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INTUITION, REASON AND FAITH IN SCIENCE¹

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FROM the earliest times scientific ideas even when erudely conceived have been of immeasurable importance, not only for man's material advancement and control over nature, but also in modifying and expanding his philosophic and religious outlook. In the effort to obtain a better understanding of his place in the cosmos, he is compelled to proceed largely by considerations of analogy based upon supposed or actual fact. And so he turns more and more toward the ever-widening vistas suggested by science in its continual discoveries of new truth.

To-day the significance of science as a principal source of revelation is almost universally recognized. Thus recently, on behalf of Pope Pius XI, Cardinal Pacelli spoke before the Pontifical Academy of Sciences concerning the enlightenment that comes from

¹ Address of the president of the American Association for the Advancement of Science, Richmond, Va., December 27, 1938.

"the potent streams of the natural and rational sciences and the great river of revealed wisdom."² He said that the former are found "wherever man looks for and finds truth." As for "the great river of revealed wisdom," is it not to be found in all the absolutely sincere utterances of poets, philosophers and prophets, based on the relevant knowledge of their day and made after deepest meditation? It would seem that such utterances are in essence similar to the pronouncements of the scientist. Is not the vague, prophetic conjecture of Pythagoras that nature is mathematical as true as Newton's more precise law of gravitation? From this point of view, the great streams of revelation seem to merge insensibly into one.

Nevertheless, the immediate effect of scientific advances is often very disquieting. The strong opposition long shown to the Darwinian theory of evolution ² See SCIENCE, 86: 2238, 470-472, November 19, 1937. bears witness to this fact. Similarly at the present day the ever-increasing number of uncoordinated theories and mechanical inventions confuses and chills many of us. Man is felt to be a mere tragic detail in a vast incomprehensible whole, and our old sense of values seems to become less and less real.

To persist in such an attitude of discouragement is unjustified. Every individual has implanted within him the desire to understand his rôle in the existing order. He feels an inalienable right to find out his duties and privileges as a citizen of the universe. By the light of any new knowledge he is always certain to gain deeper insight into his position. The wise advice of our own great Emerson comes to mind: "Fear not the new generalization. Does the fact look crass and material, threatening to degrade thy theory of spirit? Resist it not: it goes to refine and raise thy theory of matter just as much."

What, then, are some of the larger points of view which are suggested by science to-day? In attempting a reply I can of course only offer a personal interpretation, inevitably reflecting the fact that I speak as a mathematician having some acquaintance with physics.

Let us observe in the first place that the universe presents antipodal aspects-the objective and the subjective, the impersonal and the personal. If we take the objective aspect as more fundamental we put our emphasis on the notion of reality; and if we start from the subjective, we prefer to speak of knowledge. In either case we are able to discern a kind of naturemind spectrum; for there appears a roughly given hierarchy of five ascending levels-mathematical, physical, biological, psychological and social. Each level has its appropriate special language. The basic corresponding concepts are respectively: number at the mathematical level; *matter* at the physical level; organism at the biological level; mind at the psychological level; and society at the social level. If we choose to select one of these as somehow more real than the others, a great distortion arises in our point of view. For instance, if we regard the physical level as the most fundamental, we become materialists. But why make such an unnecessary choice? The languages of the various levels are essentially independent of one another, and the observed laws are best expressed in their own natural terms. Why mix up the levels of knowledge unnaturally? Does it clarify our idea of social justice to try to explain it in terms of the reactions between protons and electrons in the brain?

These considerations bring us to a first general point of view towards the levels of knowledge: It is desirable to accord reality in equal measure to all kinds of knowledge everywhere, and so to view the universe as broadly and impartially as possible.

Another very important observation is that in order

to understand the various facts and their interrelations we must always use abstractions, that is, conceptual tools of a logical or mathematical nature. Contrary to opinions which prevailed until recently, any abstraction serves only limited specific ends. At best it will enable us to grasp more clearly some small fragment of reality. For example, by use of the abstraction of Euclidian geometry, and in that way alone, we understand the nature of space with a considerable degree of exactitude; and yet to-day scarcely any physicist would ascribe objective reality to space in itself. It has been Einstein more than any one else who has taught the scientific world the true rôle of Euclidian geometry by means of his theories of space-time and relativity. More generally, we have come to realize that our only approach to a better understanding of the world is by means of a widening succession of abstract ideas, each explaining imperfectly some aspect of the stupendous whole. This is a second synthesis deserving of especial emphasis.

