

for which  $\sigma$  is infinite. It is usually pointed out that the median of this universe is  $x=0$  and that the standard deviation of the median of random samples of  $n$  drawn therefrom is obtainable from the usual formula

$$\sigma_M = \frac{1}{2\varphi_M \sqrt{n}} \quad (2)$$

where  $\varphi_M$  is the value of  $\varphi$  at the median  $M$ , or, here,  $\sigma_M = \pi/(2\sqrt{n})$  and is finite.

That the formula (2) for the standard deviation of the median can not be universally valid like the formula  $\sigma/\sqrt{n}$  for the mean may be seen from considering the two functions

$$\varphi = \frac{3}{2} x^2, |x| \leq 1 \quad (3)$$

$$\varphi = \frac{1}{4} \frac{1}{\sqrt{|x|}}, |x| \leq 1 \quad (4)$$

in the first of which  $\varphi_M = 0$  so that (2) would make  $\sigma_M$  infinite, although it is surely less than 1, and in the second of which  $\varphi_M = \infty$  so that (2) would make  $\sigma_M = 0$ , although that is highly improbable.

As a matter of fact, if one refers to the proof of (2) as ordinarily given, one sees that it depends on the formula  $1/(2\sqrt{n})$  for the standard deviation of the fraction  $(\frac{1}{2})$  of the  $n$  values in the sample which fall to one side of the median of the universe, and that this deviation  $1/(2\sqrt{n})$  is converted into a deviation of the median of the sample itself by assuming that the area  $\varphi_M \sigma_M$  is  $1/(2\sqrt{n})$ . When the function  $\varphi$  is changing very rapidly at its median  $M$  it would seem that a better statement of this assumption would be

$$\int_{-\sigma_M}^{\sigma_M} \varphi dx = \frac{1}{\sqrt{n}},$$

where the origin for  $x$  had been taken at the median of  $\varphi(x)$ . Indeed if this be used in the two cases above it will appear that

$$\sigma_M = \frac{1}{\sqrt[3]{n}} \quad \sigma_M = \frac{1}{n} \quad (5)$$

respectively, instead of  $\infty$  and 0, and that the standard deviation of the median need not vary inversely as the square root of the number in the sample.

However, though the result is better, it is not perfect. It would be necessary to have recourse to a true expression for  $\sigma_M$ . If  $n$  be odd, as  $n=2k+1$ , the median of the sample will be the middle element. If

$$F(x) = \int_{M=0}^x \varphi(x) dx$$

the chance that the median of the sample be at  $x$  is

$$\psi(x) dx = \frac{(2k+1)!}{(k!)^2} \left(\frac{1}{2} + F\right)^k \left(\frac{1}{2} - F\right)^k \varphi dx$$

The mean value of the median of random samples will not in general be at the median of the universe (taken as origin), though it must be so for a symmetrical universe, to which we shall here confine our attention; then

$$\sigma_M^2 = \int x^2 \psi(x) dx, \quad (6)$$

where the integration is extended over the whole range of the function  $\varphi$ .

If this formula be applied to (3) and (4) we find, respectively,

$$\sigma_M^2 = \frac{1}{\sqrt{\pi}} \frac{\Gamma\left(\frac{n}{2}+1\right) \Gamma\left(\frac{5}{6}\right)}{\Gamma\left(\frac{n}{2}+\frac{4}{3}\right)} \quad \text{and} \quad \sigma_M^2 = \frac{3}{(n+2)(n+4)}$$

where  $\Gamma$  denotes the gamma function. If Stirling's formula be applied to the  $\Gamma$ -functions, it may be shown that for  $n$  large the variation of  $\sigma$  is according to the powers of  $n$  indicated in (5) but that the coefficients are not unity but approximately 0.90 in the first case, whereas it is 1.73 in the second.<sup>1</sup>

If (6) be applied to (1) it may be shown that for random samples of  $n=3$  drawn from that universe the standard deviation of the median would be infinite, but for  $n=5,7,\dots$  would be finite. Finally, the application of (6) to

$$\varphi = \frac{4}{(e+|x|)[\log(e+|x|)]^2}$$

will show that the standard deviation of the median of samples of  $n=2k+1$  will be infinite, no matter how large  $n$  may be,<sup>2</sup> although the function  $\varphi$  has a sharp beak ( $\varphi_M=4/e=1.5$ ) at  $x=0$ , and appears to fall away very rapidly as  $x$  increases, being less than .007 at  $x=\pm e$ .

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## THE PUBLICATION OF ISIS

THE publication of *Isis*, an international and polyglot quarterly devoted to the history and philosophy of science, was begun in Belgium in 1912-13. Hardly had five issues appeared (Volume 1 and the first half of Volume 2) when the publication was stopped by the German invasion. After the war Volume 2 was completed and distributed without extra charge to the

<sup>1</sup> The general case  $\varphi = C|x|^{-p}$ , with  $|x| < 1$  and  $p < 1$ , leads to  $\sigma_M^2 = 1/n^\alpha$  where  $\alpha = 1/(1-p)$  by the area argument, whereas by the usual formula it leads to 0 or  $\infty$  according as  $p$  is positive or negative, but by (6) to the value

$$\sigma_M^2 = \frac{\Gamma(\alpha + \frac{1}{2}) \Gamma(\frac{n}{2} + 1)}{\Gamma(\frac{1}{2}) \Gamma(\frac{n}{2} + 1 + \alpha)} = \frac{\Gamma(\alpha + \frac{1}{2}) 2^\alpha}{\Gamma(\frac{1}{2}) n^\alpha}$$

(approximately for  $n$  large).

