough together the circles bound two minimal surfaces of revolution generated by catenaries. One of these surfaces is of minimum type and the other of non-minimum type.

We have not gone into great technical detail in our paper. We have rather presented general fundamental ideas by way of typical examples.

These ideas and examples we believe may be a prelude to far-reaching applications both in mathematics and in mathematical physics.

TECHNICAL PROGRESS IN AVIATION[1]

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THE increasing importance of the airplane in our normal social and economic life is just now overshadowed by its dominant part in our national security. When recent advances in the aeronautical sciences are being tested on a grand scale in the proving ground of war, we naturally lose interest in the possibilities of those same advances in applied science as they might assist our peace-time communications. If we hope to live in a kind of world in which individual living is worth while, our thought must be focused on the airplane as an instrument of air power; destructive or protective.

That important technical progress has been made in the development of the airplane is obvious from the reports of dive bombings, night bombings and aerial torpedo attacks on the one hand, and from reports of effective defensive action by fast fighting planes on the other hand. Guns, armor, radio, fuels, meteorology, metallurgy and photography are also contributing through technical advances to the effectiveness of the airplane in war. Attack and defense for colossal stakes stimulate development in every branch of applied science.

Technical progress in the development of the airplane as a vehicle of the air has been especially marked in the past ten years. Some steps in this progress have been abrupt, because they were the consequence of inventions. True inventions are unpredictable, but it is our experience that when new knowledge obtained by research and experiment becomes generally known, the invention necessary for its practical application soon follows. Whether the invention can then be applied by designers to realize a technical improvement in the airplane depends on whether the state of the art is ripe for it.

For example, the aerodynamic advantages of an unbraced monoplane wing were demonstrated by Fokker in the days of braced biplanes, but the safe construction of such wings had to wait for the availability of light aluminum alloys. Likewise the advantage of retracting the landing gear and wheels was recognized at an early date and mechanisms for retraction were invented, but no designer would bother with them until speeds were high enough to make it worth while to accept the added cost and complication. The designer of transport planes, moreover, needed a thick cantilever wing to afford space enough to house the wheels in the retracted position. Consequently, there was a lag of some ten years in the general adoption of this improvement.

Though some technical improvements are thus proposed before the art has sufficiently advanced to permit their use, other improvements come about as a

[1] Address of the retiring vice-president and chairman of the Section on Engineering of the American Association for the Advancement of Science, Philadelphia, December 31, 1940.

result of difficulties created by advance in the art. One example of these is the mass balancing of control surface to prevent unstable vibration or "flutter" of cantilever wings. Serious flutter trouble was not encountered until experience was had with high-speed monoplanes and particularly with dive bombers. The cure for the trouble was discovered after the trouble was disclosed.

When airplanes fly very fast, moreover, roughness of the external surfaces is found to be extremely costly in power. As a result, we now have flush rivets, spot-welding techniques and other means to help make a smooth wing.

In general, the results of methodical research and experiment lead to new knowledge and to technical progress. But technical progress, of itself, discloses new difficulties whose solution requires further research. From this solution, further technical gains may result.

Under the stimulus of war, this self-generation of technical advances is unnaturally accelerated, but the direction of the progress is likely to be toward immediate objectives. For fighting planes every effort will be made to improve the vital performance characteristics of speed and climb, regardless of expense in power and fuel. Bombers must take off with a maximum load of bombs without regard to how such an overloaded plane could safely be landed again. One assumes that the bombs will be dropped somewhere remote from the home field, or the fuel consumed to lighten the plane before landing.

The design of military airplanes is fundamentally controlled by military requirements and will be specialized as these requirements are specialized. Thus we have heavy bombers of long range to carry very heavy loads, light bombers of high speed, dive bombers and torpedo planes with special features of control, interceptor-fighters of extreme speed and climb but short range and escort-fighters with long range and heavier armament.

Such airplanes are not very well adapted for the economical transport of passengers and goods, yet the knowledge of the aeronautical sciences which makes them possible can be applied to the design of commercial airplanes. From the technical advances now being made in military aircraft, we may safely forecast corresponding advances in future civil aircraft for our air lines.

Without invading regions of official secrecy, it is evident that progress is now being made in certain aspects of aeronautical science and that one is justified in venturing some opinion as to the general nature of future technical improvement in both military and commercial aircraft.

To begin with, we must realize that the airplane is the creature of the internal combustion engine. The gasoline engine of the beginning of our century made possible both the automobile and the airplane. Progress in the airplane has paralleled the technical development of its engine. The most spectacular improvement in the engine has followed better knowledge of the nature of the combustion of the fuel in the cylinder. From such knowledge came scientific methods for testing the "octane number" of a fuel as a measure of its detonation characteristics. Midgley's introduction of tetraethyl lead to raise the octane number of a fuel has permitted engine designers to go to higher compression ratios with consequent improved power and economy. Military aircraft must have fuel of the highest octane rating procurable; military pressure will create additional capacity for such special fuels, and will stimulate the production of fuels of even higher octane rating than the best now available. Likewise, the national defense program is requiring engines of even greater power for take-off with large bomb loads and greater economy in order to obtain a longer radius of action. The new engines must use the improved fuels.