Thirdly, I would state a fundamental truth about the social level, which in some sense is the highest level of all: The transcendent importance of love and good-will in all human relationships is shown by their mighty beneficent effect upon the individual and upon society.

Thus I have begun by presenting very briefly three important articles of my personal faith. These are not verifiable experimentally or strictly demonstrable, so that any one is free to agree or to disagree. Against my belief that the levels of knowledge are to be taken as equally real, one may set for instance an opposing belief that every fact is ultimately expressible in purely physical terms. If my position is natural for the mathematician with his abstract point of view, the other may be preferred by the toughminded physicist, the biologist with mechanistic inclinations and the psychologist with a behavioristic outlook. The future will probably show that both of these beliefs are partly true and partly false.

Similarly, against my conviction that any particular abstraction is merely a useful tool enabling us to understand certain facts, some will contend that one particular abstraction will prove to be final and absolute. Here my attitude springs from an extensive acquaintance with mathematical abstractions and their numerous applications, whereas the theoretical physicist, for example, tends to believe that the ultimate theory of atomic structure is soon to be obtained.

Likewise some will declare that, much more than love and good-will, it is devoted loyalty to the state which is important; and I can imagine that under certain conditions such an assertion might be justified.

It is my especial purpose to show how this phenomenon of faith arises inevitably in the mind of the scientist whenever he tries to evaluate technical conelusions in his special field. In doing so I shall discuss the role of intuition, reason and faith in science, first at the mathematical and physical levels, and then more briefly at the biological, psychological and social levels. This will lead me in conclusion to formulate two other items of my personal creed in the hope that they may be worthy of your attention.

By way of definition it must be indicated first what is meant by intuition. There are certain elementary notions and concepts which come spontaneously to the minds of all who observe, experiment with and reflect on a specified range of phenomena. Such generally accepted ideas or intuitions constitute the consensus of reaction of intelligent men to a definite part of the world of fact. John Stuart Mill has said. "The truths known by intuition are the original premises from which all others are inferred." It is in this sense that I shall refer to intuition. By reason I shall mean the rational superstructures which may be erected upon the basic intuitive ideas by means of deductive or inductive reasoning. These superstructures will also be accepted by all who are able to follow the sequence of logical steps involved. By faith I shall mean those heuristically valuable, more general points of view, which are beyond reason, and sometimes in apparent contradiction with one another, but which to the individual concerned seem of supreme importance as he endeavors to give his conclusions the widest possible scope.

It is clear that in this way we obtain a basic classification of knowledge into three easily distinguishable types. Let us consider the occurrence of these types at the various levels of knowledge.

By continual crude experimentation with classes of concrete objects, man has come gradually and inevitably into the possession of certain numerical ideas. In particular he has been led to think of the positive integral numbers 1, 2, $3 \ldots$ as entities which exist in almost the same sense as the objects themselves. This concept finds its realization in the designation of the integers by corresponding marks 1, 2, $3 \ldots$ Such integers are found to be subject to certain simple arithmetic laws, and these laws are regarded as intuitively true.

The integers form the basis of a great part of mathematics. For it is found that with their aid one may construct fractions and, more generally, real and imaginary numbers. In the course of the centuries mathematicians have thus built by processes of pure reason the elaborate structures of algebra, the theory of numbers and analysis. An extensive array of beautiful and useful theorems has been deduced.

Similarly in geometry—which in its origin may be regarded as the most elementary branch of physicswe experiment with rigid material objects and arrive readily at the notions of idealized small rigid bodies or "points" and of idealized "lines" and "planes." Then we observe that certain postulates hold, such as the familiar ones of Euclid. By means of these postulates, which embody our intuitions, we are able by deductive reasoning to arrive at other geometrical theorems, including such results as the celebrated Pythagorean theorem which shows us in particular that a right triangle with legs of 3 units and 4 units in length has a hypotenuse of exactly 5 units in length. The vast mathematical domain called "geometry" has arisen from these elementary geometrical facts as a primary source.

There are many other abstract mathematical structures besides those just alluded to. In all cases it is found that they are made up of certain accepted intuitions (or postulates) and their logical consequences.