It may be noted that for no value of  $p$  (except  $p=0$  and  $\varphi \equiv \frac{1}{2}$ ) in this sequence of frequency functions does the standard deviation of the median vary inversely with the square root of  $n$ .

<sup>2</sup> This would appear to be true for any symmetrical  $\varphi$  for which

$$\int_x^\infty \left( \int_x^\infty \varphi(x) dx \right)^k x^2 \varphi dx$$

diverges for every value of  $k$ .

early subscribers,<sup>1</sup> and further volumes were gradually issued; they were edited in the United States, but printed in Brussels, later in Bruges. On January 12, 1924, the History of Science Society was founded in Boston, in order to guarantee and promote the publication of *Isis*. The editor undertook to give the subscribers each year a volume of about 600 pages. In fact during the last sixteen years (1924-39) the members received not sixteen volumes, but twenty-five (Volumes VI-XXX).

This year Belgium was again invaded by the same enemy, and the publication of *Isis* was again interrupted. At the time of this second invasion several issues of *Isis* were in various stages of printing, to wit, No. 84 completing Volume 31 and the whole of Volume 32.

No. 83, though dated November, 1939, was so much delayed that it reached America only in 1940; we count it as the first number of 1940 (outside of it, our members received 648 pages in 1939). The corrected page proofs of Nos. 84 and 85 were mailed to Belgium on March 27 and May 8, respectively. The neutrality of Belgium was violated by Germany on May 10.

Members of the History of Science Society<sup>2</sup> having paid their subscription for 1940 will receive at least No. 84 without charge, as soon as it appears.

From 1941 on (Volume 33 ff.) *Isis* will be printed in the United States. The first American number will include a list of all the papers and reviews which were scheduled to appear in the last Belgian issues (No. 84 and Vol. 32). Authors of these papers and reviews are welcome to publish them in other journals, but they should warn the editors to whom they apply that publication in *Isis* is not abandoned and may occur sooner than we think.

The capacity of the American *Isis* will probably be smaller than that of the Belgian predecessor, but the editorial policy will remain essentially the same. In the face of the moral and social chaos endangering the whole world it is more necessary than ever to study as well as possible our most precious heritage, the heritage not of one nation but of the whole of mankind. It is equally necessary to interpret the philosophy and inwardness of science, of which our machines, whether they be useful or destructive, give us only a very imperfect and deceiving idea. The main function of the scientist is to approach nearer to the truth and to publish the results of his investigations as faithfully as he can. The purpose of *Isis* is to explain our past efforts in that direction and thus help us to continue them in the same spirit of devotion to truth and humanity.

Let me add a few words about *Osiris*, which was

founded in 1936 in order to relieve *Isis* of the longer papers, and thus to increase its capacity, variety and attractiveness. *Osiris* serves the same purpose as *Isis* but is restricted to longer papers; it is not supported by the History of Science Society, and hence subscriptions to it or correspondence relative to it should not be addressed to the secretary of the society, but to Dr. Alexander Pogo, Harvard Library, 189, Cambridge, Massachusetts. At the time of the German invasion two volumes of *Osiris* were being printed, to wit, Volume 8, dedicated to Paul Ver Eecke, historian of Greek mathematics, and Volume 9, dedicated to Max Meyerhof, historian of Arabian medicine. These two volumes will appear in due time, and we trust that they will be followed by many others.

GEORGE SARTON

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### ZOOLOGICAL SOCIETY OF LONDON

THE effect of the war upon scientific societies in Great Britain is being felt in various ways, chiefly in reduced membership, accompanied by higher paper and printing costs for their scientific publications. In normal times the Zoological Society of London spends about £3,000 a year on its scientific publications—comprising the *Transactions* and the two series of *Proceedings*. But the society draws by far the greater part of its revenue from the shillings and sixpences of the general public who visit the London Zoo and Whipsnade.

During the war, the attendance of visitors has fallen to about one quarter of normal, so that, in spite of the utmost economy in running costs, and in spite also of the loyalty of its fellows (the total is still over 8,000, only a few hundred less than a year ago), it is operating at a very heavy loss. Thus, since the outbreak of war, it has only been able to publish about half the normal volume of work, and this will have to be further cut down in the near future. Meanwhile, papers are still being submitted in almost peace-time quantity, so that the publication of valuable work is being seriously delayed.

The society has stocks of its scientific publications—"Proceedings," "Transactions," "Zoological Record" and "Nomenclator Zoologicus"—for sale. Those desirous of completing their files of these works will, by purchasing lacunae now, materially assist the society in the continuance of its activities in these difficult times. *Bis dat qui cito dat*: help would seem to be urgently required if the society is not to break its long record of well over a century and suspend scientific publication (as well as some of its other activities) during the continuance of the war.

JULIAN S. HUXLEY

ZOOLOGICAL SOCIETY OF LONDON

<sup>1</sup> SCIENCE, 49: 170-71, 1919.

<sup>2</sup> Secretary-treasurer, Dr. H. R. Viets, 8 The Fenway, Boston.