the established form of epidemic influenza is called Influenza A, outbreaks caused by virus of the Lee type are to be designated Influenza B.

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THE VIBRATION OF CIRCULAR PLATES

THE equation for the nodal lines of a circular plate was worked out by Kirchhoff in 1850, having the form

$$w = J_n(kr)\cos(n\theta - \alpha) \qquad (1)$$

The first factor of this equation will give circles of different radii, while the second represents diameters. For nearly one hundred years this equation has been considered to be the most general possible solution so that the conclusion seemed inescapable that only circles and diameters could appear upon a circular plate. When plates are actually vibrated, however, many symmetrical patterns appear upon them which are certainly not circles. These variations from the theory were thought to be due to asymmetries in the plates, such as varying thickness, different tensions and so forth. If the assumption is made that a circular plate can give out two or more notes at the same time, then the equation of Kirchhoff may be extended in the form

$$w = AJ_n(kr)\cos n(\theta - \alpha_n) + BJ_m(k'r)\cos m(\theta - \alpha_m) \qquad (2)$$

When this equation is zero (the condition for nodal lines), the resulting figures are much more complicated than mere circles and diameters. It will be proved that they correspond to the figures produced by experiment.

In Fig. 1 are shown tracings from photographs of circular vibrating plates. The originals will be found in another article by one of the writers.[1] Below them in Fig. 2 are the mathematical curves resulting from substitution in Equation 2 of the proper numerical values. The exact equations for these are in order:

$$\left.\begin{array}{lr}
3J_0\left(\frac{12.53r}{a}\right) + J_4\left(\frac{11.95r}{a}\right)\cos 4\theta = 0 & (2a)\\
J_0\left(\frac{12.53r}{a}\right) + 2J_4\left(\frac{11.95r}{a}\right)\cos 4\theta = 0 & (2b)\\
J_2\left(\frac{5.937r}{a}\right)\sin 2\theta - 5J_6\left(\frac{11.00r}{a}\right)\sin(6\theta) = 0 & (2c)\\
J_2\left(\frac{5.937r}{a}\right)\cos 2\theta + J_3\left(\frac{3.497r}{a}\right)\cos 3\theta = 0 & (2d)
\end{array}\right\} (3)$$

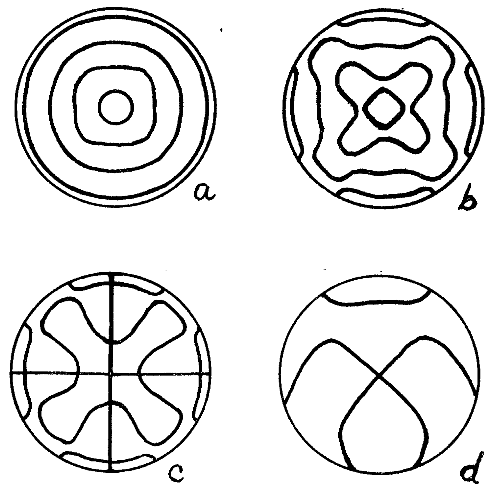

FIG. 2. Theoretical curves calculated.

The values of A and B are found by trial. After considerable experience one becomes more or less expert in picking out the number of circles and diameters in the Bessel and cosine functions which must be combined to produce any given figure. For instance, Fig. 2(c) is obtained as shown in Equation 3, for $A = 1$, $B = -5$. If these ratios are changed, another equation appears whose plot resembles 2(c) but differs from it in some important details. In Fig. 3 are shown two curves obtained from the equations

$$\left.\begin{array}{l}
J_2\left(\frac{5.937r}{a}\right)\sin 2\theta - 10J_6\left(\frac{11.00r}{a}\right)\sin 6\theta = 0\\
J_2\left(\frac{5.937r}{a}\right)\sin 2\theta - 10J_6\left(\frac{11.00r}{a}\right)\sin(6\theta - 5^\circ) = 0
\end{array}\right\} (4)$$

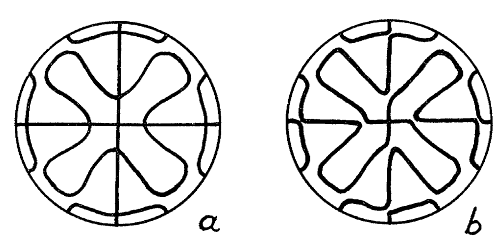

FIG. 3.

Simply varying the angle by 5° in Equation 4 causes the lines to split up. Both curves resemble that of Fig. 2(c) because similar functions are combined, although in different proportions.