Now what I desire particularly to point out is that the mathematician goes far beyond such generally accepted clean-cut assumptions and conclusions, in that he holds certain tacit beliefs and attitudes which scarcely ever find their way into the printed page. Yet these form none the less part of a considerable oral tradition. For instance, he believes in the existence of various infinite classes such as that made up of all the integers. He believes also that the whole body of strict logical thought called mathematics is self-consistent: in particular when he finds that the number π admits of diverse forms of expression, as, for example,

and

$$\pi = 2\sqrt{3} \left(\frac{1}{1} \frac{1}{1} - \frac{1}{3} \frac{1}{3} + \frac{1}{5} \frac{1}{9} - \ldots\right)$$

 $\pi = 4 \left[\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots \right]$

he feels absolutely certain that if the unending calculations could be fully carried out, the results would be exactly the same in all cases. Furthermore, when he recalls that in the past the most difficult mathematical questions have been ultimately answered, he is inclined to believe with the great German mathematician, Hilbert, that every mathematical fact is provable. Besides all this, he attributes certain values to his results and their mathematical demonstrations: some theories seem important; some proofs are regarded as elegant, others as profound or original, etc.

Such somewhat vague ideas illustrate what I would call mathematical faith. Nearly all the greatest mathematicians have been led to take points of view falling in this broad category, and have attached the deepest significance to them.

What I wish to emphasize concerning this generally overlooked aspect of mathematical thought is that, on the one hand, the beliefs involved have been of the utmost heuristic importance as instruments of discovery, and, on the other hand, when examined in detail they generally turn out to involve ideas which are held true or false, according to the specific definitions which may be subsequently adopted.

Suppose, for instance, that we turn to the first question of the existence of infinite classes. There was no hesitation about the unconditional acceptance of such classes until within a few decades, although a few, like the ancient Greek philosopher Zeno and the German algebraist Kronecker, profoundly distrusted the use of the infinite in mathematical reasoning. To-day, however, due primarily to the theory of transfinite aggregates created by Georg Cantor about fifty years ago, mathematicians have come to realize that such an infinite class may exist in the so-called "idealistic" sense but not in the sense of explicit constructibility. Thus the class of all collections of positive numbers less than 1 exists in the idealistic sense, but not in the alternative, more concrete sense.

A similar situation has arisen in the detailed study of the self-consistency of mathematics. It has appeared that very limited parts of mathematics can be proved self-consistent. But such a general assertion as that "the whole of mathematics is self-consistent" would be considered to-day not to be sufficiently precise; and each time that the proof of self-consistency is extended further, a definite logical price has to be paid in that certain so-called metamathematical ideas are tacitly employed, which need themselves to be investigated in the same respect. For instance, work prior to the "Principia Mathematica" by Whitehead and Russell (1910) showed that if the notion of class was not restricted, certain logical paradoxes would inevitably result. For this reason a theory of the "hierarchy of types" was devised by them, which limited the notion of class and so avoided the apparent inconsistencies. We are thus entitled either to say that mathematics as of the year 1900 was self-consistent or was not, according to the point of view which is adopted. In any case the belief in question has led us to a much deeper insight into the nature of logic.

With regard to the unlimited power of mathematical demonstration, it has been recently proved by the Austrian mathematician Gödel that, if we restrict ourselves to reasoning of an ordinary type, there exist explicit "undecidable" theorems, while from a higher metamathematical point of view such a theorem might be demonstrable. Hence Hilbert's affirmation is in one sense false. But despite this fact the open question on which he focused attention is much better understood than ever before.

Likewise in the question of value in mathematics, such as the importance of theories, or the elegance, profundity and originality of proofs, it is clear that these obscure ideas depend in large measure upon the momentary state of the science. Thus the theory of functions of an imaginary variable and classical geometry were regarded as extremely important a quarter of a century ago; while to-day the theory of functions of real variables and the basic kind of geometry called analysis situs have respectively displaced these subjects in general mathematical esteem. It would be hard to explain adequately the reasons for this change, but the increasing role of discontinuous quantity in physical theory and the relativistic point of view towards space and time have certainly been contributing factors.

An excellent instance of the power of individual mathematical faith in bringing about creative advance has been afforded by an American mathematician, the late Eliakim Hastings Moore, past president of this Association. Moore was a thorough-going abstractionist who believed that mathematics itself should be reorganized from a still higher point of view, by the dissection of essential common parts out of apparently different abstract fields. His point of view was strongly confirmed by the analytic work of Hilbert and Erhardt Schmidt near the beginning of this century. And so Moore was led to create his "General Analysis" in 1906. This aimed to embody his conviction that "The existence of analogies between the central features of various theories implies the existence of a general theory which underlies the particular theories, and unifies them with respect to these central features."