Obviously, our air lines will become eventual beneficiaries of the bigger and better engines and the fuel to go with them. The commercial industry could never afford to subsidize the fuel and engine development costs that are involved. With more powerful engines, the Army will develop long-range multi-engine bombers and the Navy long-range multi-engine flying boats. These aircraft will, of course, not be equipped for passengers, but requirements for habitability for their crews will force consideration of adequate heating and insulation, sound-proofing, de-icing, radio communications and navigation, and special arrangements for substratosphere flying. For large military craft there will have to be developed suitable tires, wheels, brakes, servo controls and many types of special navigational and radio equipment which do not now exist.

Our designers and manufacturers, as a result of their experience with the defense program, should be in an excellent position to adapt these aircraft for commercial purposes. The military aircraft may be too fast to be economical and may have unsafe take-off and landing characteristics for immediate conversion to air-line use, but their designers certainly will know what alterations are needed. The basic technical problem and one solution of it will be known. The designer will then have only to work out a new solution making use of a proved method.

Another field in which rapid technical progress is being made is in the aerodynamics of the airplane itself. This work, done in order to obtain higher speed from the same power, is all to the advantage of the air transport operator who must lower his costs and his fares as his business grows.

The reduction in drag by the use of smoother surfaces exposed to the air has already been mentioned.

Roughness of surface is associated with the breakdown of the initially smooth laminar flow of air in the boundary layer next to a surface. This alteration of laminar flow into a turbulent régime of much higher drag is delayed if the pressure gradient along the chord of a wing is falling. Shapes of wing section in which the pressure gradient is more favorable to the maintenance of laminar flow are being studied. It is to be expected that, by the use of new wing sections made very smooth by new techniques of fabrication, substantial reductions in drag may be realized in future.

One of the largest sources of wasted power or drag comes from the necessity for cooling the engines. The liquid-cooled engine must have a large radiator and the air-cooled engine must have a strong flow of air over its cylinders. Recent research has shown that the drag of this cooling air can be very much reduced if the principles of fluid mechanics are applied to the ducting system that leads the cooling air in and out of the airplane. In addition to the direct cooling of the engine, moreover, an internal air flow is required for the carburetor and the oil radiator and, if the airplane is designed for high altitudes, for the intercooler of the supercharger.

An internal combustion engine, of course, develops power from the combustion of fuel in the air that fills the cylinder. At an altitude of about 18,000 feet the oxygen content of a cylinder full of air is only half of that at ground level, and consequently only half as much fuel can be burned in this lighter air. The power of the engine, therefore, falls off very rapidly with altitude, unless the engine drives a compressor or supercharger to supply itself with air at ground-level pressure. The supercharger heats the air adiabatically as it compresses, and the air is further heated by the turbulence and eddies created by its passage through the surpercharger. In order to attain ground-level conditions in the cylinder, a radiator, called the intercooler, is required to cool the compressed air before it passes into the engine.

Drag caused by the intercooler means wasted power, and the power taken from the engine to drive the supercharger is also wasted as far as carrying pay load is concerned. Present efforts to improve the efficiency of the supercharger as a pump should be helpful, and very real savings can be made in the ducting of the air. An alternative solution, which saves the power taken from the engine to drive the supercharger is obtained if the supercharger is driven by an exhaust gas turbine. Such turbines spin faster as the airplane goes higher, and have ideal characteristics for the purpose. However, the turbine is an extremely difficult apparatus to make, since it must run in very hot gas and at very high revolution.

The national defense program will inevitably do much to bring about technical progress in supercharging, gas turbines and associated control apparatus which our air lines badly need when they fly "above the weather."

Another aspect of the effort to get high speed results in belated attention to the energy of the exhaust. This is about equal to the energy developed usefully by the engine. When airplanes flew at 200 miles per hour at moderate altitudes, the exhaust problem was merely one of disposal. If no flame blinded the pilot and no gas got into the cabine, disposal was satisfactory. At high altitude and high speed, however, it is possible to recover considerable forward thrust by passing the exhaust out through suitable nozzles. This is in effect "jet propulsion," long known to be a very uneconomical scheme at ordinary speeds. At high speeds, the jet is really effective and efficiency is of little consequence since the waste energy of the exhaust is being used. It is estimated that as much as a 10 per cent. gain in effective thrust of the engine can be obtained from the exhaust. Whether commercial air lines will develop an interest in jet propulsion will depend on whether their future requirements include much higher speeds of flight.

