

York City, where he will study the early stages of cancer in animals. The appointment of Dr. Cramer has been made possible by an anonymous donor, who provided sufficient funds to bring him to the United States for a year.

NEWTON B. DRURY, of the California State Park System, has been appointed to succeed Arno B. Cammerer as director of the National Park Service. Mr. Cammerer has been transferred to Richmond, Va., where he will serve as regional director. Miner R. Tillotson will be transferred from the directorship of Region One to that of Region Three, covering the Southwestern states, with headquarters at Santa Fe, New Mexico. Colonel John R. White, now regional director at Santa Fe, will be transferred to San Francisco, Calif., to take up there the work of regional director of Region Four, in the Far West, and Frank A. Kittredge, regional director of Region Four, will become superintendent of Grand Canyon National Park, Arizona.

Nature reports that Dr. S. S. Bhatnagar, professor of chemistry at the University of Lahore, has been lent for two years to the Government of India as director of scientific and industrial research.

ACCORDING to the London *Times* the trustees of the Lady Tata Memorial Fund have decided that if circumstances permit grants will be made during the academic year beginning on October 1 to defray the expenses of research in blood diseases, with special reference to leukemia. Grants have therefore been tentatively awarded to Dr. M. P. J. Guérin, Paris; Professor K. Jármai, Budapest; Professor E. L. Opie and Dr. J. Furth, New York; Dr. A. H. T. Robb-Smith, Oxford; Dr. Werner Jacobson, Cambridge.

PHILIP E. PRATT, who recently received the doctorate in organic chemistry at the State University of Iowa, and Allison S. Burhans, a graduate of Duke University, have been appointed members of the research and development staff at Bloomfield, New Jersey, of the Bakelite Corporation, Unit of Union Carbide and Carbon Corporation.

DR. LOUIS A. KAZAL, of Rutgers University, has been appointed a member of the staff of the Biochemical Laboratory of the Medical Research Division of Sharp and Dohme, Glenolden, Pa.

DR. ENRICO FERMI, of Columbia University, recently gave a lecture on "Energy Production in Stars" at the University of Chicago, where he is visiting professor of physics.

DR. M. RUIZ CASTANEDA, director of the Department of Medical Research of the General Hospital, Mexico City, has returned to Mexico after visiting scientific institutions in the United States. He gave lectures on his work on typhus before the Rockefeller Institute for Medical Research, New York City; the Hoagland Laboratories, Brooklyn; the Institute of Medicine, Chicago, and the Mayo Foundation, Rochester, Minn.

THE council of the British Medical Association has decided, in view of the present situation, not to hold the annual representative meeting provisionally arranged for July 19 and 20.

THE Semicentennial of the Biological Laboratory at Cold Spring Harbor, now of the Long Island Biological Association, was celebrated on June 29, 1940. Addresses were made by Arthur W. Page, president of the association; Professor Harold C. Urey, of Columbia University, and Dr. Robert Cushman Murphy, of the American Museum of Natural History. After a tea at Blackford Hall a series of exhibits was shown at the John D. Jones Laboratory, including among others the electric potentials of the electric eel, the living frog heart and of a marine algal cell, *Valonia*, and the application of electrophoresis to protection against allergies. In his address Dr. Urey stressed the importance of the Cold Spring Harbor Symposia and the appreciation of men of science of the people of the community who as patrons of the laboratory have given it financial aid and active interest. Dr. Murphy emphasized the special value of the laboratory due to its location inside the metropolitan district and the part it has played in improving biological instruction.

THE Massachusetts Institute of Technology has established a department of building engineering and construction, of which Professor Walter C. Voss has been appointed chairman. Associated with him on the staff will be Professor Dean Peabody, Jr., Howard R. Staley and Albert G. Dietz. The program of the new department will be based on the work of the course in building engineering and construction, which for several years has been given in the department of civil and sanitary engineering.

DISCUSSION

THE SAMPLING ERROR OF THE MEDIAN

IF σ be the standard deviation of a universe about its mean, the standard deviation of the mean of random samples of n drawn from the universe is always $\sigma/\sqrt{n}$,

no matter how small n may be. The classic example of a universe for which the sampling error of the mean is infinite is

$$\varphi = \frac{1}{\pi} \frac{1}{1+x^2} \quad (1)$$

for which σ is infinite. It is usually pointed out that the median of this universe is $x=0$ and that the standard deviation of the median of random samples of n drawn therefrom is obtainable from the usual formula

$$\sigma_M = \frac{1}{2\varphi_M \sqrt{n}} \quad (2)$$

where φ_M is the value of φ at the median M , or, here, $\sigma_M = \pi/(2\sqrt{n})$ and is finite.