As time has elapsed, the deep truth of Moore's contention has been amply sustained. Indeed one of the most active schools of contemporaneous mathematical thought follows the higher abstract point of view adopted by Moore. But it has been found necessary to modify Moore's program, in that, instead of a single "General Analysis" serving as an *omnium gatherum*, it has been desirable to employ a few typical forms. In this way his faith in the power of higher abstraction has been largely and yet not fully justified.

A good many mathematicians are seriously hampered by lack of the ardent positive faith which Moore showed. This type of deficiency is generally due to a strong development of purely critical powers and to over-specialization. Several times I have observed this lack in myself, only to be counteracted by definite effort. For example, I did not make active use of the fundamental integral of Lebesgue for a long time, and so was prevented from pursuing to their natural conclusion certain ideas which finally led me to establish the basic "ergodic theorem" in 1931. Here I was finally converted, as it were, to the use of this tool by the important advances of Koopman and von Neumann, and in particular by the latter's proof of the "mean ergodic theorem." It is worthy of note that the related ergodic hypothesis goes back in its origins to the physicists Boltzmann and Maxwell.

Let us turn next to the physical level where the corresponding situation is at least equally interesting.

If we accept the ordinary conceptions of space and time, which seem destined always to play a basic role in workaday physics, we find that the simplest physical ideas are those which arise through the manipulation of massive bodies. As these ideas have become clarified, they have been given abstract formulation in terms of such concepts as those of mass, force, etc. Newton's celebrated three fundamental laws of motion embody the final form of the refined intuitions thus arrived at. With these as a basis and the acceptance of certain further special observed laws, one may deduce by mathematical reasoning the theory of mechanics as applied, for example, in the solar system.

Similarly, through experimentation with electrified bodies, electric currents, magnets, etc., there was developed by Faraday the intuitive ideas of electric and magnetic lines of force which are now generally accepted. Later Maxwell incorporated these ideas in the appropriate electro-magnetic equations. Upon this basis all classical electro-magnetic theory has been logically constructed: Furthermore, by means of the identification of the light wave and the electro-magnetic wave, due to Maxwell, an adequate theory of light has been obtained.

Thus we see the important role which intuition and reason have played in two fundamental branches of physics-mechanics and electro-magnetism. A cursory survey of the various other branches of the subject would show that a similar situation holds throughout, except in the rapid developments of quantum mechanics during the last decade or so. In this strange theory the physicist begins indeed with a planetary model of the atom, reminiscent of Niels Bohr's earlier theory. But a flying leap is made from this temporary scaffolding to what is thenceforth regarded as the only basic reality—the wave equations of Schrödinger and, better still, of Dirac. Once having arrived at these mathematical equations the physical theorist proceeds to show how he can predict innumerable facts previously out of his range by use of this arbitrary ad hoc machinery. The process involved somehow reminds me of a record sea voyage made through a fog! I can not but anticipate that a more intuitive and natural approach to essentially the same results will be found later on. An analogous earlier instance in physics is perhaps to be found in the unmotivated theory of cycles and epicycles entertained by the ancient astronomers. This explained the motions of the heavenly bodies with considerable success, but was

destined to be completely displaced by the intuitively reasonable, gravitational theory of Newton.

The fact remains, however, that the recent development of quantum mechanics forms one of the most astounding and important chapters of all theoretical physics.

It is interesting to recall how this great advance came about through the faith of the German physicist Planck at the outset of the present century. His direct experience with the phenomena of radiation had led him to believe that there were discontinuous processes at work, not to be explained by any modification of the time-worn classical theories, and so he was led to formulate his celebrated quantum hypothesis in 1900. It was this daring concept of Planck, more than anything else, that has freed the minds of physicists from the shackles of too conventional thinking about atomic phenomena, and so has made possible the quantum-mechanical quest of which the end is not yet in sight.

