wände gegen die menschliche Abstammungslehre,"[2] but even he[3] endeavors to support his scientific opinion politically, asserting that, should the origin of man be questioned, the adversaries of the national socialist "rassenhygienische" tendencies could cite the uncertainty of science.

OTTO HAAS

THE AMERICAN MUSEUM OF NATURAL HISTORY

SCIENTIFIC BOOKS

GEOMETRY OF CURVES AND SURFACES

Metric Differential Geometry of Curves and Surfaces. By ERNEST PRESTON LANE, professor of mathematics, University of Chicago. Pp. viii + 216. Chicago: The University of Chicago Press. 1940. $3.00.

THIS book is quite elementary in character. Plane, analytic geometry and the usual first-year course in calculus are sufficient prerequisites. In addition some knowledge of three-dimensional analytic geometry (direction cosines, equations of straight lines and planes in space, etc.) will be helpful, but this can be developed as the need arises in any class using this book as a text.

As mentioned in the preface by the author, most of the material included is classic. Consequently, the scope of the book is perhaps sufficiently well indicated by an enumeration of the chapter headings with a few additional comments. Chapter I, "Curves": This includes a discussion of arc length, curvature, torsion, the osculating sphere and circle and the Frenet formulas. Chapter II, "The Moving Trihedron," contains an application of the moving trihedron to such topics as involutes, evolutes, parallel curves and Bertrand curves. Chapter III, "Surfaces": Here the first fundamental form is introduced, the envelopes of surfaces, developable surfaces and ruled surfaces are considered. Chapter IV, "Curves on Surfaces": This contains a discussion of such topics as minimal curves, asymptotic curves and the second fundamental form, conjugate nets, lines of curvature and geodesics. Chapter V, "Curvature": This includes among other topics the radius of normal curvature, principal normal curvatures, geodesic curvature and geodesic torsion. Chapter VI, "Transformation of Surfaces," contains a discussion of conformal representation, applicability and parallel surfaces.

The reviewer was very favorably impressed by the simple and easily understandable style in which Professor Lane has written this book. An important and very desirable feature of the book is the abundance of definitions. By encouraging students to reason upon carefully worded definitions, Professor Lane has performed an essential mathematical service. Definitely stated theorems expressing the results obtained are also much in evidence throughout the book. In these respects Professor Lane is strongly to be congratulated.

In the opinion of the reviewer, if this book were used as a text in an undergraduate class composed of students who have completed their first or second course in calculus, it would constitute an interesting application of their previous mathematical work and be a source of inspiration toward further mathematical activity. But in the preface Lane states that the book is designed as a text for first-year graduate students. Here the author and reviewer are in decided disagreement and as the matter is an essential one the reviewer would like to make his position clear.

The book is essentially "undergraduate" in character. For example, in the very first sentence of the book (p. 1) we find ". . . curves and surfaces in ordinary three dimensional space." Now this is the way we speak to undergraduates. When a student has entered upon graduate work (and this includes presumably only such as are genuinely interested in the subject) he will ask what is "ordinary three dimensional space" in the mathematical sense. He will ask this question and he will be entitled to an answer. But he will not find the answer in Lane's book. Again in deriving the formula for the arc length of a curve (equation (3.6) on p. 8) we find that the arc length Δs appears in the derivation. Surely a graduate student is entitled to have this formula (which in reality defines the arc length) presented to him in such a way that the concept of arc length is not inherent in the derivation. No doubt other such objections could be found, but the reviewer has not sought them, and they are mentioned here only as an indication of the "undergraduate spirit" in which this book is written.

But the main objection to this book as a graduate text lies in another direction. The subject of tensor analysis can best be introduced and illustrated as part of a course on differential geometry. Moreover, the ideas and methods of the tensor analysis enter basically in the theory of relativity, are useful in the calculus of variations, mechanics, hydro- and aerodynamics and have appeared in certain phases of such widely divergent fields as engineering and topology. In the opinion of the reviewer one should continually seek in graduate instruction to enlarge and enrich the viewpoint of the student and in failing to base his text on the invariant formulation of the tensor analysis Professor Lane has lost not merely an excellent opportunity to introduce his readers to a point of view useful in other fields

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[2] *Verhandl. d. Deutsch. Ges. f. Rassenforschg.*, Vol. X, Sonderheft, 1940, pp. 96–99.

[3] *L.c.*, p. 98.

of mathematical work but has deprived them at the same time of a type of analysis which forms in fact a necessary mode of presentation of the underlying concepts of differential geometry itself. For these reasons the reviewer does not consider this text desirable for graduate instruction in any American university.