Another kind of technical progress is based on research in aerodynamics. With improved wing sections, the ratio of lift to resistance to forward motion has been notably increased in recent years. This increase should make for higher speeds with the same power, but the so-called high-speed wings are not good weight carriers and would entail too high a landing speed to be practical for commercial aviation. By means of slots and trailing edge flaps, it is now possible when approaching a landing to convert the wing temporarily to a high-lift type. Since the development of improved high-lift devices of this sort is of great importance to bombers as well as to commercial planes, we may expect military pressure to accelerate it. Wing loadings have increased recently from 30 to more than 40 pounds per square foot, and the limit is not in sight.

With higher wing loading, the length of run at take-off requires large airports, and we shall see a considerable enlargement of our American airports as part of the defense program. This will aid the extension of our already rapidly expanding civil air transport industry.

As to the importance of one feature of technical progress, even the so-called experts have been badly misled. I refer to questions about the ability of the American type of radial air-cooled engines now universally used by our air lines. Since European engines are liquid-cooled and are used in high-speed pursuit machines like the British Spitfire and German Messerschmitt, there has been wide acceptance of the idea that this country can not build high-speed fighting airplanes until it has liquid-cooled engines of the European type. Furthermore, since certain stripped

German airplanes broke the world's speed records twice in 1939—a Heinkel in March at 464 miles per hour and a Messerschmitt in April at 469 miles officially and 481.4 unofficially—the momentary speeds developed have been accepted as inherent in the liquid-cooled engine used. This is an entirely false inference.

At the time that the Germans were claiming the world's speed record, moreover, American planes equipped with American air-cooled engines of similar rated power output were making a high speed in normal flight of about 330 miles per hour, while British planes with liquid-cooled engines of the same power output were credited with 360 or more miles per hour. The case for the liquid-cooled engine seemed to be very convincing. Yet it was based on a false conception of the aerodynamics of the matter. There was also an element of propaganda in it.

The test of war has shown that the German fighting planes have a real speed of about 350 miles per hour and that their British opposite numbers make the same speed or a little more. In the meantime, progressive improvement in the installation of the American air-cooled engines has brought up the speed of our fighter planes.

The National Advisory Committee for Aeronautics in its Langley Field wind tunnel has developed means of streamlining (ducting and cowling) the American radial air-cooled engine, so that its drag can be made as low as that of the best liquid-cooled engine installation. Thus recent technical progress has enabled American airplane builders to demonstrate airplanes with larger air-cooled engines at speeds exceeding 400 miles per hour. There now appears to be nothing to choose, as to speed, between the two types of engines when each is properly installed. This statement could have been true several years ago, but the results of research were available only recently.

The largest liquid-cooled European type engines now used by Germany and England develop about 1,200 horse power, and this year—1941—we shall have similar engines in quantity production here. Two new types of American air-cooled engine are already developed and in production in the 2,000 horse-power size. Nowhere else in the world are engines of such power available. The demand for armor, protected gasoline tanks and heavier armament can safely be predicted to make the 1,200 horse-power engine inadequate in the near future on account of the increased weight involved. Also there is always a demand in war for more speed. This country, however, can meet these demands by the development of a 2,000 horse-power fighter to compete with the 1940 European 1,200 horse-power plane. We have the engines and the knowledge of how to install them for high speed.

Production for war is not like production for domestic consumption. The business man trained in commerce may insist on "freezing" the design and going into production for three years on the same article in order to recover his investment in engineering costs. In war, technical progress on the part of the enemy can make such a three-year program disastrous.

We now have the interesting situation of a very large production program for 1,200 horse-power fighter airplanes based on European liquid-cooled engines which is getting well under way at about the time our own engineers present us with two 2,000 horse-power engines, and our scientists show us how to streamline them in an airplane installation to secure extremely high speed.

The position seems to be that we are using our great productive capacity for an immediate program of fighting airplanes that are the equal of anything known in Europe, and yet we have the potential ability to make obsolete all fighter airplanes in the world to-day, including our own. This position has come about by recent technical progress in engines and fuels, combined with new knowledge of the aerodynamics of air flow. When we should take advantage of this recent technical progress is a delicate question involving a balance between strategic and tactical considerations. It is the old dilemma of quantity now versus quality later and can be decided wisely only when we know whether it really is later than we think.

PROFESSOR LAWRENCE AND THE DEVELOPMENT OF THE CYCLOTRON[1]

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I HARDLY need remind this distinguished gathering that it is the father of the cyclotron whom we are delighted to honor to-day. When I first met Lawrence, in June, 1931, the cyclotron was a very small baby. In fact, the only examples that had been made to work up to that time were still barely out of the sealing wax and string stage in which all good physical apparatus is first realized in material form. My memory of that

[1] An address on the occasion of the presentation of the Duddell Medal of the Physical Society of London at the dinner of the American Physical Society and Section B of the American Association for the Advancement of Science, Philadelphia, December 27. The medal was to have been presented by Lord Lothian. In view of his death the presentation was made by Mr. Neville Butler, chargé d'affaires of the British Embassy.