There has always been an abundance of faith among the physicists. Every one knows how Newton and others have found confirmation even for their religious beliefs in the lawful character of physical phenomena. It is not hard to understand why the tendency towards dogmatic affirmation among the physicists has been stronger than among the mathematicians. For the physicist with considerable justice feels that he is exploring the mysteries of the only actual and very exciting universe; whereas the mathematician often appears to live in a purely mental world of his own artificial construction. A good illustration of this tendency of the physicists is afforded by their changing attitudes towards the wave theory versus the corpuscular theory of light. Over a considerable period the corpuscular theory of Newton held sway; then this was displaced by the wave theory of Huyghens, the Dutch physicist; and nowadays a kind of vague, uncertain union of the two is generally accepted.

In this connection it is especially interesting to recall the scientific beliefs to which Faraday was led in his fundamental work on electricity and magnetism. From his experimental results in this field, he saw that there was obeyed here as elsewhere the law which he called the "conservation of force" and which we today would call the "conservation of energy." He saw that this energy was localized in space, and he could only conceive of it as being propagated in time; and so he was led to the belief that electro-magnetic energy is also propagated with finite velocity. Thus in an article, "On the Conservation of Force," published in 1857, he expressed himself as follows: "The progress of the strict science of modern times has tended more and more to produce the conviction that 'force [energy] can neither be created or destroyed' . . .;" "time is growing up daily into importance as an element in the exercise of force; to inquire, therefore, whether power acting either at sensible or insensible distances, always acts in *time* is not to be metaphysical." By way of justification of the rather mathematical direction in these thoughts, Faraday said further, "I do not perceive that a mathematical mind, simply as such, has any advantage over an equally acute mind not mathematical . . .;" "it could not of itself discover dynamical electricity nor electromagnetism nor even magneto-electricity, or even suggest them." But the achievements of the more mathematical Maxwell were later to show that Faraday had underestimated the power of pure reason.

It is thus clear that through an act of faith Faraday attained to a kind of deeper insight; for the existence of the electro-magnetic wave has long since been established experimentally. However, the beliefs of Faraday in this connection can not be regarded as absolutely true, since according to present-day conceptions the notion of energy which he accepted is only roughly valid as a statistical approximation. Nevertheless, Faraday certainly penetrated more into the nature of electrical and magnetic phenomena than any of his contemporaries; and it is difficult to see how, with the limited mathematical and physical knowledge at his disposal, he could have gone any further in the way of prophetic conjecture.

The intimate relation between philosophical-scientific points of view and actual advances in theoretical physics has been admirably illustrated by Einstein's gravitational theory of 1915. Taking as his starting point the bold but reasonable hypotheses that matter must condition space and time, and that, in parts of space remote from matter, elementary particles move with uniform velocity in a straight line, he arrived at his field equations as the most elegant mathematical embodiment of these ideas. Thus there was obtained a quasi-geometrical theory of gravitation which in certain respects is more natural than the celebrated theory of Newton, while the predicted differences, although excessively minute, are in favor of the new theory. But Einstein's theory can not be regarded as true in any absolute sense, since it gives us at best a partial, highly idealized view of the physical universe.

It is hardly too much to say that, since the beginning of the present century, the main advances in theoretical physics have been the outcome of a similar kind of mathematical guesswork, in which, however, the mathematician himself has taken little or no part! The guessing of the physical theorist is guided almost entirely by considerations of subtle mathematical analogy.

This peculiar situation has led naturally enough to the feeling that pure mathematics almost suffices with-

out much recourse to the results obtained in the physical laboratory. Sir Arthur Eddington has embodied the extreme point of view in his recent book, "The Relativity Theory of Protons and Electrons," thus taking a position antipodal to that of Faraday. Eddington says: "Unless the structure of the nucleus has a surprise in store for us, the conclusion seems plain-there is nothing in the whole system of laws of physics that can not be deduced unambiguously from epistemological considerations. An intelligence, unacquainted with our universe but acquainted with the system of thought by which the human mind interprets to itself the content of its sensory experience, should be able to attain all the knowledge of physics that we have attained by experiment.... For example, he would infer the existence and properties of radium, but not the dimensions of the earth."

I would comment upon this mystical conjecture of Eddington as follows: It is no doubt partially true that in some respects we need the laboratory less than we did before, due to the fact that we live surrounded by all manner of scientific instruments and machines, with whose properties we have become acquainted. In other words, we live in a transformed world which is a kind of huge laboratory. Yet I doubt whether any individual, however intelligent, who was not acquainted with such instruments and machines, would be able, through analysis of ordinary sensory experience, to go very far. On the other hand, I would agree with Eddington that the starting point from which known physical laws may be deduced is likely to depend on only a few intuitive ideas; and perhaps a sufficiently powerful mathematical intelligence would realize that the facts of sensory experience could only be simply explained in this way.