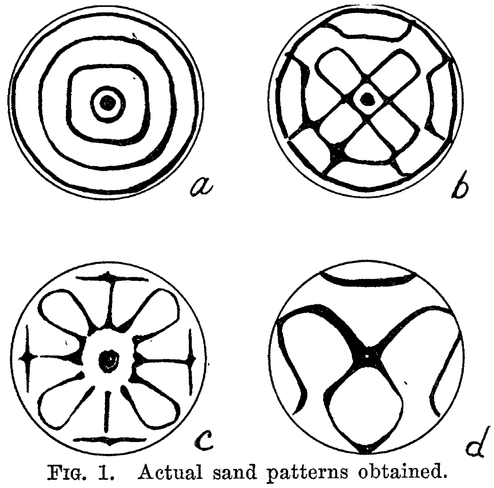

FIG. 1. Actual sand patterns obtained.

[1] Robert C. Colwell, *Jour. Franklin Inst.*, 213: 373–380, 1932.

Three functions may be added instead of two, or two functions may be multiplied together. The latter mathematical process indicates that two separate and distinct sets of lines may occur on a single plate at one time. Several such curves will be found in the last two pages of the article referred to above.

A fairly complete discussion of the method may be of interest whereby the mathematical curve of Fig. 2(*c*) is obtained from the Chladni plate shown in Fig. 1(*c*). To begin with it is usually best to take $A = B = 1$; by doing this one is often able to recognize related configurations which previously had escaped notice. In the Chladni plate Fig. 1(*c*), there are seen six diameters and at least one circle. This reduces the total possibilities to a limited number, and it is finally found by trial that the two modes of vibration which produce this Chladni pattern correspond to one circle and six diameters combined with one circle and two diameters. Now the correct values of the *k*'s for these circles are found in tables of Bessel functions. From these the equation

$$AJ_6\left(\frac{11.00r}{a}\right)\cos 6\,\theta + BJ_2\left(\frac{5.937r}{a}\right)\cos 2\,\theta = 0 \qquad (5)$$

is obtained.

When this equation is plotted the two predominant diameters are on the 45° and 135° lines. These can be related to the 0° and 90° lines by changing the cosine to sine. This changes Equation 5 to

$$AJ_6\left(\frac{11.00r}{a}\right)\sin 6\,\theta + BJ_2\left(\frac{5.937r}{a}\right)\sin 2\,\theta = 0 \qquad (6)$$

This is the first equation of the two shown in Equation 4, in which $A = 10$ and $B = -1$.

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SCIENTIFIC APPARATUS AND LABORATORY METHODS

A METHOD FOR MEASURING AND RECORDING CONTINUOUSLY THE pH OF THE CIRCULATING BLOOD[1]

WITHIN recent years the glass electrode has been found readily adaptable to the continuous measurement and recording of the pH of the circulating blood in experimental animals. Of the more recent descriptions of a method of this type is that presented by Nims, Marshall and Burr.[2] Our report briefly describes a method similar in principle, but differing in details.

The units of the assembly employed consist of a chamber containing the glass electrode, a Cameron detector and pH meter, a potentiometer, a Leeds and Northrup mirror type galvanometer, a light source and a kymographic camera.

The essential features of the glass electrode assembly are shown in Fig. 1; during its use the ends of the severed blood vessel are attached to side inlet and outlet tubes, directly in case the vessel is large enough or by means of glass cannulae and rubber tubes. From the bottom of the chamber there is a recurved tubular extension to which there is attached a stiff rubber tube connecting with the KCl chamber containing the reference electrode. The electrode stem is held firmly in a tapered glass stopper which is ground to fit accurately the mouth of the chamber. The stopper is secured against arterial pressure by stopcock grease or rubber bands stretched between glass ears on both the stopper and chamber. The chamber with the electrode in place has a volume of approximately 1 cc.

For measuring the pH of the circulating blood the method of operation of the Cameron detector and meter is the same as that employed in the determination of the pH of any other fluid. The pH may be read directly from the dial; however, provision has been made for recording the electrode potentials by

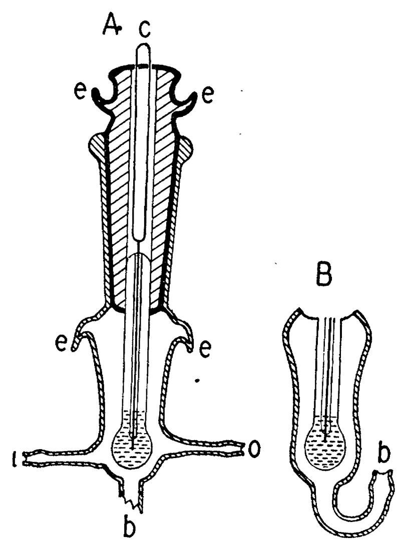

FIG. 1. A, cross section of the glass electrode and chamber, about two thirds natural size; b, position of salt bridge tube; c, wire contact; e–e, glass ears; i, inlet tube; o, outlet tube. B, cross section of the lower part of the chamber, showing the salt bridge tube more in detail. This tube lies in a plane at right angles to the planes of the inlet and outlet tubes.

[1] This work was aided by a Fluid Research Fund from the Rockefeller Foundation.

[2] Nims, Marshall and Burr, SCIENCE, 87: 197, 1938.