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A MATHEMATICAL THEORY OF EQUILIBRIUM WITH APPLICATIONS TO MINIMAL SURFACE THEORY[1]

By Professor MARSTON MORSE

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THE theory of equilibrium points or critical points of functions appears in fragmentary form in the work of Poincaré, Maxwell and Kronecker. Birkhoff introduced the minimax principle and applied it in dynamics. A systematic study of the critical points of functions of n-variables was begun by the author in 1922. A. B. Brown made important contributions. In 1924 the calculus of variations in the large was introduced and developed as an extension of the theory of critical points of n-variables. The integrals used were ordinary and were regarded as functions of the curves along which they were evaluated.

[1] Address of the retiring vice-president and chairman of the Section on Mathematics of the American Association for the Advancement of Science, Philadelphia, December 27, 1940.

In 1937 the theory was put on a more general basis with the function defined on an abstract metric space. For details the reader may refer to the author's fascicule on "Functional Topology and Abstract Variational Theory," published by Gauthier-Villars, Paris. In this general theory one is free from any limitation of dimension. It is immaterial whether the independent variable is a point in a Euclidean space, a curve, a surface or a more general configuration. A critical point of a function of n-variables is a point at which all the partial derivatives vanish. When the function is an integral in the calculus of variations—for example, the integral of length on a surface or the area of a surface bounded by a curve—a critical point (curve or surface) is one satisfying the Euler-La-

It will be seen that the glycerin-buffer trypsin solutions stored in the icebox are stable over long periods. By slightly increasing the incubation time for digestion (at the most by two or three minutes), the identical end-point may be obtained for an indefinite period (three years at least) with a single standard solution of trypsin.

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PURIFICATION OF DIPHTHERIA ANTI-TOXIN

A PROTEIN has been obtained from equine antidiphtheria plasma which is highly antitoxic and which satisfies the criteria for a pure protein. The preparation contains about 700,000 to 1,000,000 anti-toxin units per gram protein nitrogen by the Ramon flocculation test and about 700,000 anti-toxin units per gram by guinea pig protection test. This is about 20 times the activity of the original antisera and agrees approximately with that reported by Pappenheimer and Robinson[1] and by Pope[2] for anti-toxin prepared by a different method. The material is completely precipitated by diphtheria toxin.

Dr. Alexandre Rothen has examined a sample of the protein in the ultra-centrifuge and in the Tiselius electrophoresis cell. Both methods show the presence of only one protein at pH 7.3. A slow migration to the anode is observed at this pH ($\mu \backsimeq 4 \times 10^{-6}$ cm^2/sec. volt). The rate of sedimentation ($S^{26} = 5.77 \times 10^{-13}$) indicates a figure of about 80,000 for the molecular weight. At pH 8.0 the protein migrates also slowly to the anode and is homogeneous. At this pH the material shows some aggregation in the ultra-centrifuge. Salt precipitation shows that some denaturation occurs at pH 8.0 and becomes marked in slightly more alkaline solution.

The solubility of this sample varied slightly, however, with different quantities of solid indicating the presence of more than one protein. A second more highly purified preparation showed constant solubility, independent of the quantity of solid present. The protein therefore satisfies the criteria for a pure protein.

Several samples have been obtained in the form of fine, poorly formed needles and thin plates. The protein has been prepared from three different lots of plasma precipitated with four different lots of toxin.

The method of preparation consists in precipitating the toxin anti-toxin complex. This precipitate is dissolved by the addition of acid and the toxin destroyed by digestion with trypsin at about pH 3.7. The anti-toxin is then purified by fractionation with ammonium sulfate at pH 7.2. The purified antibody is soluble in 0.50 saturated ammonium sulfate pH 7.4 and insoluble in 0.65 saturated ammonium sulfate.

Twenty-five to 50 per cent. of the original antibody may be recovered after the removal of the toxin. This crude antibody has a titre of from 300,000 to 500,000 units per gram protein nitrogen. It exhibits only one moving boundary in the electrophoresis cell, but there is marked spreading and the solubility test shows this crude preparation to be far from homogeneous.

The toxin and anti-toxin were furnished by the Biological Laboratories of E. R. Squibb and Sons. The animal protection tests were carried out by Dr. W. E. Bunney, at E. R. Squibb and Sons, New Brunswick, N. J.

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[1] A. M. Pappenheimer, Jr., and E. S. Robinson, *Jour. Immunol.*, 32: 291, 1937.
[2] C. G. Pope, *Brit. Jour. Exp. Path.*, 19: 245, 1938.

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HILLS, E. SHERBON. *Outlines of Structural Geology.* Pp. 172. 105 figures. Nordeman. $2.25.

NEWBY, WILLIAM W. *The Embryology of the Echiuroid Worm Urechis Caupo.* Pp. xiv + 219. 85 figures. American Philosophical Society. $2.00.

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SMITH, CHARLES M. and MARY M. ROOS. *A Guide to Guidance.* Pp. xvi + 440. Illustrated. Prentice-Hall. $3.00.

WEYL, CHARLES, S. REID WARREN, JR. and DALLETT B. O'NEILL. *Radiologic Physics.* Pp. xvii + 459. Illustrated. Charles C. Thomas. $5.50.