An equally remarkable conjecture was expressed by Dr. Charles Darwin in a vice-presidential address, "Logic and Probability in Physics," before the British Association last summer. In this address he said, "The new physics has definitely shown that nature has no sharp edges, and if there is a slight fuzziness inherent in absolutely all the facts of the world, then we must be wrong if we attempt to draw a picture in hard outline. In the old days it looked as if the world had hard outlines, and the old logic was the appropriate machinery for its discussion." He therefore suggested "that some day a real synthesis of logic will be made" leading to "a new reformed principle of reasoning."

Here I can agree with Darwin to the extent of admitting that there always exists a metamathematical fringe in logic. But it seems obvious that in logic there has been a record of continual advance by critical and profound diversification rather than by any essential alteration of point of view.

In my own limited experience in mathematical

physics I have also seen how natural it is to take a positive attitude on open questions. Thus a good many years ago I showed mathematically that mere spatial symmetry about a center necessitates a *static* gravitational field. This led me to believe that the Einstein field equations were probably too inelastic to fit the facts, but I did not put forth this opinion. Shortly afterwards Lemâitre, in trying to explain the expanding (non-static) stellar universe found it necessary to modify the field equations, in part because of my result; and so my belief was to this extent justified.

Again, I have had during the last few years a feeling that a conceptual space-time model for quantum mechanics is likely to be found, although theoretical physicists would in general disagree. Nevertheless, my faith is so strong that my recent researches lie principally in this direction. I have already found interesting results, and am confident that these efforts will not be wasted, since the possibilities of the conceptual approach need to be more carefully explored.

In ending these remarks about the role of intuition, reason, and particularly of faith, at the physical level, it is to be observed that the physicist as such systematically ignores the phenomena of life, for it is dead and not living matter with which he concerns himself in his laboratory.

All in all, it is a faith in the uniformity of nature which remains the guiding star of the physicist just as for the mathematician it is a faith in the self-consistency of all mathematical abstractions, although these faiths are more sophisticated than ever before. The minds of both are tinged with an unwavering belief in the supreme importance of their own fields. The mathematician affirms with Descartes, omnia apud me mathematics fiunt—with me everything turns into mathematics; by this he means that all permanent forms of thought are mathematical. The physicist on his part is apt to think that there is no reality essentially other than physical reality, so that life itself is finally to be fully described in physical terms.

Although I have no especial acquaintance with the biological, psychological or social domains, it seems clear to me that a similar situation prevails in them. In the biological field the intuitions upon which one depends are those associated with the concept of the organism and its evolution. These intuitions can not be formulated conclusively and completely in simple postulates, as is possible at the mathematical and physical levels. It is rather through an acquaintance with an immense array of interrelated, analogous facts that the biologist finds himself able to deal with novel situations. By means of the geological record on the one hand and the results obtained in the field and laboratory on the other, he acquires a better and better understanding. His principal weapon is always inductive reasoning. It seems certain that a deductive treatment of biology is at least very remote and if ever accomplished will be utterly different from anything which we can imagine to-day. There are, however, a few special fields like the theory of heredity, in which a considerable mathematical structure has been developed. In this theory, by means of the "chromosomes" and their corresponding abstract "genes," it has been possible to explain a complicated array of facts.

The faith of the biologist generally tends in the direction of a mechanistic theory of life or of some opposing vitalistic theory. In fact, he is forced to employ the principle of physical causation in his efforts to understand biological phenomena and does not yet know of definite limitations in its use. Recently there has been some indication of a return to vitalism, so that once more a considerable group of biologists are convinced that not all the phenomena of living matter are to be accounted for by ordinary physical and chemical law. The controversy involved has long been a burning one, and accordingly one naturally suspects that the question is really meaningless. In any case, however, special mechanistic hypotheses have so far pointed the way to new creative advances.

It is interesting to remark that the insufficiency of a rigorously deterministic theory of the living organism admits almost of mathematical demonstration in the following manner. A genuinely mechanistic universe would have to be free of any infinite factors. For example, if one accepts a simple Newtonian theory, there might be reaching the earth from infinite space unknown quantities of matter and energy, so as to change arbitrarily the course of events upon the But in any completely mechanistic system, earth. free of such infinite factors, it is not difficult to prove that there will necessarily be a kind of eternal Nietzchean recurrence. For instance, we are here together this evening considering a particular topic. The strict adherence to the deterministic point of view would entail the consequence that in the eons yet to come this same scene will be re-enacted infinitely often. I submit that this is dramatically improbable!