The binding and printing are excellent, and the University of Chicago Press is to be congratulated for its part of the work.

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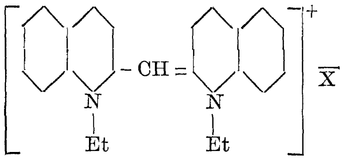
SPECIAL ARTICLES

THE STRUCTURE OF THE MESOMORPHIC PHASE OF CERTAIN CYANINE DYES

BASED partly on extensive work to be presented by the writer and colleagues, a structure is proposed for the aggregated phase of *di-ethyl-ψ-cyanine* and related dyes which give a remarkable new absorption (and fluorescence) band. Discovered by E. E. Jelley[1] and independently by G. Scheibe,[2] it is convenient to term it a Z-band, and the corresponding aggregation of the dye, a Z-state.

On silver halides[3] the dye cations are adsorbed edge-on at the ionizable hydrophile (ventral) aspect of the planar molecule. The counter-ion is furnished by the silver halide lattice. At high adsorption densities the dye cations form packets of parallel ordered molecules, like card-packs, but with an interval of ca 3.7 Å between adjacent cations. These "card-packs," but with an important modification, are regarded as constituting the separately sensitizing dye "aggregates" of Leermakers, Carroll and Staud[4] and the "nematic" filaments discovered by Jelley in aqueous suspension. From optical observations Jelley showed that the new absorption band corresponds to a light vibration *parallel* to the axis of the filament, and in the z-axis of the resultant crystal. In our model thus far presented this means a light-vibration perpendicular to the planes of the units of the card-packs. Conclusions similar in essentials were reached by Scheibe,[5] but from adsorption experiments on mica he considered that alternate parallel molecules are staggered. He has suggested that the forces orienting and holding the units in parallel apposition are inductive forces of the p-electrons of the (heterocyclic and aromatic) nuclei and of the chain joining these. The structure proposed here postulates a very different type of linkage, *viz., water molecules,* coordinated intermolecularly between *opposite* terminal nitrogen atoms of the parallel resonance chains. A reversible effect of moisture on the production and disappearance of the Z-band for ψ-cyanine adsorbed to silver bromide was observed by Leermakers, Carroll and Staud,[4] and confirmed by Sheppard, Lambert and Walker;[3] the fundamental importance of water for the Z-state of unadsorbed dye has been shown by Jelley[6] and equally recognized by Scheibe.[5]

The ψ-cyanine has the formula:



where $\overline{X}$ is a univalent anion, and the large cation derives its positive charge from the resonance hybrid between states having each N-atom alternately in the quaternary state. We will symbolize such a molecule as $N_p \sim N_p$, where the subscript represents the nuclear portion (and *its electromeric effect*) and is changed accordingly. Thus for a non-symmetrical cyanine we may write $N_{p'} \sim N_q$, where p' denotes the same nucleus as p, but changed because joined with q. In the Z-state we supposed these cations arranged in parallel, and between each successive opposite pair of molecules at least one water molecule—H_2O—coordinated between each pair of opposite N-atoms. Any adjacent pair of dye molecules form a cell or doublet; in the extended filament $(N_p \sim N_p)n$, where n is the number of cells, if there are just two H_2O molecules per cell, the ratio of H_2O/dye molecules will vary from 1:1 for $n=1$ to 2:1 for $n=\infty$ and similarly for other basic H_2O values per cell. These interstitial "hydrate" molecules furnish the mobile electrons permitting (in the ground state) inter-molecular or cross-resonance between the N-atoms of adjacent molecules, which evidently must be coupled, or in phase relationship, with the intramoleculer resonance of the component dye molecules. Hence, a condition of alternation or pulsation will obtain between successive adjacent cells, which is important as permitting undamped transfer of excited states throughout a filament. *These intermolecular resonance linkages through "hydrate" water molecules are supposed to furnish the characteristic*

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[1] E. E. Jelley, *Nature,* 138: 1009, 1936; *ibid.,* 139: 631, 1937.

[2] G. Scheibe, *Angew. Chem.,* 50: 51, 1937.

[3] S. E. Sheppard, *Atti. X. Congress Internat. Chim.* (Rome, 1938, V. I. pp. 235–283; S. E. Sheppard, R. H. Lambert and R. D. Walker, *Jour. Chem. Phys.,* 7: 265, 1939.

[4] J. A. Leermakers, B. H. Carroll and C. J. Staud, *Jour. Chem. Phys.,* 5: 878, 1937.

[5] G. Scheibe, *Koll. Zeitschr.,* 82: 1, 1938; *Angew. Chem.,* 52: 631, 1938.

[6] Unpublished work, these laboratories.