Roger Bacon and the "ancients" were probably both right in so far as they reported the results of individual experiments. They were both wrong in generalizing from insufficient experimental data. I do not presume to say what Dr. Thompson was.

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SCIENTIFIC BOOKS

THE THEORY OF NUMBERS

Modern Elementary Theory of Numbers. By LEONARD EUGENE DICKSON. vii + 309 pp. Chicago: The University of Chicago Press. 1939.

THIS book provides an account of the essentials of the classical theory of numbers, and of such special topics as Diophantine equations of the second degree, the representation of integers by quadratic forms in three or more variables (with integer coefficients and variables), Waring's problem for cubes and fourth powers, and other problems of representation by quadratic, cubic or quartic polynomials, in so far as these questions can be handled by elementary (*i.e.,* non-analytical) methods.

The word "modern" in the title is justified by the inclusion of new or recent results and methods, due in the main to the author and his pupils. Thus the theorem that every positive integer can be expressed as a sum of nine non-negative integral cubes (the classical solution by Wieferich and Kempner of the "universal" Waring's problem for cubes) is treated as a particular case of representation by $\sum_{i=1}^{9} h_i x_i^3$ (where the h_i are assigned positive integers and the variables x_i are restricted to non-negative integer values), and the author gives an extensive list of cases in which this form is "universal," *i.e.,* represents all positive integers. As another extension of Waring's problem for cubes the representation of positive integers by $\sum_{i=1}^{9} \varphi(x_i)$ is discussed for various integer-valued cubic polynomials $\varphi(x)$. Wieferich's theorem that every positive integer can be expressed as a sum of 37 positive or zero fourth powers (the nearest approach by elementary methods to the 19 which is believed to be the "correct" number) is proved by a method which shows in addition that 20 of the 37 fourth powers can be taken equal in pairs.

The sections on quadratic forms include a discussion of "universal" forms (*i.e.,* positive definite or indefinite forms which represent all positive integers or all integers respectfully), and of "zero forms" (*i.e.,* indefinite forms which represent the number zero with integer values of the variables not all zero).

An appendix is devoted to Dirichlet's theorem on primes in an arithmetical progression, a result frequently required though not "elementary" in the strictest sense.

It may perhaps be felt that the scope of the book hardly justifies so comprehensive a title, for the elementary theory of numbers has made important advances in other directions in recent years. A notable example is the additive theory of the "density" of sequences of integers, created in the last decade. Within its own limits, however, the book is a storehouse of information on a variety of topics not easily accessible hitherto, and the author has made the proofs about as simple as can be expected from the nature of the problems. This is not to say that the book will be found easy reading by anyone who sets out to work conscientiously through its 309 compactly written pages of text and examples (some of which are quoted in the text), for many of the discussions necessarily involve much intricate detail and lengthy enumeration of cases. The reader interested in a particular topic can, however, read selectively; and the reader desiring to pursue any subject further will find the essential references scattered through the text.

A. E. INGHAM

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A MATHEMATICAL CLASSIC

Topological Groups. By L. PONTRJAGIN (translated by Emma Lehmer). ix + 299 pp. Princeton, 1939.

PROFESSOR PONTRJAGIN'S book on "Topological Groups" is more than distinguished; it is a mathematical classic. The subject of topological algebra, occurring as it does at the junction of two great streams of thought, has been one of the most fascinating mathematical developments of the past fifteen years. The present volume by the brilliant blind Russian scientist is likely to remain far and away the most important work on this subject for many years to come.

The main theme of the book is the "analyticity" and representability by ordinary matrices of compact and (locally compact) commutative topological groups. One's enjoyment of the beautifully clear exposition of this theme is enhanced by the realization that many of the principal ideas are due directly to the author. The discussion is also rounded off at the end by an appropriate selection of related special topics from the theory of Lie groups.

Although the book generalizes a theory which has proved of great importance in quantum mechanics, it is probably too abstract and technical for the non-mathematician. But it should be an ideal text for graduate courses in mathematics, both because of the value of the contents and as an introduction to modern mathematical ideas.

GARRETT BIRKHOFF