That the formula (2) for the standard deviation of the median can not be universally valid like the formula $\sigma/\sqrt{n}$ for the mean may be seen from considering the two functions

$$\varphi = \frac{3}{2} x^2, |x| \leq 1 \quad (3)$$

$$\varphi = \frac{1}{4} \frac{1}{\sqrt{|x|}}, |x| \leq 1 \quad (4)$$

in the first of which $\varphi_M = 0$ so that (2) would make σ_M infinite, although it is surely less than 1, and in the second of which $\varphi_M = \infty$ so that (2) would make $\sigma_M = 0$, although that is highly improbable.

As a matter of fact, if one refers to the proof of (2) as ordinarily given, one sees that it depends on the formula $1/(2\sqrt{n})$ for the standard deviation of the fraction $(\frac{1}{2})$ of the n values in the sample which fall to one side of the median of the universe, and that this deviation $1/(2\sqrt{n})$ is converted into a deviation of the median of the sample itself by assuming that the area $\varphi_M \sigma_M$ is $1/(2\sqrt{n})$. When the function φ is changing very rapidly at its median M it would seem that a better statement of this assumption would be

$$\int_{-\sigma_M}^{\sigma_M} \varphi dx = \frac{1}{\sqrt{n}},$$

where the origin for x had been taken at the median of $\varphi(x)$. Indeed if this be used in the two cases above it will appear that

$$\sigma_M = \frac{1}{\sqrt[3]{n}} \quad \sigma_M = \frac{1}{n} \quad (5)$$

respectively, instead of ∞ and 0, and that the standard deviation of the median need not vary inversely as the square root of the number in the sample.

However, though the result is better, it is not perfect. It would be necessary to have recourse to a true expression for σ_M . If n be odd, as $n=2k+1$, the median of the sample will be the middle element. If

$$F(x) = \int_{M=0}^x \varphi(x) dx$$

the chance that the median of the sample be at x is

$$\psi(x) dx = \frac{(2k+1)!}{(k!)^2} \left(\frac{1}{2} + F\right)^k \left(\frac{1}{2} - F\right)^k \varphi dx$$

The mean value of the median of random samples will not in general be at the median of the universe (taken as origin), though it must be so for a symmetrical universe, to which we shall here confine our attention; then

$$\sigma_M^2 = \int x^2 \psi(x) dx, \quad (6)$$

where the integration is extended over the whole range of the function φ .

If this formula be applied to (3) and (4) we find, respectively,

$$\sigma_M^2 = \frac{1}{\sqrt{\pi}} \frac{\Gamma\left(\frac{n}{2}+1\right) \Gamma\left(\frac{5}{6}\right)}{\Gamma\left(\frac{n}{2}+\frac{4}{3}\right)} \quad \text{and} \quad \sigma_M^2 = \frac{3}{(n+2)(n+4)}$$

where Γ denotes the gamma function. If Stirling's formula be applied to the Γ -functions, it may be shown that for n large the variation of σ is according to the powers of n indicated in (5) but that the coefficients are not unity but approximately 0.90 in the first case, whereas it is 1.73 in the second.¹

If (6) be applied to (1) it may be shown that for random samples of $n=3$ drawn from that universe the standard deviation of the median would be infinite, but for $n=5, 7, \dots$ would be finite. Finally, the application of (6) to

$$\varphi = \frac{4}{(e+|x|) [\log(e+|x|)]^2}$$

will show that the standard deviation of the median of samples of $n=2k+1$ will be infinite, no matter how large n may be,² although the function φ has a sharp beak ($\varphi_M = 4/e = 1.5$) at $x=0$, and appears to fall away very rapidly as x increases, being less than .007 at $x = \pm e$.

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THE PUBLICATION OF ISIS

THE publication of *Isis*, an international and polyglot quarterly devoted to the history and philosophy of science, was begun in Belgium in 1912-13. Hardly had five issues appeared (Volume 1 and the first half of Volume 2) when the publication was stopped by the German invasion. After the war Volume 2 was completed and distributed without extra charge to the

¹ The general case $\varphi = C|x|^{-p}$, with $|x| < 1$ and $p < 1$, leads to $\sigma_M^2 = 1/n^\alpha$ where $\alpha = 1/(1-p)$ by the area argument, whereas by the usual formula it leads to 0 or ∞ according as p is positive or negative, but by (6) to the value

$$\sigma_M^2 = \frac{\Gamma(\alpha + \frac{1}{2}) \Gamma(\frac{n}{2} + 1)}{\Gamma(\frac{1}{2}) \Gamma(\frac{n}{2} + 1 + \alpha)} = \frac{\Gamma(\alpha + \frac{1}{2}) 2^\alpha}{\Gamma(\frac{1}{2}) n^\alpha}$$

(approximately for n large).

It may be noted that for no value of p (except $p=0$ and $\varphi \equiv \frac{1}{2}$) in this sequence of frequency functions does the standard deviation of the median vary inversely with the square root of n .

² This would appear to be true for any symmetrical φ for which

$$\int_x^\infty \left(\int_x^\infty \varphi(x) dx \right)^k x^2 \varphi dx$$

diverges for every value of k .