Recent advances in the chemical knowledge of large organic molecules seem to indicate an innate hospitality of actual matter toward the evolution of the living organism. In this way a plausible genetic account of the origin of life is suggested, which, however, can scarcely be called mechanistic. It begins to seem possible that we are on the verge of further refinements in our concept of matter, such as Emerson anticipated in the quotation made above.

The situation at the psychological level is even less

amenable to precise treatment. All of us have a lifelong experience with ourselves and other human beings. This automatically gives rise to a vast complex of intuitive psychological notions. We all are aware of course that there are concomitant physiological processes going on in the body, nervous system and brain. Now it is the business of the professional psychologist to give exact definition and interpretation to these crude ideas; and he finds his greatest illumination in the facts of abnormal psychology, with which most of us are unacquainted. However, in the case of either layman or professional the processes of reasoning are mainly by analogy. Even the psychiatrist, familiar with many concrete cases, must treat each new patient by the inductive method. There are too many psychological intuitions and too few exact laws for any imposing edifice of pure reason to be erected.

In certain restricted psychological domains, formalization is to some extent possible. Thus I have ventured to formulate a theory of "esthetic measure," by explicit numeration and weighting of esthetic fac-This aims to explain certain simple esthetic tors. facts in our enjoyment of visual and auditory forms. The theory has been to some extent substantiated by experiments made at Harvard and elsewhere. But in any case, no matter how successful the theory might prove, it would be wholly absurd to try to set up an elaborate logical structure on the basis of the fairly arbitrary and inexact assumptions involved. Generally speaking, as we proceed from the more objective to the more subjective levels of thought, we find that elaborate logical structures seem to be of less and less utility.

The basic belief of the professional psychologist is in the completeness of the physiological accompaniment of every psychical fact; and he formalizes the observed facts by means of the parallelism. But there is a conflict between this attitude of the technician towards mind, for whom the individual is a complex of neurally characterized components, and that of the ordinary man—equally an expert though of a different kind—who sees all sorts of permanent values in personality, not adequately characterized in neural terms. The second attitude leads nearly all of us to have deep affections and abiding personal loyalties, whether or not we are psychologists!

Here again I think that these apparently opposing points of view are both more or less true; and I incline all the more to this opinion because of my conviction that as yet we know relatively little about the phenomena of personality. For it seems certain to me that the extent of hidden organization in our universe is infinite, outside as well as inside of space and time; such a conviction is very natural to a mathematician, since the three ordinary spatial dimensions and the single temporal dimension are for him only particular instances of infinitely many other conceivable dimensions! If this be true, any broad conclusions concerning the nature of personality would seem altogether premature.

At the social level the most serviceable intuitive ideas eluster around the concept of societal evolution. It is of course the comparative study of human institutions which furnishes the principal interest. The analogy between forms of society and evolving organisms is a deep-lying one. Here again the useful logical structure which can be built around the very complicated facts is exceedingly simple. Even in such a formalized field as ethics, dealing with the behavior of the individual as a member of society, logic plays an almost negligible role.

Belief here seems to gather principally around the idea of societal progress. Progress—or its non-existence—serves as our fundamental tenet. Some believe that society can improve indefinitely, tending toward a perfect society. Such a belief is of course a fundamental one in most religious systems. Others find this idea too naive. They stress the gregarious instinct in man and tend to think of societal changes as taking place in various directions strongly conditioned by changing physical environment. All would admit, however, that without the concept of dynamical social processes, social theorizing would be stale and unprofitable.

Let us turn now to consider some further conclusions, towards which this brief survey of intuition, reason and faith at the various levels seems to point.

As far as intuition and reason are concerned, these are the common property of all competent individuals. The narrow, closely articulated chains of deductive reasoning serviceable at the earlier levels are more and more replaced by loose webs of inductive reasoning at the later levels, as we pass from the objective to the subjective. At the same time the basic intuitions change from the simple and precise types employed in mathematics and physics to the increasingly complicated and diverse forms characteristic of biological, psychological and social phenomena.

However, it is just as necessary to clarify and to formalize our knowledge at these later levels as at the earlier ones. The processes of systematic reasoning, whether inductive or deductive, have always a definite prophylactic value, and in particular enable us to avoid the dangers of prejudiced and intolerant points of view. It may be observed in passing that the careful application of impartial thoroughgoing analysis is as important for everyday living as it is in the study and the laboratory.

The striving for rational comprehension is one of

the noblest attributes of man. In his agelong difficult struggle he has been able to secure greater freedom only through a better technical mastery of his environment. No other method of liberation has been vouchsafed to him. But this increased mastery has brought with it automatically new intellectual responsibilities and a more complex way of life. In consequence, unforeseen and threatening dangers arise from time to time; and there is thus imposed on him the necessity to advance still further, which is to-day more urgent than ever before.

• A new injunction has been laid upon the spirit of man, to know and to understand ever more broadly and deeply.³

Now along with the increase in scientific knowledge there appear certain crudely expressed, deeper insights, not completely true or false, some in opposition to others, but all supremely valuable nevertheless. These are embodied in beliefs which seem the inevitable accompaniment of all creative thought.

Thus in the daring effort of the scientist to extend knowledge as far as possible, there arises an aura of faith. It is this spontaneous faith which furnishes the most powerful incentive and is the best guide to further progress.

Such are some of the very general points of view to which a considerable mathematical and scientific experience has led me. If they are worthy of serious attention it is not because of their novelty, but rather because in their aggregate they rise above the details of the numerous specialized fields of knowledge and sustain the scientist in his unceasing and ardent search after truth.

Doubtless many of you are ready to ask the ever more insistent question: If science has thus profoundly modified the general outlook and way of life of mankind, is it not the especial duty of such an association as ours to point out constructive remedies for the ensuing maladjustments? In the "Part II: Science and Warfare" of his admirable address as president of the British Association last August Lord Rayleigh closed by expressing the hope that our two associations could cooperate in such a way as to "bear useful if modest fruit in promoting international amity." In this hope all of us will deeply concur. The presence of Sir Richard Gregory with us at the Richmond meeting is the first token of the projected closer relation between the parent British Association and ourselves. It is much to be desired that this action will encourage further unification of the whole scientific world. I am sure that practically all our joint membership would agree with me that it is the wider diffusion of "the steady light of scientific truth" which holds out most hope of a better understanding among men.

SCIENTIFIC EVENTS

THE PENNSYLVANIA CHEMICAL SOCIETY

A GROUP of Pennsylvania chemists received on December 14 a charter as "The Pennsylvania Chemical Society." Included among the incorporators are Dr. Edward R. Weidlein, director of the Mellon Institute at Pittsburgh, and Dr. Frank C. Whitmore, dean of the School of Chemistry and Physics at the Pennsylvania State College.

The society is incorporated "for the purpose of encouraging in the broadest and most liberal manner the advancement of chemistry as a science and as a profession in the Commonwealth of Pennsylvania, especially in fostering public welfare and education in matters involving chemistry, and aiding the development of industry and promoting the health, happiness and prosperity of the people of the Commonwealth. The society will carry forward the important role which chemistry has played in Pennsylvania from earliest Colonial times. Even prior to the establishment of this nation when the colonies and the early states were mainly dependent upon other foreign countries for many advances in science and also for most of their chemical necessities, there was formed in Penn-

³ From my circular Association letter of 1936.

sylvania what appears to be probably the first organization on the American Continent for the production of chemical products upon an industrial basis. Pennsylvania has mothered American chemical industry and been the seat of much distinguished work in the profession. Pennsylvania has led in the formation of institutions of learning from which there have gone forth innumerable chemists to teach others throughout the land how best to make use of the science and how to serve the commonwealth, the nation and themselves in an adequate capacity."

The officers of the society are as follows:

President—Dr. Jos. W. E. Harrisson, consulting chemist, member of the firm of LaWall and Harrisson of Philadelphia, assistant professor at the Philadelphia College of Pharmacy and Science.

Vice-president—Dr. Nelson W. Taylor, of the School of Mineral Industries, Pennsylvania State College.

Secretary and Treasurer—Dr. Elliott P. Barrett, member of the staff of the Mellon Institute for Industrial Research, Pittsburgh.

The society will shortly hold a meeting for formal acceptance of the articles of incorporation and will actively proceed with its corporate purposes.